

PSG COLLEGE OF ARTS & SCIENCE  
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2018  
(First Semester)

Branch – MATHEMATICS

CLASSICAL ALGEBRA & TRIGONOMETRY

Time : Three Hours

Maximum : 75 Marks

SECTION-A (20 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 x 2 = 20)

- 1 Find the limit of sequence defined by  $a_n = \frac{(-1)^n}{2+n} + 1$ ; apply the definition.
- 2 When does a monotonic decreasing sequence tends to minus infinity?
- 3 State Raabe's test.
- 4 What is conditionally convergent series? Give an example.
- 5 When can an equation have atleast one positive root and atleast one negative root?
- 6 Transform  $x^n + p_1x^{n-1} + p_2x^{n-2} + \dots + p_n$  into another whose roots are same as those of the above but opposite in sign.
- 7 Expand  $\tan 7\theta$  in terms of  $\tan \theta$ .
- 8 Write the coefficients of the terms in the expansion of  $(x + \frac{1}{x})^6$ .
- 9 Prove  $\log \frac{a+ib}{a-ib} = 2i \tan^{-1} \frac{b}{a}$ .
- 10 Using Gregory's series evaluate  $\frac{\pi}{4}$ .

SECTION - B (25 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 5 = 25)

- 11 a Find whether the series in which  $u_n = (n^3 + 1)^{\frac{1}{3}} - n$  is convergent or divergent.  
OR  
b Show that  $\left\{ \frac{n}{n+1} \right\}$  is monotonic increasing sequence.
- 12 a Examine the convergence of  $1^2x + 2^2x^2 + 3^2x^3 + \dots$   
OR  
b Discuss the convergence of  $1 + \frac{(1!)^2}{2!}x + \frac{(2!)^2}{4!}x^2 + \frac{(3!)^2}{6!}x^3 + \dots$
- 13 a If  $\alpha, \beta, \gamma$  are roots of the equation  $x^3 + ax^2 + bx + c = 0$ , form the equation whose roots are  $\alpha\beta, \beta\gamma$  and  $\gamma\alpha$ .  
OR  
b Solve  $x^4 - 8x^3 + 14x^2 + 8x - 15 = 0$ , being given that sum of two of the roots is equal to the sum of the other two.
- 14 a Prove  $\frac{1 + \tan \lambda x}{1 - \tan \lambda x} = \cos \lambda 2x + \sin \lambda 2x$   
OR  
b Evaluate  $\text{Lt}_{x \rightarrow 0} \frac{\sin x - x \cos x}{x^3}$

15 a If  $\tan \log(x+iy)=a+ib$  where  $a^2 + b^2 \neq 1$  prove  $\tan \log(x^2 + y^2) = \frac{2s}{1-a^2 - b^2}$

OR

b Find the sum of the series  $\sin \alpha \sin 2\alpha + \sin 2\alpha \sin 3\alpha + \dots + n$  terms

**SECTION - C (30 Marks)**

Answer any **THREE** Questions

**ALL** Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Show that the series  $\sum \frac{\{(n+1)r\}}{n^{n+1}}$  is convergent if  $r < 1$  and divergent if  $r \geq 1$

17 Find the limit of the sequence  $\{a_n\}$  where  $a_n = \left(1 + \frac{1}{n}\right)^n$

18 Find the condition that the general biquadratic equation  $ax^4 + 4bx^3 + 6cx^2 + 4dx + e = 0$  may have 2 pairs of equal roots.

19 Expand  $\cos 6\theta$  in terms of  $\sin \theta$

20 Find the general value of  $(x+iy)^{\alpha+i\beta}$  and show that the sum of the moduli of the values less than unity of  $(1+i)^{(1+i)}$  is  $\frac{1}{\sqrt{2}} e^{3\pi/4} \operatorname{cosec} \pi$

Z-Z-Z

END