

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2018
(Third Semester)

Branch - MATHEMATICS

PARTIAL DIFFERENTIAL EQUATIONS & FOURIER TRANSFORMS

Time : Three Hours

Maximum : 75 Marks

SECTION-A 120 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 x 2 = 20)

- 1 Solve $p = \log(px - y)$.
- 2 Solve $\frac{d^2z}{dxdy} = \sin x$.
- 3 Find the general solution of $2p + 3q = 1$.
- 4 Solve $pxy + pq + qy = yz$.
- 5 Find the constant a_0 of the Fourier series of the function $f(x) = k, 0 < x < 2\pi$.
- 6 Write the formula for half range cosine series in $(0, \pi)$ of a function $f(x)$.
- 7 State the Fourier integral theorem.
- 8 What is meant by reciprocal with respect to Fourier transform? Give an example.
- 9 Write finite Fourier sine transform of $f(x)$.
- 10 Prove that $F_c \{ f(x) \} = (-1)^p f(1) - f(0) + -y U_s(p)$.

SECTION - B (25 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 5 = 25)

- 11 a Solve $xyp^2 + p(3x^2 - 2y^2) - 6xy = 0$.
OR
b Solve $p + q = npq$.
- 12 a Solve $(mz - ny)dx + (nz - lz)dy + (ly - mx)dz = 0$.
OR
b Solve $pz + qy = x$.
- 13 a Find the Fourier series for the function $f(x) = e^x$ defined in $(-\pi, \pi)$.
OR
b Obtain the Fourier series for the function $f(x) = |x|$ in $(-\pi, \pi)$.
- 14 a Find Fourier sine transform of $\frac{x}{a^2 + x^2}$ and Fourier cosine information of $\frac{1}{a^2 + x^2}$.
OR
b Find Fourier sine transform of $\frac{1}{x}$.
- 15 a Find the finite Fourier Sine and Cosine transforms of $f(x) = x^3$ in $(0, 1)$.
OR

Solve $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t}$ for $0 < x < 10, t > 0$ given the conditions $u(0, t) = u(10, t) = 0$
& $\frac{\partial u}{\partial x} = 0$

for $t > 0$ and $u(x, 0) = 10x - x^2$ for $0 < x < 10$.

SECTION - C (30 Marks)Answer any **THREE** Questions**ALL** Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 i) Obtain a partial differential equation, by eliminating the arbitrary function from, $cp(x^2 + y^2 + z^2, lx + my + nz) = 0$.

ii) Determine the complete integral of $9(p^2z + q^2) = 4$.

17 Solve $(p^2 + q^2)y = qz$ by using Charpit's method.

18 Express $f(x) = (TC - x)^2$ as a Fourier series of a period $2n$ in the interval $0 < x < 2TC$. Hence deduce the sum of the series $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$

19 Find the Fourier transform of $f(x) = 1 - |x|$ if $|x| < 1$ and hence Find the value

$$\int_0^t \frac{1}{t} dt = 0 \text{ for } |x| > 1.$$

20 Solve $\frac{du}{dt} = K \frac{d^2u}{dx^2}$ for $0 < x < \infty$, $t > 0$ given the conditions

i) $u(x, 0) = 0$ for $x > 0$

ii) $\frac{du}{dx}(0, t) = -a$ (constant)

iii) $u(x, t)$ is bounded.

Z-Z-Z

END