14MAU01

PSG COLLEGE OF ARTS & SCIENCE

(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2018

(First Semester)

Branch - **MATHEMATICS**

CALCULUS

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks!

Answer **ALL** questions

ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 If x^3+y^3-3axy, find $\frac{dy}{dx}$

2 Define Centre of Curvature.

3 What is the radius of Curvature of a (i) Straight line (ii) Circle.

4 Define evolute of a curve.

5 If f(x) is an odd function of x then prove that $\int_{-a}^{a} f(x)dx = 0$.

6 Evaluate J(logx) dx.

7 Find the value of $\int_{0}^{a}\int_{0}^{b}(x^2 + y^2)dxdy$.

8 Evaluate $\int_{0}^{1}\int_{1}^{2}\int_{0}^{3} xy^2 z\,dz\,dy\,dx$.

9 Prove that (3(m,n)= (3(n,m).

10 Prove that T(1 / 2) = *yfn.*

SECTION - B (25 Marks)

Answer **ALL** Questions

ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Find the maximum and minimum values of the function $f(x,y)=xy-x^2-y^2-2x-2y+4$.

OR

b If $u=x^3y^2$ where $x^2-xy+y^2=a^2$. Find du/dx.

12 a Prove that the radius of Curvature of any point of the Cycloid is $x=a(9+\sin 9)$ and $y=a(1-\cos 9)$ is $4a\cos\frac{A}{2}$.

OR

b Find the radius of the curvature of the curve $4ay^2 = (2a - x^3)$ at (a,-|).

13 a Prove that $\int_0^{9} \sin^3 9\,d9 =$

OR

b Find the reduction formula for $\int \tan^{11} x\,dx$.

14 a Evaluate $\iint (x^2 + y^2)dxdy$, over the region for which x,y are each>0 and x+y<l.

OR

b Evaluate jjjxyz dx dy dz, taken through the positive octant of the sphere $x^2+y^2+z^2=a^2$.

Cont...

15 a Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$.

OR

15 b Evaluate $\int x^4 e^{-x}\, dx$.

SECTION - C (30 Marks)

Answer any **THREE** Questions
ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Using Lagrange's method of multipliers, to find the minimum value of $u=a^3x^2+b^3y^2+c^3z^2$ where $l/x+l/y+l/z=l$.

17 Find the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$.

18 Derive the reduction formula for the $\int x^m (\log x)^n dx$ integral and hence evaluate $\int (\log x)^3 x^4 dx$.

19 Change the order of integration in the following integral and evaluate it:

$$\int_0^a \int_{x^2}^{2a-x} xy\, dy\, dx.$$

20 Derive the relation between Beta and Gamma function.

Z-Z-Z END