14MAU20

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2018
(Sixth Semester)

Branch – **MATHEMATICS**

COMPLEX ANALYSIS

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Define differentiability.

2 Which is the Laplace's equation?

3 Define Jacobian of a transformation.

4 What are inverse points with respect to a circle?

5 Define partition.

6 What is complex integrals?

7 Define primitive.

8 What is residue at a pole?

9 State Cauchy's residue theorem.

10 What is evaluation of integral?

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Prove that continuity is a necessary but not a sufficient condition for existence of a finite derivative.

OR

b If the real part of an analytic function f(z) is a given harmonic function u(x, y) show that f(z) = 2u(z/2, z/2i) – u(0, 0).

12 a Prove that for the transformation $w = \sqrt{x^2 + y^2} - i$, determine the region D' of the w-plane corresponding to the region D of the z-plane given circular disc $x^2 + y^2 \leq 1$.

OR

b Show that the transformation $w = e^{\frac{i\pi}{4}} z$ determine the region in the w-plane corresponding to the triangular region bounded by the lines x = 0 y = 0 and x + y =1 in the z-plane.

13 a Let f(x) be continuous on a contour L of length I and $|f(x)| \leq M$ on L, show that $\left|\int f(z)dz\right| \leq ML$.

OR

b State and prove Cauchy's integral formula.

Cont ....

14 a Expand the following function in a Taylor's series about $z = 0$ and determine the region of convergence in each case (i)e^z, (ii) $\sin z$.

OR

b State and prove Schwarz lemma.

15 a Show that $\int_0^{\pi} \frac{1+2\cos\theta}{5+3\cos\theta} d\theta = 0$.

OR

b Evaluate $\int_0^{\infty} \frac{\cos ax}{(x^2+b^2)} dx$, $(a > 0, b > 0)$.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Derive polar form of Cauchy – Riemann equation.

17 State and prove Necessary condition for $w = f(z)$ to represent and conformal mapping.

18 State and prove Morera's theorem.

19 Find the nature and location of the singularities of the function $f(z) = \frac{1}{z(e^z - 1)}$. Prove that $f(z)$ can be expanded in the form $\frac{1}{z^2} - \frac{1}{2z} + a_0 + a_2 z^2 + a_4 z^4 + ...$

20 Prove that $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^6 + 10x^2 + 9} dx = \frac{5\pi}{12}$.

Z-Z-Z END