Environmental and Resource Economics
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Natural Resources Economics and Dynamic Optimization Part - 6

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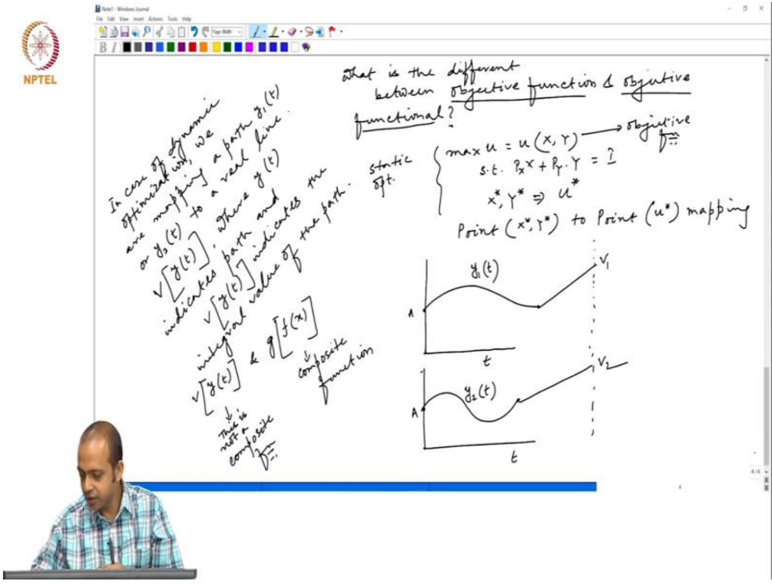

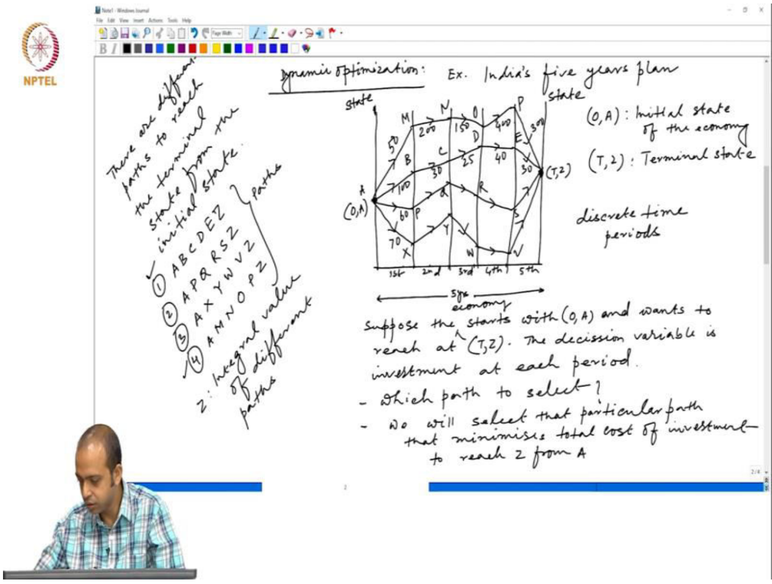

So what is the difference between function and functional that we need to understand. What is the difference between objective function and objective functional? Now, when you are trying to this objective function is basically, relevant as I said in case of static optimization, for example, when you are maximizing u equals to u function X and Y subject to P X into X plus P Y into Y equals to I. So here you are getting X star and Y star and that will lead to u star.

Now, if you look at this X star and Y star if you put then you will get u max. So that means here X star and Y star is some amount of quantity may be 5 unit of X and 10 units of Y and this is also some amount of utility, which can be measured. So that means in case of static optimization, this case of static optimization, there is a, this is a point to point mapping, point; X star Y star is point to point mapping.

So static optimization basically involves point to point mapping. So that means I have purchased 10 units of X and 15 units of Y this 10 and 15, these two points actually, one particular consumption bundle. One particular consumption bundle is mapping towards another particular utility. So that is called point to point mapping. That is why, this is called objective function. So this maximize u. This is called objective function.

But here in the context of dynamic optimization, if you look at, if you go back to the diagram, what we are mapping here? We are actually mapping a path this A B C D E and Z. We are mapping a path to a real line. We have several paths, to achieve this terminal state. So that means this Z is basically, the sum total of this path, that means integral value of this path.

You have invested different amount at different time period and at the end you are getting Z. So you can think Z is basically the integral, so what you can say Z is basically the integral value of different paths.

What are the paths? A B C D E Z, A P Q R S Z, A X Y W V Z. So these paths are actually, mapping in a real line. This is the real line, where we are achieving Z amount of capital stock. So obviously we can think that Z is the integral value of different paths. So that is why in objective, in dynamic optimization, we are mapping a path to a real line.

So in dynamic optimization, what is happening? We are mapping a path. For example, let us say, this is, let us say, in this axis we are measuring T and here this is the initial state A. So this is a path, which is denoted by let us say, Y 1 T and let us say, this is a real line. So this value goes to V 1. Let us say V 1. Then we have another, we may have another path. And if you map this, then this goes to V2, let us say, this path is Y 2 T.

So we are basically mapping the path into a real line. If you follow this path, then you are achieving V 1. If you are following this path, this is also from here, you are achieving V 2. This is a real line. There are different numbers V 1, V 2, V 3, like that. So here that means, in case of dynamic optimization, we are mapping a path either Y 1 T or Y 2 T to a real line. A real line, which is basically V.

So real line that means, what is that real line? V of Y T. So V of Y T is basically the integral value of this path. Where Y T indicates path and V of Y T indicates the integral value of the path. Now this V of Y T, it looks like a composite function. V of Y T that means if you think of this V of Y T and let us say you are thinking of g of f X, they are not same. They are not same.

Here basically, what we are saying? g of f X is a composite function. But this is not a composite function rather, this is the integral value of Y T. You understand? Here g is again a function of f X. So ultimately g is also a function of X. But here I am not saying, V is a function of Y T, Y is a function of T, and thereby, V is also a function of T. No. Rather I am saying V is basically the integral value of Y T. Total Y T is mapped into the real line.

So here this is a composite function. But it is not. This is not a composite function. So do not get confused with this V of Y T and do not think it is just like g of f X. g of f X is a composite function. Where f is a function of X, g is again a function of f X. So ultimately g is also a function of X. Here V is only a function of Y T. That means we can only say, V is the integral value of Y T. Which path you will select, depending on that, we will either get V 1 or V 2. That is why V is a function of Y 1 T or V 2 is a function of Y 2 T.

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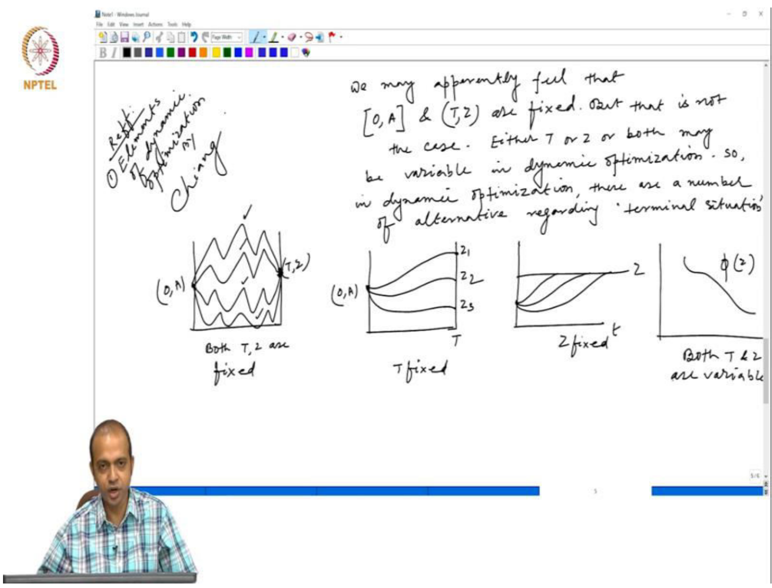

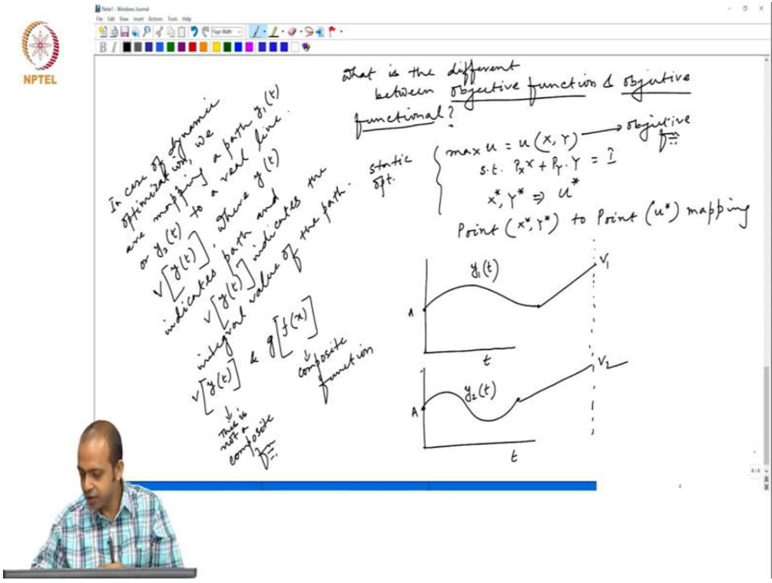

Now, from this discussion it may look like, from this discussion it may apparently, we may apparently feel that 0 A that means the initial state and T Z are fixed, but that is not the case. 0 A and T Z, in this example, what we have assumed that is basically a specific example.

But this is not the case, either T or Z or both can, both may, vary, may be variable in dynamic optimization. And depending on, whether T or Z or both are variable, we will get different context. So in dynamic optimization, there are a number of alternatives regarding the terminal state or terminal situation.

So let us take example. So you are starting with here, 0 A, and let us assume that my T is fixed. This is T. T is fixed. So T is fixed here. It is only let us say, 5 years, 10 years, or 15 years. Now, in this terminal state, I am telling you that you start with A amount of capital stock, I am giving you five years time and you achieve wherever you can.

So, if T is fixed then what is the variable here? You can achieve either here, which is Z 1. You can achieve here, which may be Z 2. Or you can achieve here, which is Z 3. So depending on, which particular path we will select, how you will invest, you will achieve at Z 1, Z 2, Z 3.

In the previous case, that is a standard example, where T and Z both are fixed. I have given T years time, I am giving you, telling you that this Z amount of capital stock, and then you are selecting your, by way of your decision variable, you will achieve that terminal state T Z, you are trying to minimize this.

But here our objective is T is fixed, and depending on, how will travel, you will achieve as Z 1, Z 2, Z 3. So we have to see that depending on the values of Z 1, Z 2, Z 3, what is the maximum, where we can achieve at minimum cost. This is one example. Let us say there is another example, where Z is fixed. T is not. So this is let us say, Z. And you are starting from here. So everyone will achieve Z only. Z is fixed.

So what we need to do is, in this axis this is time, so I will select that path that minimizes time. So which path should I select so that I can achieve the terminal state of capital at minimum amount of time? That is, that might be also a case. So the standard case, what we have discussed there is this. 0 A and T Z. So here both T and Z are fixed.

So that is why, you are selecting several paths like this. There might be another case, where basically, both T and Z are variable. So we will not discuss this case actually. So we will generally take the simple case, where T and Z both are fixed. But you must keep in mind that this is not the only case in dynamic optimization. Depending on your objective, we may get several cases. So that means there are different types of terminal situations in dynamic optimization.

So that means this today's discussion basically gives you a basic starting point of dynamic optimization, which is quite different from the static one. In static optimization basically, we are trying to optimize at a single time period, if it is a consumption, then we are basically, trying to optimize the consumption of X and Y, so that our utility is maximum at that point of time.

But if you think of you are maximizing a lifetime utility, then each and every time period, we need to optimize our consumption bundle, so that at the end of your life, you feel that my lifetime utility is maximum. So that lifetime utility maximization requires dynamic optimization technique. And the example, what we have taken? In an economy starts with the initial stock of capital, wants to achieve terminal stock of capital and decision variable is investment at each period of time, at each year, so that I can achieve the terminal state of capital at minimum cost.

Then we discussed about some important elements of dynamic optimization, which are initial state and terminal state. Then a decision variable. We must always identify, what is the decision variable? In this case the decision variable is, how much to invest? In the context of utility maximization, how much to consume? How much to consume? So these are all decision variables.

When this is a resource, let us say, you have one barrel of oil or one ton of coal, the decision variable would be optimum rate of extraction. At each period, you want to extract some amount of resource, so that your intertemporal benefit that you derive out of that resource, is maximum. At each year, you have to decide, whether you need to extract 5 tons of coal or 10 tons of coal, depending on the market condition, so on and so forth.

Depending on the market condition, you need to optimize at each and every period. So that at the end the sum total of the benefit is basically maximum. So you need to have a decision variable. Then there are different paths to achieve the terminal state. And we must have one objective functional, which is quite different from objective function.

Objective function is relevant for the static optimization but objective functional is relevant in the context of dynamic optimization. Since static optimization is in one sort game, there basically, you are mapping a point to another point, depending on your X star and Y star, you will achieve at u star, which is point to point mapping.

But here in the context of dynamic optimization, you are basically mapping several paths to a real line. I will select that path, whether this path or this path or this path or this path, depending on my terminal state. So that means which path will help me reaching T Z at minimum cost. So that means here, in terms of this diagram, you can easily understand, here you are mapping a path to a real line, depending on the values of V 1 or V 2, I will select either Y 1 T or Y 2 T.

So that is why, here, it is V of Y T, in the context of dynamic optimization, which is quite different from g of f X, that is what I said. While g of f X is a composite function because f is a function of X and Z is a function of f X, so ultimately Z is also a function of X, which is not the case here because here I am mapping the entire Y T, the entire path to a real line.

So this is basically the introduction to dynamic optimization. And the reference for this would be, first one reference is Elements of Dynamic Optimization by Chiang. But the textbook for dynamic optimization is mathematically quite involved. Little difficult to understand. That is why, what I will do? I will try my level best to collect the specific information from that Alpha C. Chiang, another book, to make it more lucid, which should be, which you can understand easily.

So that is why for this dynamic optimization, discussion on dynamic optimization, I suggest, you fully concentrate on this and try to understand from my lecture only. Once you are thorough with this lecture, then you can just look at this Chiang's reference on Elements of Dynamic Optimization.

But this technique you must learn, which would be useful in your future career. This is a quite interesting and useful technique but difficult to understand from a textbook. That is why please follow my lecture. Wherever you have doubt please ask me then and there. Thank you.