

**Environmental & Resource Economics**  
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**Optimum extraction of renewable resources and Tragedy of Commons Part - 3**

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- By joining points like M, N and so on, we get a locus which shows different level of harvest based on stock  $x$ .
- This locus is called catch locus.
- If we assume cost of harvest is low / fishes are highly valued, the R curve will be very steep and cost curve will shift downward and catch locus will shift upward.
- Now let us superimpose the catch locus on the growth for discussed earlier.

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Optimum extraction  
of a renewable resource

$$\dot{f}(t) - \frac{\partial C(t)}{\partial x(t)} + f(t) \cdot \frac{\partial g(t)}{\partial x(t)} = r f(t)$$

At steady state,  $\dot{f}(t) = 0$

$$\Rightarrow r f(t) + \frac{\partial C(t)}{\partial x(t)} - f(t) \cdot \frac{\partial g(t)}{\partial x(t)} = 0 \dots \text{--- (1)}$$

$$\dot{x}(t) = 0$$

$$\Rightarrow g(x) = y(t) \dots \text{--- (2)}$$

Along the optimum harvest path  $f(t) = f(t) + \frac{\partial C(t)}{\partial y(t)}$  ... from F.N.C we suppose at points M & N the above condition is satisfied.

$y_1^*$ : Optimum harvest when  $x = x_1$   
 $y_2^*$ : Optimum harvest when  $x = x_2$

*we assume  $x_2 > x_1$  we can differentiate harvest cost wrt depending on stock  $C(x, y)$*

*At steady state,  $\dot{f}(t) = 0$*

*R curve*

*catch locus*

*$C(x, y)$*

*$\frac{\partial C(x, y)}{\partial y}$*

*$y(t)$*

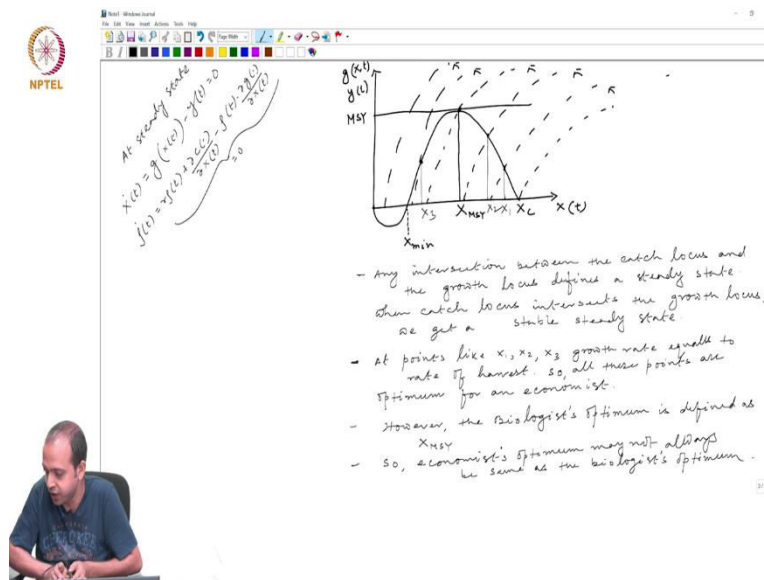
*$x_1^*$   $x_2^*$*

So, what will happen? Cost curve will seep downward and catch locus will shift upward. So, this is what we have to understand the dynamics from the catch locus. I will go back to the diagram once again to understand the dynamics better. So, here what we are assuming that if the features are highly valued obviously, the revenue would be more, this revenue curve will shift upward. Similarly, if the cost of extraction goes down because increase in stock then this cost curve will keep on sifting downward.

As a result of which this catch locus will seep upward that is the point that I would like to mention here. So, catch locus will shift upward. So, that means, we will have different types of catch locus in this dynamic setup, when there is a change in cost of extraction or cost of harvests, when there is a change in price in the fish market that will give an upward push to this catch locus, catch locus will shift upward. Now, what we will do?

Now let us superimpose catch locus on the growth pad that we have discussed earlier, the growth function discussed earlier. What is the growth function? What we will do we will go back and draw that diagram once again. So, let us do it in the next page.

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So, this was our diagram if you recall this was our diagram in the x axis what we are measuring? We are measuring stock and in the y axis we are measuring growth, which is a function of x and t and also rate of extraction, and then this was about growth path, this is  $x_c$ , this we call is  $x_{min}$ ,  $x_{mean}$ .

Now in this diagram and this is the maximum this is  $x_{MSY}$  that means stock corresponding to maximum sustainable yield and this is I will say maximum sustainable yield. Now, what we will do, we will superimpose the catch locus in this diagram. There will be various type of, so catch locus is shifting upwards. Starting from here, we can understand catch locus is shifting let us say this is the dynamics.

So, that means the moment it shifts. Why the catch locus is shifting? For two reasons; fishes are highly valued and then the cost of extraction is going down as a result of which the optimum

extraction is becoming more and more. Because, how we have defined catch locus? Catch locus is the optimum rate of harvest on those costs function.

So, obviously, when the cost function shifts downward fishes are highly valued then catch locus will shift upward that is what is happening here. At steady state what we know that  $\dot{x} = g(x) - y = 0$  and also  $\dot{r} = r - \delta x = 0$ , these are the two things that happens.

So, any intersection between the catch locus and the group locus defines a steady state, but all steady states are not stable. So, what are the stable state? When catch locus intersects the growth locus, we get a stable steady state. So, all these intersection point where the growth locus is actually intersecting with the catch locus these are all economist optimum. So, let us assume that  $x_1$  is the sustainable stock where growth rate and harvest rate is same.

Let us say that all this intersection let us say this is  $x_1$ . This is let us say  $x_2$ , then it is  $X_{MSY}$  this is let say  $x_3$  and this is  $x$ -mean. So, let us say that at points like  $x_1, x_2, x_3$  what happens, growth rate equals to rate of harvest. So, all these points are optimum for an economist. Because economist optimum is defined by the steady state where growth equals to rate of harvest  $g(x) = y$ ,  $\dot{x} = 0$ .

But how is the biologist optimum defined? Biologist optimum however the biologist optimum is defined as  $X_{MSY}$ . So, economist optimum may not always be same as the biologist optimum. Because biologists have only a single optimum, while economists have many optimum points.

All these point  $x_1, x_2, X_{MSY}, x_3$  all these are optimum for economies while biologist optimum is only  $X_{MSY}$ . Interestingly through this  $X_{MSY}$  this  $X$ , is also a point where a growth locus is intersecting with the catch locus. So, that means, if economists optimum happens at this point at the peak of the group then only biologist optimum will converge with that of the economist own. Please try to understand what I am saying.

There are multiple optimum point for the economists where growth is truly equals to harvest  $x_1, x_2, x_3$  they are all economist optimum, but there is only one point where the economist optimum and biologist optimum is converging to each other, what is that  $X_{MSY}$ , because it is derived from a point where growth locus, sorry, catch locus is intersecting the growth locus

from the below and that is happening at the peak of the growth locus also. Then from here, we can actually derive the condition for economist optimum and biologist optimum to converge each other.

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So, conditions for biologist optimum to converge with the economist one. And how we will derive the condition? Same steady state equilibrium  $\dot{r} = 0$ ,  $\dot{r} = 0$  equals to  $r$  into  $\dot{r}$  plus  $\frac{\partial c}{\partial x} \cdot \dot{x}$  minus  $\dot{r}$  into  $\frac{\partial g}{\partial x} \cdot \dot{x}$  equals to 0. Let us assume that  $r$  equals to 0 and  $\frac{\partial c}{\partial x} \cdot \dot{x}$  is also equals to 0.

So, that means, for the entire thing to become 0 therefore, so, for  $\dot{r} = 0$  we need  $\frac{\partial g}{\partial x} \cdot \dot{x}$  to be 0 because  $\dot{r}$  cannot be 0. Now, this basically implies at the point where I will say that where growth rate is 0. So, that means, this is the point where peak of the growth locus. Peak of the growth locus is happening.

And if we have the peak of the growth locus then only economist optimum is converging to biologist optimum. Hence, we see that if  $r$  equals to 0 and if  $\frac{\partial c}{\partial x} \cdot \dot{x}$  equals to 0 then and only then biologist optimum and economist optimum economists' optimum converges with the biologist one. Now, that means, basically what we are saying that economist optimum will converge with the biologists optimum when there is no discount rate, and there is no stock effect on the cost of extraction, these are the two conditions.

Now in this condition let us say that let us assume  $x(t)$  is actually greater than  $X_{MSY}$ . So, since  $\frac{\partial c}{\partial x} \cdot \dot{x}$  is actually 0 there is no point in holding a stock, which is greater than  $X_{MSY}$ ,

MSY there is no stock effect why should I hold more to stock than what is X, MSY. So, the stock will come down to X, MSY only. Hence no benefit from more stock and extra stock so, more benefit.

There is no benefit for holding extra stock, so that is why it is more rational, so it is more rational to harvest more and stock will come down to X, MSY only. So, if at any point of time the stock is more than these since there is no stock effect on cost of extraction, it is better to harvest more and more and it will, the stock will come down to this.

Similarly, if  $r$  equals to 0 then what happens? Then, that means, that implies there is no discount rate, no discount. So, it is better to consume to consume today than preserving it for tomorrow. So, stock reduce to X, MSY. But this condition, if you see, both these conditions are highly restrictive, that means, it is difficult to assume that there is no stock effect on cost of extraction.

Similarly, it is difficult to assume that there is no stock effect actually, there is no discount. So, these are all very restrictive condition. And as a result of which we say that actually there is economist optimum will converge to the biologist optimum only in this very, very specific scenario. We will discuss the remaining thing in our next class. Thank you.