

PSG COLLEGE OF ARTS & SCIENCE

(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2018

(Second Semester)

Branch - MATHEMATICS /

ANALYTICAL GEOMETRY OF 3D & VECTOR CALCULUS

Time : Three Hours

Maximum : 75 Marks

SECTION-A (20 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 x 2 = 20)

- 1 Write down the equation of a Pointer Sphere,
- 2 Find the equation of the sphere with centre (-1,2,-3) and radius 3 units.
- 3 Define a cone.
- 4 What is the condition for the plane $lx+my+nz=0$ to touch the quadric cone?
- 5 Define a right circular cylinder.
- 6 Write down the equation of tangent plane at (x_i, y_i, z_i) .
- 7 Show that $r \times \frac{dr}{dt} = na \times b$ whenever $r = a \cos nt + b \sin nt$ where a, b and n are constants.
- 8 When $w \times r$ becomes solenoidal?
- 9 State Gauss's divergence theorem.
- 10 Find the value of $\text{div}(r^n \cdot r)$

SECTION - B 125 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 5 = 25)

- 11 a Find the equation to the sphere through the four points (2,3,1), (5,-1,2), (4,3,-1) and (2.5,3).
OR
b Show that the intersection of the sphere and the plane $x^2+y^2+z^2-2x-4y-6z-2=0$ and $x+2y+2z-20=0$ respectively is a circle of radius with its centre at the point (2,4,5).
- 12 a Derive the equation for intersecting straight line and quadric cone.
OR
b Show that the equation of a right circular cone whose vertex is 0, axis oz and semi-vertical angle a is $x^2+y^2=z^2 \tan^2 a$.
- 13 a Express the surface hyperboloid of two sheets and provide the condition for that surface.
OR
b Find the equations of the tangent planes to $x^2+y^2+4z^2=1$ which intersect in the line whose equations are $12x-3y-5=0$ and $z-1$.
- 14 a Find the angle between the surfaces $z=x+y-3$ and $x+y+z=9$ at (2,-1,2).
OR
b Find the unit tangent vector for the curve $r = a \cos t i + a \sin t j + ct k$.
- 15 a Evaluate $\int A dr$ where $A = (5xy - 6x^2)i + (2y - 4x)j$ and c is the curve $y = x^3$ in the xy plane from the point (1,1) to (2,8).
OR
b Find the area of the surface of the hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$.

SECTION - C (30 Marks)Answer any. **THREE** Questions**ALL** Questions Carry **EQUAL** Marks (3 x 10 = 30)

- 16 Show that the locus of the centre of the sphere OABC is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 2$ if a plane passes through a fixed point (a,b,c) and cuts the axes in A,B and C.
- 17 Find the equations of the tangent planes to the cone $9x^2 - 4y^2 + 16z^2 = 0$ which contain the line $\frac{x}{32} = \frac{y}{72} = \frac{z}{72}$
- 18 Find the equation of a right circular cylinder of radius 3 with axis $\frac{x+2}{3} = \frac{y-4}{6} = \frac{z-1}{2}$
- 19 Prove that (i) $\text{curl}(\text{grad}(\phi))=0$ (ii) $\text{div} \text{curl} F=0$
- 20 Verify Green's theorem for $\int_C [(xy + y^2)dx + x^2dy]$ where c is the boundary of the common area between $y=x^2$ and $y=x$.

Z-Z-Z

END