PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)
BSc DEGREE EXAMINATION MAY 2018
(Second Semester)

Branch – MATHEMATICS

<u>ANALYTICAL GEOMETRY OF 3D & VECTOR CALCULUS</u>

Time : Three Hours   Maximum : 75 Marks

<u>SECTION-A (20 Marks)</u>
Answer ALL questions
ALL questions carry **EQUAL** marks   (10 x 2 = 20)

1   Find the equation of the sphere with centre (-1, 2, -3) and radius 3 units.

2   Find the centre and radius of $x^2 - y^2 + z^2 - 2ux + 2vy + 2wz + d = 0$.

3   Define Cone.

4   Find the condition for the equation $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2bxy + 2ux + 2vy + 2wz + d = 0$ to represent a cone.

5   Define enveloping cylinder.

6   Write the equation of hyperboloid of one sheet.

7   Define gradient and divergence of a vector function.

8   Define an irrotational vector.

9   Define an line integral.

10   State Greek's theorem in the plane.

<u>SECTION - B (25 Marks)</u>
Answer ALL Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a   Find the equation of the sphere which has its centre at the point (6, -1, 2) and touches the plane $2x - y + 2z - 2 = 0$.

OR

   b   Find the equation of a sphere which touches the sphere $x^2 + y^2 + z^2 - 6x + 2z + 1 = 0$ at the point (2, -2, 1) and passes through the origin.

12 a   Show that the equation of a right circular cone whose vertex is origin, axis oz and semi-vertical angle α is $x^2 + y^2 = z^2 \tan^2\alpha$.

OR

   b   Show that the equation
$4x^2 - y^2 + 2z^2 - 3yz + 2xy + 12x - 11y + 6z + 4 = 0$ represents a cone.

13 a   Find the equation of the cylinder whose generators are parallel to the Z axis and the guiding curve is $ax^2 + by^2 = cz$, $lx + my + nz = p$.

OR

   b   Find the equation of a right circular cylinder of radius 2 and whose axis is the line
$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{6}.$$

14 a If $f = xz^3i - 2xyzj + xzk$, find div f and curl f at (1, 2, 0).

OR

b Find the unit normal to the surface $x^2y + 2xz^2 = 8$ at the point (1, 0, 2).

15 a If $F = (3x^2 + 6y)i - 14yzj + 20xz^2k$, evaluate $\int_c F.dr$ from (0, 0, 0) to

(1, 1, 1) along the curve $x = t, y = t^2, z = t^3$.

OR

b Evaluate $\iint_s A.n\ ds$ where $A = 4xzi - y^2j + yzk$ where S is the surface of the

cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1.

SECTION - C (30 Marks)
Answer any **THREE** Questions
ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Find the equation to the sphere through the four points (2, 3, 1) (5, -1, 2) (4, 3, -1) and (2, 5, 3).

17 Find the condition for the equation $F(x, y, z) = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$ to represent a cone.

18 Find the equation of the right circular cylinder described on the circle through the points (a, 0, 0) (0, a, 0), (0, 0, a) as a guiding curve.

19 Determine f(r) so that the vector f(r)r is both solenoidal and irrotatunal.

20 Verify stoke's theorem for a vector field $A = (x^2 - y^2)I + 2xyj$ in the rectangular region of the z = 0 plane bounded by the lines x = 0, x = a, y = 0 and y = b.

Z-Z-Z   END