Environmental & Resource Economics
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Dynamic Optimization and Renewable Resources Part - 4

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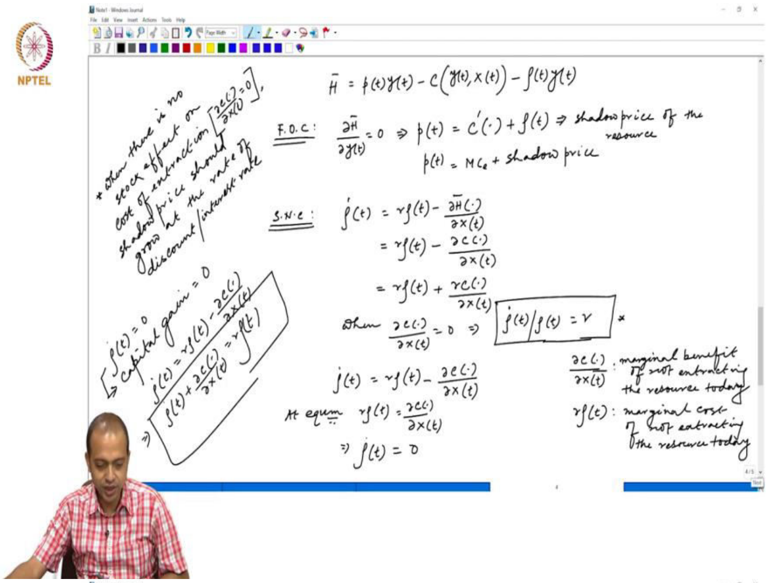

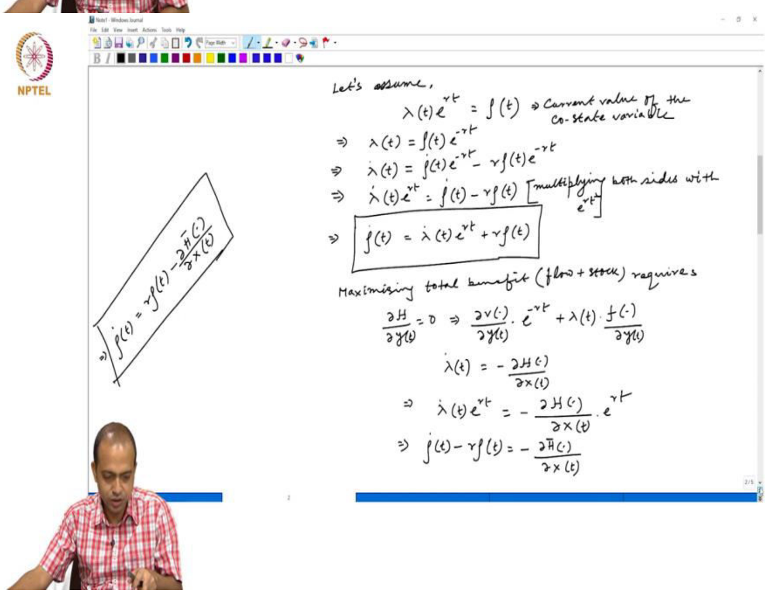

The current value Hamiltonian H bar would be pt into yt minus c into yt xt minus rho t into yt. So, this is very simple this is net revenue but in the context of a non-renewable resource apart from the cost of extraction what we said that since this resource cannot be replicated so easily apart from this cost of extraction. We need to subtract the opportunity cost of extracting the resource today. That is what we say.

So, now, first that condition is, what is the first order condition differentiating this function with respect to yt and set equals to 0 and that implies what you will get pt equals to c plus rho t, this rho t that means price equals to marginal cost of extraction plus rho t where rho t is known as shadow price of the resource.

So, that means price equals to marginal cost of extraction what we can write plus shadow price pt equals to this. Now second order necessary condition is what we have derived already what is the secondary order necessary condition rho dot t equals to r into rho t minus del H dot del x t which we have derived earlier if you go back I can just show you this condition rho dot t equals to r into rho t minus del H bar del x t this is the condition.

Now from here what you can write r into rho t and these del H bar del xt if you differentiate this function with respect to x t. So, that means x is appearing only here in the c function what I will get del H bar sorry, this I will get del c del x t because there is no other place where x is happening x is only here. So, this is we can think of del c del x itself is negative because this is called stock effect on cost of extraction.

So, that means we can write this is equals to r into rho t plus del c del x t. When del c del x t equals to 0 what you can get that rho dot t divided by rho t equals to r. So, that means, when there is no stock effect on cost of extraction shadow price should grow what is rho dot t divided by rho t this is growth of shadow price.

So, when there is no stock effect on cost of extraction shadow price should grow at the rate of discount. So, this we can write from here what I can write that when there is no stock effect on cost of extraction that means, what I am saying del c dot del x t equals to 0 shadow price should grow at the rate of discount or interest rate.

Now, this condition rho dot t equals to r into rho t this we need to again examine. So, rho dot t equals to r into rho t plus del c dot del x t this condition we need to analyze. So, here these del c dot del x t what we can think of this is stock effect on cost of extraction that means, indirectly we can say this is marginal benefit of not extracting the resource today. If I do not extract the resource today, then only the stock will be more in the next period and cost of extraction would be less very simple.

More stock means less cost of extraction. So, this del c dot del x t then we can think of marginal benefit of not extracting the resource today and what is r into rho t, see rho t we said this is shadow price. So, that means this is some kind of price of the resource we can think of. Since there is no direct price we are thinking up the shadow price concept here of the resource.

If I extract the resource today and keep the money in bank then bank will give me some interest and what would be the total interest gain r into rho t. Because one unit of resource the price is rho t at t th point of time. So, one unit of resource if I extract today, I will get rho t amount of money and that same rho t amount of money if I keep it in bank, then bank will give me a rate of interest r so the total gain from that is r into rho t.

So, that means when I am not extracting the resource today I am sacrificing this amount. So, that means we can think of r into rho t is marginal cost of not extracting the resource today. So, this is marginal benefit of not extracting the resource today and this is marginal cost of not extracting the resource today at equilibrium these two things should be equal.

So, here what you can write we can say that this condition we can write as negative also as the original this thing. So, at equilibrium r into rho t should be del c dot del x t which implies rho dot t should be 0. Now, what is rho dot t so, rho dot t when I am saying rho dot t equals to 0. What is the meaning of this see rho dot t we can think of change in price that means, borrowing from the finance literature we can say that rho dot t is capital gain.

So, that means, at equilibrium capital gain is 0 if there is a capital gain then the resource owner will always try to reallocate the resource from today to tomorrow instead of extracting today they will extract it tomorrow. So, then there would be some kind of benefit. So, this capital gain that means, from this equation rho dot t equals to this when I am saying rho dot t equals to r into rho t minus del c dot del x t from here what I can write rho dot t plus del c dot del x t equals to r into rho t. Now, this condition rho dot t I will write it again in the next page.

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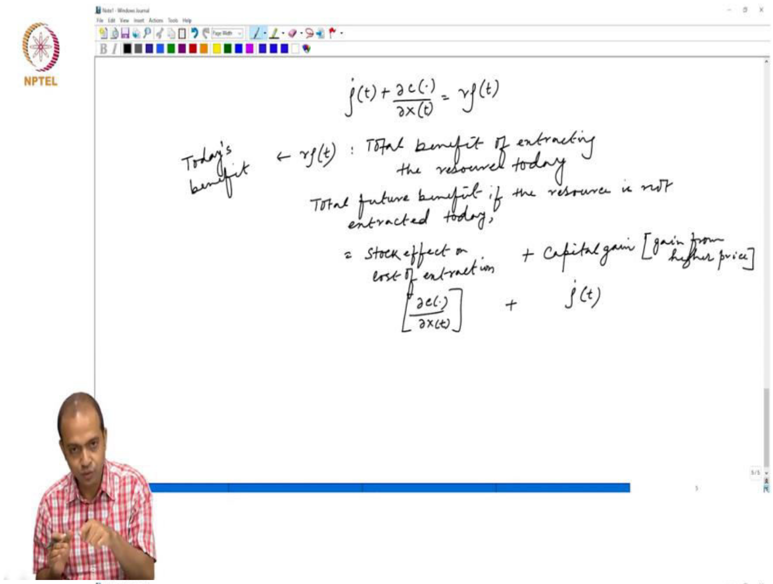

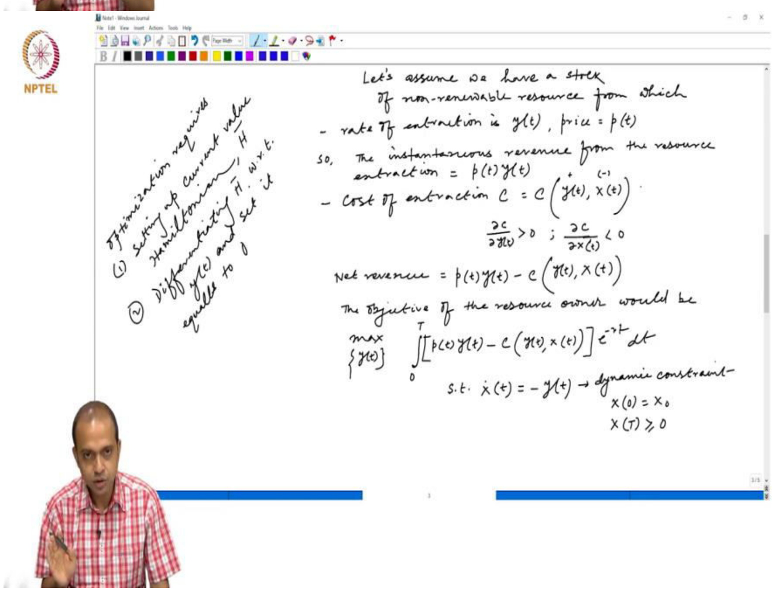

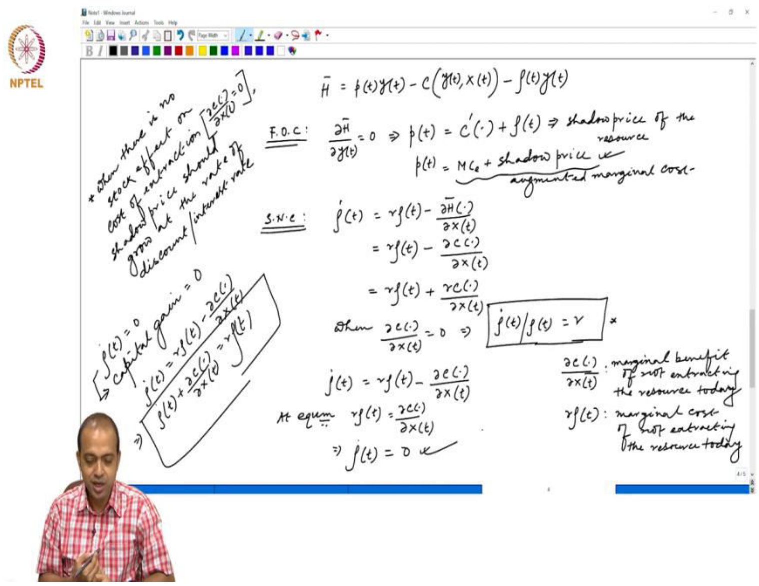

Rho dot t plus del c dot del x t equals to r into rho t. So, r into rho t is total benefit of extracting the resource today. How, because if I extract the resource rho t is the money what I will get I will keep it in bank. Bank will give me an interest at the rate r that is the reason r into rho t is total benefit of extracting the resource today.

Now, total future benefit if the resource is not extracted today is a summation if I keep the resource for tomorrows use, what will happen tomorrow there will be more stock and that will give me an impact on the cost of extraction which is called stock effect on cost of extraction.

So, I will get this benefit stock effect cost of extraction which is denoted by del c del x t plus there would be another benefit if I do not extract the resource today and keep it for tomorrow there would be another benefit what is the benefit tomorrow, we may expect that price of the resource will increase. So, that is called capital gain.

So, this is gain from higher price capital gain and that is denoted by rho dot t. So, that means at equilibrium what is happening, today's benefit is equals to tomorrow or future benefit. Today's benefit is this, this is today's benefit, given by r into rho t and tomorrows benefit is summation of these two benefits stock effect on cost of extraction and capital gain denoted by rho dot t. So, this is a very simple rule that we have derived from the cost of extraction of a non-renewable resource we have derived the price condition.

So, that means, if we go back to the optimization problem, we said that this is the rest we have a stock of resource for which yt is the rate of extraction price is pt. So, the there is instantaneous some revenue if we extract one unit of resource that would be pt into yt and cost of extraction c which is a function of rate of extraction and the stock.

We assume the cost of extraction positively related with the rate of extraction that is why del c del yt is greater than 0 while del c del x t is less than equals to 0 and net revenue is pt yt minus c function yt into xt an objective of the resource owner is to maximize this benefit over a period of time that is why integration 0 to t pt yt minus this.

And optimization requires two steps to be involved first of all setting up current value Hamiltonian H bar and then we need to like any other standard optimization differentiating the current value Hamiltonian with respect to your decision variable yt and set it equals to 0. If we do so, if we follow these two steps.

So first step what we did, we set up the Hamiltonian and how do we have Hamiltonian this is simply net revenue minus opportunity cost of extracting the resource today rho t into it amount we have subtracted and rho t is we call shadow price of the resource. So, if you differentiate first order condition we will get price equals to marginal cost of extraction plus the shadow price over and above the marginal cost of extraction something should be added.

We have discussed initially and we said that that is equals to user cost whatever you can think of and everything taken together this is called augmented marginal cost. Marginal cost is augmented by an user cost and the user cost or shadow price these are all similar concepts earlier what we defined as user cost here in this context I am saying it shadow price.

So, marginal cost of extraction plus the shadow price that is what we got from this condition in the context of non renewable resources. Since, it is not easily replicable optimum price should be determined by this rule, where price equals to marginal cost of extraction plus the shadow price and the shadow price concept is equivalent to marginal user cost MUC and taken together this you can think up augmented marginal cost. And the second order condition, we have already derived earlier rho dot t equals to r into rho t minus this.

So, that means, we need to differentiate that this function current value Hamiltonian with respect to x and if you do so, x is appearing only here in the cost function. So, that is why rho dot t minus this since this is negative we made it positive you can keep it as it is and we said that when this is actually 0 del c dot del x t equals to 0 then shadow price should grow at a rate of discount a very simple.

Because, if you do not keep the resource then there would be a capital gain price would be changed and what is a rate of change price rho dot t minus this and overtime at least that price would grow with the rate of interest otherwise, what is my incentive to preserve the resource. So, rho dot t minus rho t the change in price should be at least equal to r that is what we say.

And from there we derive rho dot t equals to r into rho t minus this, this you can conveniently keep positive or negative if I keep it negative then also it will become positive because this is negative. So, at equilibrium these two are these equals to this r into rho t equals to this, what is del c del xt this is stock effect on cost of extraction which is nothing but marginal benefit that means, if I do not extract that resource today, keep it for tomorrow. Tomorrow only I am going to realize the stock effect on cost of extraction due to higher stock.

So, that means that is marginal benefit or not extracting that today. But if you do not extract the resource today, there is a cost also, what is the cost you will lose some amount of interest which you could have earned by extracting the resource today and keep the money in bank and what is that amount r into rho t that is why marginal cost of not extracting the resource today at equilibrium these two are equal and that makes rho dot t equals to 0.

So, that means the resource owner is indifferent when there is no capital gain. So, this condition rho dot t equals to this then later on we said that what we derive this condition rho dot t plus del c dot del x t equals to r into rho t it gives, it tells us that at equilibrium todays benefit of extracting the resource should be equals to tomorrows benefit if it is not extracted today.

So, todays benefit is r into rho t because I will extract the resource get the rho t amount of money keep it in bank. Bank will give me interest at r so r into rho t is todays benefit and total future benefit if I do not extract the resource today is two components. Firstly, I will have some stock effect which is given by del c dot del xt and I will have some capital gain. So, that is what we learned from today’s discussion. Thank you.