

**Environmental & Resource Economics**  
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**Optimum extraction of renewable resources and Tragedy of Commons Part - 2**

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Welcome, once again to our discussion on Natural Resource Economics. We were specifically discussing about the optimum extraction of renewable resource taking fishery, as an example. And then what we discussed if you recall in our last class that we discussed a biological model and then we discussed an economist model.

So, when there is no human being in the picture, that means, we assumed a lake full of fish but that is undiscovered by human being. So, when it is undiscovered and undisturbed by human beings then it is a purely biological setup, and that is why we discussed about a biological growth model in the context of that fishery.

And we said that, in this biological setup, it is a stock-dependent growth. There are two important elements in that stock-dependent growth model, basically the critical amount of species or critical amount of fish that is required for its regenerating capacity to be exhibited, we call it  $\bar{x}$ .

And then we say that the growth of this species has basically two routes, one is at the  $\bar{x}$ -mean and another also another point at  $x_c$ . Why two routes? Because in that growth path we have a maximum or optimum point in the growth path, where growth rate is 0 and if the stock exceeds beyond that we said that there would be much more competition among these species

for food and oxygen and that is why the stock will come down and again it will become 0, and then we derived the optimality condition.

And if you recall at last, so basically this is what we are discussing is let us say optimum extraction of a renewable resource. So, the last condition if you remember we derived this condition  $\dot{\rho} = \rho - \frac{\partial c}{\partial x} \dot{x} + \rho \int \frac{\partial g}{\partial x} dx = r \rho$ . And we said that this  $r \rho$  is basically the total benefit for preserving the resource for tomorrow, and it has several components.

The first component we say that this  $\rho$  is basically the capital gain then  $\frac{\partial c}{\partial x} \dot{x}$  this is the stock effect on the cost of extraction, and then the third component is the benefit due to biological growth. And we say that at steady state  $\dot{\rho} = 0$  and that implies at steady state  $\rho = \frac{\partial c}{\partial x} \dot{x} + \rho \int \frac{\partial g}{\partial x} dx = 0$ , so that means,  $r \rho + \frac{\partial c}{\partial x} \dot{x} - \rho \int \frac{\partial g}{\partial x} dx = 0$ .

And at steady state we also know that growth of extraction  $\dot{x}$  is also 0, so this is let us say condition 1 at steady state, and  $\dot{x} = 0$  implies that growth stock dividend growth equals to rate of extraction. So, this is also these two I will say that these are the two condition 1 and 2 that happens at steady state. Now, from these steady state condition today, we are going to introduce another concept, which is called catch locus.

Let us say that you will introduce catch locals with a simple diagram. Let us say that this is the revenue curve,  $r$  curve, revenue that you will generate by catching fish. And let us say that, we have this is let us say cost curve, cost of extraction that depends on your rate of extraction and let us say your stock.

So, from this curve, what you can see, that cost curve is a function of two things, cost of extraction and the rate of extraction and the stock. If you have more stock then obviously, cost would be less, if you have less stock then cost would be more. Suppose keeping rate of extraction constant stock of the resource let us say this is  $x = x_1$  stock of the resource increases.

So, what will happen? When stock of the resource increases, so obviously, cost curve will come down, so this is a cost corresponding to  $y$  but  $x = x_2$  where do we assume that  $x_2$  is actually greater than  $x_1$  this is the assumption. And the difference between cost and the revenue is a profit. So, actually we can draw different harvest cost function based on the

stock. So, we can draw different harvest cost or cost of extraction we call it harvest, harvest cost function this is  $c$  is actually harvest cost function. You can say this is  $c_1$  this is  $c_2$  you can call it harvest cost function depending on stocks.

So,  $c_1$  is the first harvest cost function  $c_2$  is the second harvest cost function. Now, let us assume that along the optimum harvest path, what you will get  $p_t$  price should be equals to  $\rho_t$  plus  $\Delta c / \Delta y_t$ . So, price equals to marginal cost plus the shadow price that is derived from the first order necessary condition we derived earlier that along the optimum path or harvest price should be equals to cost of extraction plus the shadow price.

Now, let us suppose at point M and N let us say this is point M and this is point N. Suppose at points M and N the above condition is satisfied. That means, this condition  $p_t$  equals to  $\rho_t$  plus  $\Delta c / \Delta y_t$  is satisfied, the above condition is satisfied. So, what will happen? We will say, that, that means what you can write is that this is optimum harvest corresponding to M and this is optimum harvest corresponding to y.

So, in this axis we are measuring harvest, rate of extraction, here we are measuring cost, which is a function of  $y$  and  $x$ . So,  $y_1$  is basically the optimum harvest when  $x$  equals to  $x_1$ ,  $y_1$  star is optimum harvest when  $x$  equals to  $x_1$  and  $y_2$  is optimum harvest when  $x$  equals to  $x_2$ . Now, if we join the points like M and N, points like M and N where this optimality condition is satisfied then we will have a locus like this.

So, if we join point like M, N we will have point like this, and that is called catch locus. So, this is catch locus. So, what is catch Locas, then what do we have defined? See, first we have drawn the harvest cost function then depending on different type of different level of stock we will have different cost function.

And then we assume, along this harvest cost function there are points like M and N where this condition, this first order necessary condition is satisfied. M is the point on the first cost harvest function where this condition is satisfied, and N is the point where the FNC is satisfied. Now,  $y_1$  star is basically giving the optimum harvest to N stock equals to  $x_1$ ,  $y_2$  is the optimum harvest to N it is stock is  $x$  is  $x_2$ .

So, by joining this type of different optimum points what we will get, we will get a catch locus. So, catch locus is the locus which shows different level of catch optimum harvests

depending on the stock. So, these are all coming from connecting out the optimum harvest point.

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- By joining points like M, N and so on, we get a locus which shows different level of harvest based on stock  $x$ .
- This locus is called catch locus.
- If we assume cost of harvest is low / fishes are highly valued, the R curve will be very steep and cost curve will shift downward.

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Optimum extraction  
of a renewable resource

$$\dot{f}(t) - \frac{\partial C(x)}{\partial x(t)} + f(t) \cdot \frac{\partial g(x)}{\partial x(t)} = r f(t)$$

At steady state,  $\dot{f}(t) = 0$

$$\Rightarrow r f(t) + \frac{\partial C(x)}{\partial x(t)} - f(t) \cdot \frac{\partial g(x)}{\partial x(t)} = 0 \dots \text{--- (1)}$$

$$\left. \begin{aligned} \dot{x}(t) &= 0 \\ \Rightarrow g(x) &= y(t) \end{aligned} \right\} \dots \text{--- (2)}$$

Along the optimum harvest path  $f(t) = f(x) + \frac{\partial C(x)}{\partial g(x)}$  ... from F.N.C we suppose at points M & N the above condition is satisfied.

$y_1^*$ : Optimum harvest when  $x = x_1$   
 $y_2^*$ : Optimum harvest when  $x = x_2$

Graph description: The graph plots  $y(t)$  on the horizontal axis and  $C(x, y)$  on the vertical axis. It shows a curve labeled 'R curve' and a curve labeled 'catch locus'. Points M and N are marked on the catch locus. The horizontal axis has points  $x_1^*$  and  $x_2^*$  marked. A note on the left says: 'we assume  $x_2 > x_1$  as we can get different harvest cost depending on stock'. Another note says: 'At steady state,  $\dot{f}(t) = 0$ '. The catch locus is a curve that starts at the origin and increases. The R curve is a curve that starts at the origin and increases more steeply. The catch locus is labeled 'catch locus' and the R curve is labeled 'R curve'. The graph shows that as the stock  $x$  increases, the harvest  $y$  also increases, and the cost  $C(x, y)$  increases.

So, what we will write here by joining points like M and N we can get a locus, by joining point like M, N so on, what we will, we will get a locus, which shows different level of harvest different level of harvest based on stock. So, this locus is called catch locus.

So, if we assume that cost of harvest is low or we can assume that fishes are highly valued then what will happen the revenue curve or the r curve will be very steep because fishes are highly valued. That means, the price of the fish is so good and cost of harvest is very low, so,

the revenue that you generate out of this fishing that would be too much revenue would be generated then the revenue will be very steep.

And what will happen, that very steep and since the cost of harvest is low, cost of harvest is also low so cost curve will steep downward. And if the cost curve steeps downward, we can under we can go back to the diagram. So, what we are assuming that cost of extraction is low because stock is increasing, so this curve will shift downward.

And as this curve cost curve shifts downwards catch locus will shift upward because you want to have more and more harvest. From the diagram also if these curves again comes down, so this point will be sifting upwards.