14MAU14

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2018
(Fifth Semester)

Branch - MATHEMATICS

REAL ANALYSIS

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10x2 = 20)

1 Define neighborhood of a point and limit point.
2 Define: Metric Spaces.
3 Prove that closed subsets of compact sets are compact.
4 Define the Cantor Set.
5 Define Cauchy Sequences.
6 State the root test theorem.
7 Define: Uniformly Continuous function.
8 Consider f(x) = fx(x rational) [0(x irrational). Is it discontinuity of the first kind or second kind?
9 Write the derivative of f where f(x) = xsin x * 0) 10(x = 0)
10 State the mean value theorems.

SECTION - B (25 Marks!
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5x5 25)

11 a Prove that every infinite subset of a countable set A is countable.
OR
b Prove that a set E is open if and only if its complement is closed.

12 a Prove that compact subsets of metric spaces are closed.
OR
b Prove that a subset E of the real line R^1 is connected if and only if it has the following property:
If x g E,y e E and x<z<y, then z e E.

13 a Prove that if E is the closure of a set E in a metric space X, then diamE = diamE.
OR
b Prove that lim(1 + —) n = e. n-»co n

14 a Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then prove that f(X) is compact.
OR
b If f is a continuous mapping of a metric space X into a metric space Y, and if E is a connected subset of X, then prove that f(E) is connected.

15 a Suppose f is continuous on [a,b], f'(x) exists at same point xe[a,b]. g is defined on an internal I which contains the range of f, and g is differentiable at the point f(x). If h(t)=g(f(t)) (a<t<b), then prove that h is differentiable at x, and h'(x)=g'(f(x))f'(x).
OR
h T et f bp defined "on fabl: if f has a local maximum at a noint xefa.bY.

SECTION - C (30 Marks)

Answer any **THREE** Questions
ALL Questions Carry **EQUAL** Marks (3 x 1 0 = 30)

16 (i) Let $\{E_n\}$,n = 1,2...be a sequence of countable sets, and put $S = \bigcup_{n=1}^{\infty} E_n$.
Then prove that S is countable.
(ii) Prove that every neighborhood is an open set.

17 Prove that every K-Cell is compact.

18 (i) Suppose {S,,} is monotonic. Then prove that $\{S_n\}$ converges if and only if it is bounded.
(ii) State and prove the Ratio Test.

19 Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f is uniformly continuous on X.

20 State and prove the L'llospital's Rule.

Z-Z-Z END