Health Economics

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Week – 06

Lecture 27- Expected Utility Theory

Welcome to our course on Health Economics. This is in continuation to the previous lecture on behavioural economics. We discussed traditional economic theory as well. However, in this lecture, we are focusing on the expected utility theorem, which is part of the traditional economic discussion and how the utility explanation, etcetera is not valid in the behavioral context. The expected utility we calculate or use to calculate may not be valid in the realistic context or through assumptions, and those calculations may not be valid. So, we discuss this lecture, we will be focusing on expected utility theory, we will discuss the problems associated with its axioms, and then only we will connect it to the behavioural economics.

We will discuss many counterexamples disproving and highlighting the failures of the axioms of expected utility theorem such as bounded rationality, the puzzle of self-torturer, the game of rock, paper and scissors, the Condorcet paradox and the St. Petersburg paradox, then Elsberg paradox etcetera. So, without discussing the background of the earlier lecture, I think it is better to discuss the expected utility theory. So, when we say expected, we discuss the extent of risk and how it is useful in understanding decisions.

When there is no risk involved, economists believe that individuals have a utility function that can convert ordinal preferences into a real-valued function. However, related to the risk contained in the utility function or in the preference function, initially, there are works emphasized by Daniel Bernoulli. This expected utility theory originally by Bernoulli in 1748, then it was expanded by John von Neumann and Oskar Morgenstern in 1947. This expected utility theory implies that the function of U is equal to the sum of its utilities as a product of their probabilities. So, $u(x_i)$ is the utility, and p_i is the risk or the extent of probabilities. So, i stands for the i^{th} state of the world, and the individual receives x_i dollars as a return.

$$U=\sum_i p_i u(x_i)$$

So, the expected utility of an act is a weighted average. That is why we discussed the weighted average of the utilities of each of its possible outcomes, where the utility of an outcome measures the extent to which the outcome is preferred or preferable to the alternatives. The utility of each outcome is weighed according to the probability that the act will lead to that outcome. Hence, the expected utility theory is normative, explaining how people should make decisions. In classical economics, expected utility theory is often used as descriptive theory and that is the theory of how people make decisions or a predictive theory that is a theory that, while it may not accurately model the psychological mechanisms of decision-making and correctly predicts people's choices.

So, the EUT is an account of choosing rationally when you are unsure which outcome will result from your acts. This basic slogan is choosing the act with the highest expected utility. By giving you the context in this illustration, you will find out how their decisions are made. Taking an umbrella is the problem here, given a sunny or rainy day. Taking an umbrella or leaving an umbrella on a sunny day or taking or leaving an umbrella if it is a rainy day.

So, in this case, let us know a student, maybe a person like Pragya, who is planning to go to our school early in the morning. She would rather not take the umbrella on a sunny day but rather face rain with the umbrella than without it. There are two acts available to her to choose by taking her umbrella or leaving her umbrella at home. Which of these acts should be chosen? So, we will explain. So, there are three sorts of entities in the example that you have just cited.

One is an outcome. The outcome is basically either dry and encumbered or wet. There are three possibilities in terms of outcomes. The states are either it raining or not raining. Acts basically take the umbrella as T and not as L.

So, we are just putting the states in matrix and trying to find out which is the preferred one. So, the act we have mentioned on the horizontal and vertical ones we have mentioned states. If it rains, then acting is like taking an umbrella; it is encumbered and dry. There are outcomes we have mentioned. Then encumbered and dry in another case, wet and dry, etcetera, all possibilities are taken.

Each column corresponds to the state of the world, each row corresponds to an act, and each entity corresponds to the outcome that results when the act is performed in the state of the world. So, we are just mentioning how the T and L are acted upon. So, the value of the outcome is measured by utility and the probability of the outcome conditional on the T and the action is taken. Hence, the expected utility of the T is the sum of the probabilities of the outcome and their respective utilities. So, I think you can follow between the lines.

$$EU\ (T) = \sum_{o\varepsilon O} P_T(o)U(o)$$

So, considering the same example of Pragya, the expected utility of taking the umbrella will be, in this case, equal to Pr times the utility of R and T that is the probability of R, in this case we are mentioning and Pn probability of not raining and the outcome. And hence, it is equal to the probability we have already mentioned for an arbitrary example. So, it will either rain or not rain. So, they are 0.6 and 0.4, respectively. So, 0.6 is the probability times the return is 5; the outcome is five plus 0.4, this is 0.4 times this 5. So, this is defined to be 5. The expected utility is 5 when the Pragya is taking umbrella, whereas in the case of leaving the umbrella, the expected utility will be 4 with the same approach. So, that means the first action, this green one, is greater than that of the second. So, that suggests that persons, through the EUT, expected utility theory to be that one should carry the umbrella. This is what is mentioned: Pragya will carry the umbrella with her.

Consider in the previous example of Pragya,

		States	
		It Rains (R) (P=0.6)	It Does Not Rain (N) (P=0.4)
Acts	Take Umbrella (T)	Encumbered, Dry (U=5)	Encumbered, Dry (U=5)
	Leave Umbrella (L)	Wet (U=0)	Free, Dry (U=10)

Expected utility of taking the umbrella, EU(T):

$$EU(T) = P_R * U(R,T) + P_N * U(N,T)$$
$$EU(T) = 0.6 * 5 + 0.4 * 5 = 5$$

Expected utility of leaving the umbrella, EU(L):

$$EU(L) = P_R * U(R,L) + P_N * U(N,L)$$
$$EU(L) = 0.6 * 0 + 0.4 * 10 = 4$$

Since EU(T) > EU(L); Pragya will carry the umbrella with her.

Suppose Pragya's classmate Prapti has a different utility structure given below. Find out whether Prapti will also carry an umbrella with her or not. From the same approach, we are just mentioning that different probabilities are the same, but the outcome is different and you can calculate the same and tell us the result. And I think you will cross-check whether you understand or not. We are just keeping for you to solve it.

The answer to Prapti's preference is that her expected utility is higher for leaving the umbrella. I am just mentioning this. You can follow between the lines and try to understand. So why it is this the case? Let us explain the axioms of the expected utility theory. So, according to the expected utility theory framework, the four axioms must hold for people to make decisions.

Suppose Pragya's classmate Prapti has a different utility structure given below. Find out whether Prapti will carry the umbrella with her or not.
(Pause the video and do on your own!)

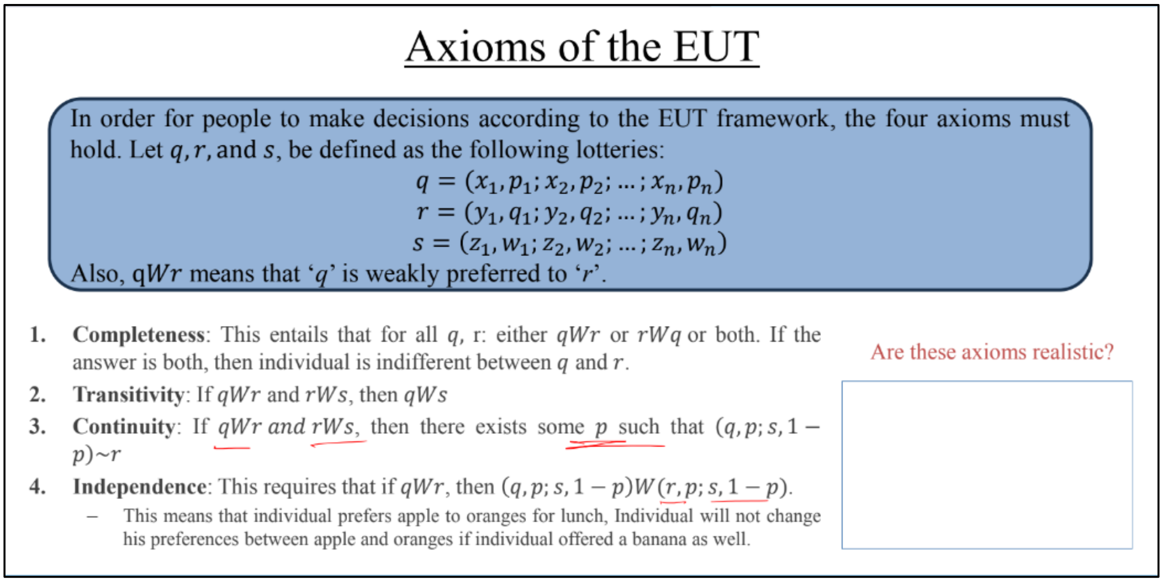
		States	
		It Rains (R) (P=0.6)	It Does Not Rain (N) (P=0.4)
Acts	Take Umbrella (T)	Encumbered, Dry (U=4)	Encumbered, Dry (U=4)
	Leave Umbrella (L)	Wet (U=2)	Free, Dry (U=8)

- The answer to Prapti's preferences is that her expected utility is higher for leaving the umbrella.
- Utility functions of Pragya and Prapti rank the outcomes in same order:
  - (Free, dry) > (Encumbered, dry) > (Wet)
- Why does expected utility theory gives different advice in two versions of problem?
  - Positive linear transformations of outcome utilities will never affect the verdicts of EUT.
  - Pragya and Prapti's outcome utility are not in sync with positive linear transformations.

  $U' = aU + b$ where, a>0

So, let q, r and s be defined as the following lotteries. So, q with its probability function r and s. So, also qWr means q is weakly preferred to r, not strictly preferred to r. So, some axioms that we usually consider in our typical decision theory are completeness, transitivity, continuity, and independence. So, completeness means q preferred to weakly preferred to r and r preferred to q, or both should be entailed.

Then, the individual is, in this case, indifferent and has already expressed all possible choice functions between q and r. In contrast, if q is preferred to r or weakly preferred to r and r is weakly preferred to s, then obviously, as per the transitivity axiom, q is weakly preferred to s. Similarly, continuity, if continuity is an axiom, this means that if q is weakly preferred to r and r is weakly preferred to s, then there exists some p such that q and its p and s and it's, 1-p etc in all events should be following the continuity function. And similarly, another one like independence, is important. This requires that if q is weakly preferred to r, then q, its probabilities, then s and its other side of it, which is 1-p, should be weakly preferred to the r and its respective p and so on.



This means that individual prefers apples to oranges for lunch. The individual would not change his preferences between apples and oranges if the individual offered a banana as well. So, independence axioms clarify the fact that if you have some interventions or some nudges in between, or you may say some forms of disturbance in between your choice function should not be altered. So, that means you still have independence in your selection and there is no pressure in your selection. So question is, are these axioms realistic? I have just clarified that these are unrealistic; some are not.

So maximizing expected utility is indeed impossible. So, maximizing expected utility is irrational. So, there are counterexamples that will clarify things. For example, we will mention counterexamples involving this transitivity and completeness. Why is it clarifying the fact that the expected utility is rational? So, we also give counterexamples involving independence, counterexamples involving zero probability events, and counterexamples involving unbounded utility.

Then, we start by explaining why maximizing expected utility is impossible. March and Simon in 1958 mentioned that maximization requires a complex understanding of available acts, possible outcomes values of outcomes, and the best act selection. Another work by McGee in 1991 mathematically mentioned this as impossible, and the bounded rationality approach aims to replace the expected utility theory with some more tractable rules. So let us start with this bounded rationality. We accept this human decision-making process to satisfy rather than optimize.

When the target is not to the bounded option that is the best option or the optimizing option then another option left out with us usually; that is the case with the consumers or in the choice function is basically just to attend the level, just to satisfy in our action. In other words, when we seek a decision that will be good enough rather than the best possible, most of the choice functions are used to be just satisfying in nature, not attending to the bounded optimum level. Bounded rationality suggests that we make decisions without all information due to cognitive and time constraints. Choices through reasonable with available data may not align with ideal outcomes. Striving for perfect decisions is really challenging and given practical limitations.

The theory of bounded rationality is developed by Herbert Simon in 1957, and this often occurs due to our short-term memory or short-term decisions and like unbounded rationality occurs like we choose just to satisfy not to reach the optimum level, still, we feel the better. This is because we do not wait for a long time to reach the best possible solution. We might be saturated at the non-optimum level. So, because of brains have limits, we use shortcuts. Those shortcuts are called heuristics, and they help people make quick decisions.

These shortcuts help, but sometimes they make us choose things that are not the very best. Counterexamples involving transitivity and completeness. We have already assumed that it has to follow the EUT traditional theory, and transitivity and completeness of preferences should be there. There are cases where rationality seems to permit failures of transitivity and completeness. Like some of the works, they mentioned the Queen's puzzle of self-torturer, then a game of rock, paper and scissors and even in Condorcet paradox in all three cases, the preferences are indeed cyclic.

Though cyclic, but it is not following either completeness or it is not following the transitivity properties. So, violation of completeness, such as completeness of preferences in health, cannot be taken for granted. So, it is not possible that we will just be attending to the complete possible status of health. You can refer to the article. We are referring to the self-torturer puzzle of Queen, he is contracting the transitivity properties.

So, referring to the work, the source is highlighted. Imagine a special device that doctors used to give tiny electric shocks to a person's body. So small that the person can't even feel them. Then that might occur repeatedly, and they may go for a higher degree of dose, but that might happen when you have given a higher dose; there is a possibility of reaction from the person side, then the person might not agree to go for the better choice. So, this suggests that medical devices have many settings to set that electric shocks ranging from 0 to 1000.

Let us talk about a person we will call the self-torturer. This person agrees to have this device attached to him. There are some rules, though. The device starts at 0 and he can try different settings like a little experiment every week. However, after the week, the setting returns to 0.

So, outside this experiment, he can only do two things: stay where he is or increase the setting by one unit again. The tricky part is that he can never go back once he reaches increases. But for every increase, he gets 10,000 dollars let it be. Even though the self-torturer can feel much difference between close settings, still there are problems. When the settings are far apart, there is a big difference in how he feels.

If he keeps going off, he might eventually reach very painful settings and he might want to stop and go back to again 0. So, the transitivity, the properties which are say that if you keep on increasing, you might end up with higher returns and then the higher is paying better. But in this case, once he reaches the other level, the final outcome might be cyclic and reaching to the 0 level. The self-torturer puzzle is realistic and does not follow the classical rationality assumption. Similarly, rock, paper and scissors, yes, in a cycle, it works.

We used to play this game in our childhood: rock, paper and scissors. It has a direction. However, when the transitivity or the linkages are broken, the transitivity might not work. Like if you prefer rock over scissors because rock crosses scissors and you prefer scissors over paper because scissor cut paper, then you should prefer rock over paper as per the transitivity.

However, this is not the case. So, this is indeed violating. Another one is called the Condorcet paradox. That is precisely called the paradox of voting. This is a phenomenon in voting theory where the overall preference of a group may be cyclic violating the transitivity property of individual preference. Condorcet paradox challenges the principles in the context of collective decision-making.

Consider a scenario where three candidates are actually contesting; let it be A, B and C. And let us say a group of voters individually express their preference. Some voters prefer A over B, others say B over C, and others prefer C over A. That does not mean C is indeed preferred than that of the candidate. So far as the candidate is concerned, C is preferred over A.

The Condorcet paradox illustrates that aggregating individual preferences into group decisions may sometimes result in inconsistency. This is a practical outcome that challenges the assumption of collective transitivity. This is referred to Marquis D Condorcet of the 18th-century contribution. There are also counterexamples to the traditional assumption of rationality through interdependence.

So, we are referring to the work of Allais and Ellsberg. We will also explain in detail in our other lecture. They also propose examples of preference that cannot be presented by the expected utility theorem. The paradox was proven to violate the interdependence axiom of expected utility theory, which requires that an individual's preference should not change when altering two lotteries by equal proportions. Even we have discussed about interdependence in our previous example. The last paradox suggests that people may overweight extreme probabilities of small events.

Ellsberg paradox suggests that people may be bosses to ambiguity as well. Hence, those do not validate the expected utility theory. We now present the Ellsberg 1961 experiment and clarify these based on urns A and B through an example. Considered two urns, each containing 100 balls.

The balls are either black or red. It is known that the first urn contains exactly 50 black and 50 red balls. In the second urn, the composition of the colors is unknown. You can see from the picture that the bets are given in red or black. The urns are presented as A and B. There are some outcomes we are just mentioning based on the action.

So, choosing red balls from the urns and red A from the A urn, the person gets 100 dollars if red is drawn from urn A; otherwise, it is 0 dollars. Similarly, bet black A if he is chosen and the person gets 100 dollars and if black is drawn from A and 0 otherwise. Similarly, red is from B, and black is from B. So, which bet would you prefer based on the returns? So either it would be red A or black A. Red A or black A since the options are given here, or red B or black B or red A or red B.

There are all possibilities: black A and black B. So, out of all sorts of things, most people prefer red A or black A or red B or black B because of its possibilities, and most people are indifferent on two bets. These two bets are indeed indifferent, but people mostly prefer these two. And then, coming to the balls, I think it is correctly spread. Out of these four, these are actually indifferent to each other.

And similarly, other two are indifferent. However, most people bet on red A and black A because they have a 50 percent chance and, respectively, bet three and bet 4. So, we have already mentioned this in the example. So, let us see what really happens. Bet red A gets 100 dollars. We have already seen that 100 dollars if red is drawn; otherwise, it is 0, and this is one of the four outcome possibilities we have mentioned. Most people prefer red A or red B; let us see which are actually indifferent.

You can see from the urns that red A and black A are indifferent, and red B and black B are indifferent. However, coming to red A and red B, obviously, it is quite certain from the basket that red is preferred to red B. Similarly, black A is preferred over black B based on the urns. If you prefer red A over red B, it means that you believe that A has more red balls. Similarly, if you prefer black A over black B, that means you feel that black balls are more in A or black B are less on B.

And so what really happens? For this reason, you should offer on B for black balls, then just have a look. So that means black balls are preferred over that of black A. However, most people select in reverse order. Thus, people's preferences are inconsistent, violating the axiom of independence. Given the odds and its possibilities, people violate independence as the axiom of the choice function.

Coming to the discussion of zero probability events, we are again citing how it is countering to the rationality choice function when there are zero probabilities. So example of irrational preference that satisfies the EUT we will just explain. Suppose Pragya is about to throw a point sized dart at a round dart board. Classical probability theory gives the situation where the dart has a probability of zero of hitting any point.

Pragya offers Prapti the following lousy deal. If the dart hits the board at its centre, Prapti will get $100; otherwise, no money will be on their hand. As you can see from the matrix, so act is accepting the deal or refusing the deal and hitting at the centre; there are returns and missing the centre it supposes refusing the deal it has zero returns. And the probability of hitting the centre is near zero or zero if it is the case. Since the expected utility is, in this case, the expected utility will be, of course, zero because probabilities are zero and other returns are zero. So, do you think Prapti will accept the deal? The expected utility theory is not a complete theory of rationality because the expected value will be zero.

When two acts have the same expected utility, they do not tell us which to prefer. The two acts are tied for having the highest expected utility; agents are required to be indifferent between them. So Skyrms (1980) points out that this but lets us also derive strange conclusions about events with probability zero. Similarly, you can follow another case of bounded rationality: a utility function U is bounded. If there is a limit to how good things can be according to utility or, more formally, if there is some least natural number, so indicate supremum such that for every A in the utility domain that will be reaching the suboptimal level, then at the supremum level.

U is bounded below to that of the top level, and if there is a limit to how bad things can be according to the U and, more formally, if there is some greatest natural number or infimum, you can refer to the original book. So, expected utility theory can run into trouble when utility functions are unbounded below or above, below or both. If it is unbounded, then the expected utility theory will be difficult to comprehend. So, here we clarify the unbounded utility and how it creates a problem in traditional theories.

So, we are referring to the Pittsburgh game by Nicholas Bernoulli. So, suppose that a coin is tossed until it lands tails for the first time. If it lands tails on the first toss, you win 2 dollars. If it lands tails on the second toss, you will win 4 dollars. If it lands tails on the third toss, you will win accordingly 8 dollars and so on.

And if it lands tails on the n^{th} toss, you will get $\$2^n$. Assuming each dollar is worth 1 util, the expected utility or expected value of the St. Peter's game is basically in the first case; it is half into 2, the probability is half, then in the next case, it will be 1/4 times 4 and so on till the nth. Finally, in every case, it tends to infinity until the infinite values are 1 plus 1 plus 1. So, this claims that a rational agent should pay millions or any hefty amount since the expected utility is infinity, which is absurd. In this game, we will find that to get the best return, one must pay an infinite amount, which is absurd.

- A utility function U is **bounded above** if there is a limit to how good things can be according to U, or more formally, if there is some least natural number *sup* (indicates supremum) such that for every A in U's domain, U(A)≤sup).
- U is **bounded below** if there is a limit to how bad things can be according to U, or more formally, if there is some greatest natural number *inf* (indicates infimum) such that for every A in U's domain, U(A)≥inf.
- **Expected utility theory can run into trouble when utility functions are unbounded above, below, or both.**

Example: **St. Petersburg game (by Nicolas Bernoulli)**

Suppose that a coin is tossed until it lands tails for the first time. If it lands tails on the first toss, you win $2; if it lands tails on the second toss, you win $4; if it lands tails on the third toss, you win $8, and if it lands tails on the nth toss, you win $\$2^n$. Assuming each dollar is worth one util, the expected value of the St Petersburg game is

$$\left(\frac{1}{2}*2\right)+\left(\frac{1}{4}*4\right)+\cdots+\left(\frac{1}{2^n}*2^n\right)+\cdots$$

Or, $1+1+\cdots+1+\cdots \rightarrow$ infinity

To claim that a rational agent should pay millions or any hefty amount since the expected payout is infinity is absurd.

- Very few people opt for this.

Counterexamples involving Unbounded Utility

So, very few people offer this because they just want to win, not just attain the maximum possible utility at the bounded level. So, the unbounded utility as a counterexample really clarifies the problems of the classical theories. So, there are developments in non-expected utility theory as well. So, like prospect theory developed by Kahneman and Tversky in 1979, then generalized expected utility theory by Machina in 1982, regret theory, rank-dependent expected utility theory, etc., we will emphasize prospect theory in the next class.

So, they all challenge the classical one. I think we have clarified with their right example. If you are still cut off somewhere or stuck somewhere, we suggest you go through the original reading you have cited, and I am sure you can clarify. Given the time constraints, it is very difficult to emphasize each of them.

So, these are the suggested readings. So, the next class will be on prospect theory. So, I think I should close here. Thank you.