

PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2018
(Third Semester)

Branch – MATHEMATICS

PARTIAL DIFFERENTIAL EQUATIONS AND FOURIER TRANSFORMS

Time : Three Hours

Maximum : 75 Marks

SECTION-A (20 Marks)

Answer ALL questions

ALL questions carry EQUAL marks (10 x 2 = 20)

- 1 Find the differential equation of all spheres whose centres lie on the z-axis.
- 2 Solve $\frac{\partial^2 z}{\partial y^2} = \sin y$
- 3 Write the auxiliary equation of $px+qy=z$.
- 4 Find the complete integral of $p=2qx$.
- 5 Define even and odd functions.
- 6 State Dirchelet's conditions.
- 7 State Fourier integral theorem.
- 8 Find the theorem transform of $f(x)=e^{ikx}$, $a < x < b$
 $=0$, $x < a$ and $x > b$
- 9 State Inversion formula for sine transform.
- 10 Find fourier sine and cosine transforms of $f^4(x)$.

SECTION - B (25 Marks)

Answer ALL Questions

ALL Questions Carry EQUAL Marks (5 x 5 = 25)

- 11 a Solve $p^2+q^2=x+y$
OR
b Solve $p(1+q^2)=q(z-1)$
- 12 a Solve $(y^2+z^2)p-xyq=-xz$
OR
b Solve $(p^2+q^2)=z^2(x^2+y^2)$
- 13 a Find a sine series for $f(x)=c$ in the range 0 to π
OR
b Expand $\pi - x$ ($0 < x < \pi$) as a half range cosine series.
- 14 a State and prove Inversion theorem for complex fourier transform.
OR
b Find fourier cosine transform of $\frac{1}{a^2+x^2}$.
- 15 a Find the finite fourier sine transforms of $f(x)=x^2$ in (0, 1)
OR
b Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ for $x > 0, t > 0$ given that
(i) $u(0,t) = 0$ for $t > 0$
(ii) $u(x,0) = 1$ for $0 < x < 1$
 0 for $x \geq 1$
and (iii) $u(x,t)$ is bounded.

SECTION - C (30 Marks)Answer any **THREE** Questions**ALL** Questions Carry **EQUAL** Marks (3 x 10 = 30)

- 16 Solve $z = px + qy + \sqrt{1 + p^2 + q^2}$
- 17 Solve $p^2 + q^2 - 2px - 2qy + 1 = 0$
- 18 Show that $x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} (-1)^n \frac{\cos nx}{n^2}$ in the interval $(-\pi \leq x \leq \pi)$
- 19 Find the fourier transform of $f(x) = 1 - |x|$ if $|x| < 1$ and hence find the value of $\int_0^{\infty} \frac{\sin^4 t}{t^4} dt$
- 20 Using finite fourier transform, solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$
given $u(0,t) = 0$ and $u(4,t) = 0$
 $u(x,0) = 2x$ where $0 < x < 4, t > 0$

Z-Z-Z

END