14MAU13

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION DECEMBER 2018
(Fifth Semester)

Branch - **MATHEMATICS**

ALGEBRA-1

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (1 0 x 2 = 20)

1 Define projection of S x T on S and T.

2 Prove that if G is a finite group and a eG then a°^ = e.

3 If <j) is a homomorphism of G into G then prove that <j>(e) = e, the unit element of G.

4 Definekernel.

5 Defineeven permutation with example,

6 Express the given permutation into cycles:

(1 2 3 4 5.6 7 8 9^s
– i_v2 3 8 1 6 4 7 5

7 Define ring.

8 Let Obe the set of all symbols $a_0 + a_1i + a_2j + a_3k$ where all the numbers a_0 ,a.i ,a_2 and a_3 are real numbers. Compute the sum and product of its elements.

9 Define Maximal ideal.

10 DefineEuclidean ring.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

1 1 a For all aeG, prove that Ha = {x *e* G / a = x mod H}.

OR

b Prove that there is a one-to-one correspondence between any two right cosets of H in G.

12 a if G is a group, N is a normal subgroup of G, then prove that G|N is a group under multiplication.

OR

b If <|) is a homomorphism of G into Q with kernel K, then prove that K is a normal subgroup of G,

13 a Let G be a group of order 99 and suppose that H is a subgroup of G of order 11. Prove that H itself is a normal subgroup of G.

OR

4 a Prove that any field F is an integral domain.

OR

b If <(> is a homomorphism of R into R[1] with Kernel I(cj>), then prove that (i) I(<j>) is a subgroup of R under addition (ii) If a e 1(4)). and r *e* R then both ar and ra are in I(4>).

5. a Let R be the ring of all the real valued continuous functions on the closed unit interval. Let M = {f(x) e R ¡ f(l/2) = 0}. Prove that M is a maximal ideal ofR.

OR

b Let R be a Euclidean ring and let A be an ideal of R. Then prove that there exists' an element a_0 e A such that A consists exactly of all a_0x as x ranges over R.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 1 0 = 3 0)

16 Let G be the set of all 2 x 2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d are real numbers such that ad - be ^ 0. Prove that G is a group under multiplication.

17 Prove that HK is a subgroup of G <=> HK = KIT.

18 Prove that 1(G) s GjN where 1(G) is the group of inner automorphism of G-and Z is the center of G.

19 If R is a ring then for all a, b e R, prove the following:
i) aO = 0a = 0 ii) a(-b) = (-a)b - -(ab) iii) (-a)(-b) = ab.

20 Prove that if R is a commutative ring with unit element and M is an ideal of R, then M is a maximal ideal of R <=> R|M is a field.

Z-Z-Z END