

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2018
(First Semester)

Branch – MATHEMATICS

CALCULUS

Time : Three Hours

Maximum : 75 Marks

SECTION-A (20 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 x 2 = 20)

- 1 Find $\frac{du}{dt}$ if $u=x^3y^4z^2$ where $x=t^2, y=t^3, z=t^4$.
- 2 Find $\frac{dy}{dx}$ if $x^3+y^3+3axy=0$
- 3 Find the Cartesian formula for the radius of curvature.
- 4 Find the coordinates of the centre of curvature of the curve $xy=2$ at the point $(2,1)$.
- 5 Find $\int_0^{\pi/2} \sin^6 x dx$
- 6 Evaluate $\int x^4 e^x dx$
- 7 Define double integral.
- 8 Evaluate $\iint xy dx dy$ taken over the positive quadrant of the circle $x^2+y^2=a^2$
- 9 Define Beta and Gamma functions.
- 10 Prove that $\beta(m,n) = \beta(n,m)$

SECTION - B (25 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 5 = 25)

- 11 a Find $\frac{du}{dx}$ if $u=x^2+y^2+a^2$ where $x^3+y^3=a^3$
OR
b Find $\frac{du}{dt}$ if $u=x^2+y^2+z^2$, where $x=e^t, y=e^t \sin t, z=e^t \cos t$.
- 12 a Prove that the radius of the curvature at any point of the cycloid $x=a(\theta+\sin \theta)$ and $y=a(1-\cos \theta)$ is $4a \cos \frac{\theta}{2}$
OR
b Find the envelope of the family of a straight lines $y+tx=2at+at^3$, the parameter being t .
- 13 a Evaluate $I = \int_0^{\pi/2} \log \sin x dx$
OR
b Obtain the reduction formula for $\int \sin^n x dx$
- 14 a By changing the order of integration, evaluate $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dx dy$
OR
b Evaluate $\iint (x^2 + y^2) dx dy$ over the region for which x, y are each ≥ 0 and

15 a Evaluate $\int_0^1 x^m (\log \frac{1}{x})^p dx$.

OR

b Evaluate the integral $\iint x^p y^q dy dx$ over the triangle $x > 0, y > 0, x + y \leq 1$ in terms of Gamma functions.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 If $u = a^3 x^2 + b^3 y^2 + c^3 z^2$ where $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, find the minimum value of u .

17 Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

18 Obtain the reduction formula for $\int x^n \cos ax dx$ (n a positive integer).

19 Evaluate $\iiint xyz dx dy dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ in terms of Gamma functions.

20 Evaluate the integral $\iint x^p y^q dx dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$ in terms of Gamma functions.

Deduce (i) the area of the circle.

(ii) the coordinates of the centroid of a quadrant of the circle.

Z-Z-Z

END