PSG COLLEGE OF ARTS & SCIENCE
(AUTONOMOUS)

BSc DEGREE EXAMINATION MAY 2018
(Fifth Semester)

Branch – **MATHEMATICS**

ALGEBRA – I

Time : Three Hours | Maximum : 75 Marks

SECTION-A (20 Marks)

Answer **ALL** questions
ALL questions carry **EQUAL** marks (10 x 2 = 20)

1 Let $\sigma: S \to T$ and $\ell: T \to U$ then prove that $\sigma \circ \ell$ is one-to-one if each σ and e is one-to-one.

2 State the Associative Law.

3 If ϕ is a homomorphism of G into $\overline{G}$ then prove that $\phi(x^{-1}) = \phi(x)^{-1}$ for all $x \in G$.

4 Define normal subgroup.

5 Find which of the following are odd permutations?
(a) (1,2,4,6,7,8) (b) (2,3,4,6,7,8,9)

6 Express the permutations as a product of disjoint cycles.
(i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 1 & 3 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 2 & 1 \end{pmatrix}$

7 State Pigeonhole principle.

8 Define Finite characteristic.

9 Define Prime element.

10 State unique factorization theorem.

SECTION - B (25 Marks)

Answer **ALL** Questions
ALL Questions Carry **EQUAL** Marks (5 x 5 = 25)

11 a. If H is a non empty finite subset of a group G and H is closed under multiplication, then PT H is a subset of G.

OR

b Prove that the relation $a \equiv b$ mod H is an equivalence relation.

12 a Prove that N is a normal subgroup of $G \Leftrightarrow gNg^{-1} = N$ for every $g \in G$.

OR

b Prove that A subgroup N of G is a normal subgroup of $G \Leftrightarrow$ the product of two right cosets of N in G is again a right coset of N in G.

13 a Prove that every permutation is the product of its cycles.

OR

b Let G be a group and ϕ an automorphism of G. If $a \in G$ of order $0(a) > 0$ then prove that $0(\phi(a)) = 0(a)$

14 a If R is a ring, then you all $a,b \in R$ and R has a unit element 1, then
(i) (-a)(-b)=ab
(ii) (-1)a=-a
(iii) (-1)(-1)=1

OR

b Prove that let F be any field then the only ideals of F are $\{0\}$ and F.

15 a Prove that let R be a Euclidean ring and $a, b \in R$. If $b \neq 0$ is not a unit in R then d(a)<d(ab)

OR

Cont...

15 b Prove that let R be a Euclidean ring. Suppose that for $a,b,c \in R$, a/bc but (a.b)=1 then a/c.

SECTION - C (30 Marks)

Answer any **THREE** Questions

ALL Questions Carry **EQUAL** Marks (3 x 10 = 30)

16 If G is a group then prove that

(i) the identity element of G is unique.

(ii) Every $a \in G$ has a unique inverse in G

(iii) For every $a \in G, (a^{-1})^{-1} = a$

17 If H and K are finite subgroup of G of order O(H) and O(K) respectively then prove that $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$

18 State and prove Cayley's theorem.

19 Prove that if U is an ideal of the ring R, the R/U is a ring and is a homomorphic image of R.

20 Let R be a Euclidean ring then prove that any two elements a and b in R have greatest common divisor d. Moreover $d = \lambda a + \mu b$ some $\lambda, \mu \in R$.

Z-Z-Z END