PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2018
(Fifth Semester)

Branch – **MATHEMATICS**

REAL ANALYSIS

Time : Three Hours Maximum : 75 Marks

SECTION-A (20 Marks)
Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Prove that the set of all integers is countable.
2 Define: Metric Function.
3 State: Weierstrass theorem.
4 Define a Connected set.
5 Define: Cauchy sequence.
6 Define: Complete metric space.
7 Define: Continuous Function.
8 Define a bounded mapping of a set E into R^K.
9 Explain: Local maximum.
10 State: Mean Value Theorem.

SECTION - B (25 Marks)
Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a Prove that every neighbourhood is an open set.

OR

b Define Closure of E. If X is a metric space and ECX, prove that $\overline{E}$ is closed.

12 a Prove that closed subsets of compact sets are compact.

OR

b Let E be a set in R^K. If E is closed and bounded, prove that E is compact.

13 a If $\overline{E}$ is the closure of a set E in a metric space X, prove that diam $\overline{E}$=diam E.

OR

b If p>1, prove that $\sum_{p=1}^{\infty}\frac{1}{n^p}$ converges.

14 a Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Prove that f(E) is compact.

OR

b Define discontinuity of the second kind and give an example.

15 a Suppose f is a real differentiable function on (a,b) and suppose $f^1(a)<\lambda<f^1(b)$. Then there is a point $x \in (a,b)$ such that $f^1(x)=\lambda$.

OR

b Let f be defined on (a, b). If f has a local maximum at a point $x \in (a,b)$ and if $f^1(x)$ exists, prove that $f^1(x)=0$.

SECTION - C (30 Marks)
Answer any **THREE** Questions
ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 Let $\{E_n\}$, n=1,2,3,... be a sequence of countable sets and let $S=\bigcup_{n=1}^{\infty}E_n$. Prove that S is countable.
17 If p is a limit point of a set E, prove that every nhd of p contains infinitely many points of E.
18 State and prove root test of a series $\sum a_n$
19 Let f be a continuous mapping of a compact metric space X into a metric space Y. Prove that f is uniformly continuous on X.
20 State and prove Generalized Mean Value theorem.