PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2018
(Sixth Semester)

Branch – **MATHEMATICS**

ALGEBRA - II

Time : Three Hours　　　　　　　　　　　　　　　Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks　　　　(10 x 2 = 20)

1　Compute (i) $\begin{pmatrix} 1 & 2 & \sqrt{2} \\ \sqrt{3} & 4 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 1 & 2 \\ -\sqrt{3} & \sqrt[3]{5} & 7 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

2　Define symmetric and skew-symmetric matrices with examples.

3　If V is a vector spice over F then prove that $\alpha\, 0 = 0$ for $\alpha \in F$.

4　Prove that the kernel of a homomorphism on vector spaces is a subspace.

5　If $U \subset W$, then prove that $A(U) \supset A(W)$.

6　Define an inner product space.

7　Define a row – reduced echelon matrix.

8　Define the rank of a matrix.

9　Define right and left invertible.

10　Computer $\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}^2$.

SECTION - B (25 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a　If the matrix products AB and BC are defined , prove that (AB)C = A(BC).

OR

b　For any square matrix A of order n, A(adj A) = (adj A) A = (det A)I_n.

12 a　Define a linear span of a non empty subsets of the vector space V. Prove that L(S) is a subspace of V.

OR

b　If $v_1, \ldots\ldots v_n$ are in V, prove that either they are linearly independent or some v_k is a linear combination of the preceding ones, $v_1, \ldots, v_{k-1}$.

13 a　If V is finite-dimensional and $v \neq 0 \in V$, then prove that there is an element $f \in \hat{V}$ such that $f(v) \neq 0$.

OR

b　State and prove the Schwarz inequality.

Cont ....

14 a Determine the rank of the matrix.

$$\begin{pmatrix} 1 & -1 & 0 & 2 & 1 \\ 3 & 1 & 1 & -1 & 2 \\ 4 & 0 & 1 & 0 & 3 \\ 9 & -1 & 2 & 3 & 7 \end{pmatrix}.$$

OR

b Prove that the characteristic roots of a hermitian matrix are all real.

15 a If V is finite-dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

OR

b In F_2, prove that for any two elements A and B, $(AB - BA)^2$ is a scalar matrix.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Show that the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{pmatrix}$ satisfies the equation on $A^3 - 23A - 40I = 0$. Hence computer A^{-1}.

17 Prove that any two finite – dimensional vector spaces over F of the same dimension are isomorphic.

18 If V is a finite – dimensional inner product space and W is a subspace of V, prove that $(W^{\perp})^{\perp} = W$.

19 Verify the Cayley – Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{pmatrix}$.

Hence compute A^{-1}.

20 a) If $\lambda \in F$ is a characteristic root of $T \in A(V)$, prove that for any polynomial $q(x) \in F[x]$, $q(\lambda)$ is a characteristic root of $q(T)$.

b) If $\lambda \in F$ is a characteristic root of $T \in A(V)$, prove that λ is a root of the minimal polynomial of T.

Z-Z-Z END