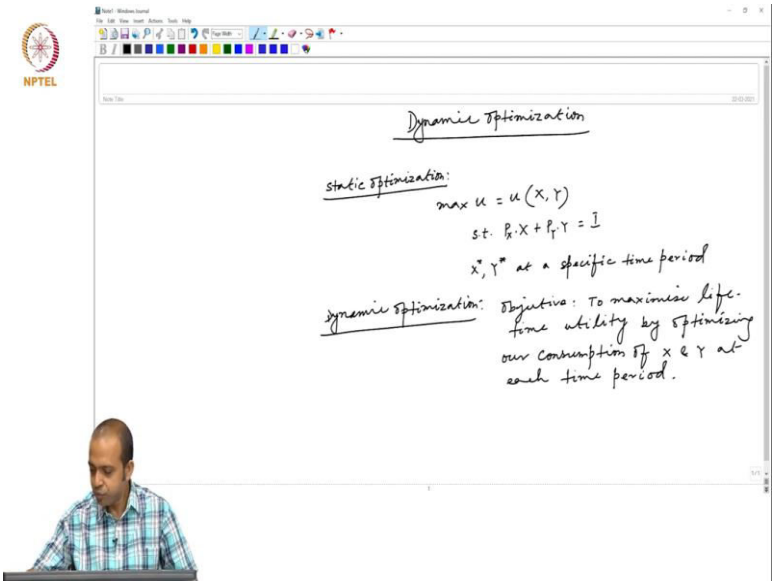
Environmental and Resource Economics
Professor Sabuj Kumar Mandal
Department of Humanities and Social Sciences
Indian Institute of Technology Madras
Natural Resources Economics and Dynamic Optimization Part - 5

Welcome to our discussion on Resource Economics. So if you recall in our last class, we were talking about the optimal extraction of a non-renewable resource and particularly, we are talking about the price path. Price path and then we said that as T tends to infinite, then price of the existing resource actually, apparently, it we may feel that price of the existing resource will become infinite, as a sufficiently longer period of time.

But that does not happen in reality, because of the availability of the backstop. Basically if you think the price path means, determining the price at each and every period, which generally comes from the optimum extraction of your resource, at each and every period of time, so that your intertemporal benefit out of that resource extraction is maximum.

So that means here basically instead of getting optimum price at one point of time, it is basically a price path that we need to determine. And to get the price path, the static optimization, which we have learned so far, does not work. What you need to know is, basically dynamic optimization and today we are going to talk, briefly, on dynamic optimization. The next couple of lectures we will be fully concentrating on dynamic optimization.

(Refer Slide Time: 02:02)

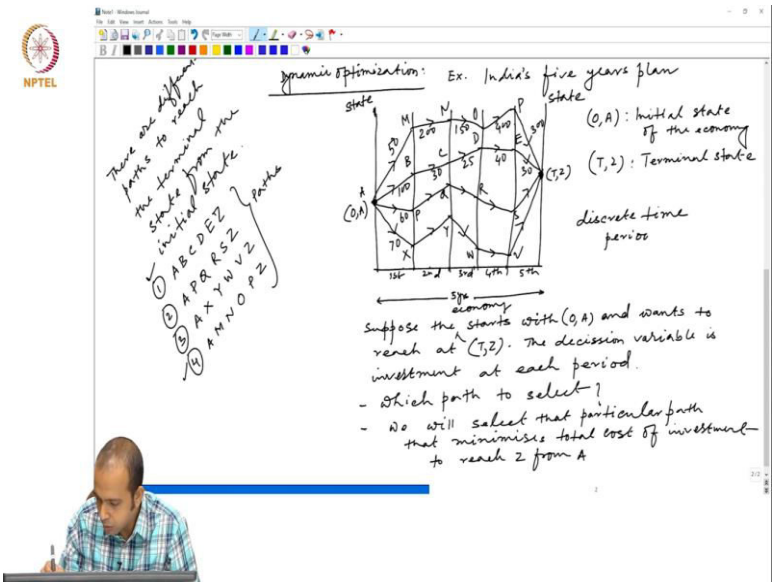


So this is dynamic optimization that we are going to start from today. Dynamic optimization. So let us first try to understand, the difference between the static and dynamic optimization. What is a static optimization? So static optimization, let us say, that a consumer is trying to maximize his or her utility, where so this is basically maximize utility, which is a function of two commodities X and Y and subject to the constraint, which is P of X into X plus P of Y into Y equals to your total income.

So here, you are basically trying to maximize your utility at one point of time and you will get ultimately your X star and Y star. These are all optimum consumption of X and Y at a specific time period. But now suppose, our objective in a dynamic optimization, what we need to do, in dynamic optimization our objective here is to maximize lifetime utility, by optimizing our consumption of X and Y at each time period. So this is called maximizing lifetime utility.

So that means you can easily understand, if we assume that we are going to live for, let us say, 70 years or 80 years, so during this entire 70 or 80 years, we need to decide, how much to consume, how much to invest, how much to save, at each and every time period each, and every year, so that my lifetime utility is maximum. That is what is called dynamic optimization. So obviously you can understand the process is much more complex than the simple static optimization, where you are basically interested in optimizing your utility at one point of time.

(Refer Slide Time: 06:20)



So in dynamic optimization, I will give you an example, better example of dynamic optimization. Let us assume that, for an example, we can think of, let us say, that Indian economy is thinking about its five years plan, next five years, Indian economy is planning for, what the economy will reach after five years time, in terms of capital stock. Because economic growth depends on the capital, is one of the major important factor as we all know.

So the economy is trying to achieve a certain stock of capital after five years. And this entire five years is divided into five small-small time period. One period means one year. So that the economy is trying to invest, decide about optimum investment at each and every time period, so that at the end of the fifth year, the economy will achieve a certain level of capital stock. And the cost of such investments should be minimum. That is what is called dynamic optimization.

So example is, let us say India's five years plan, actually the planning commission on plan planning is not there, in the context of Indian economy, before this 'NITI Aayog' came, there is a planning commission and they used to plan for each and every five years, basically to decide, how much to invest, how much to save, so on and so forth, for the economy to achieve a certain level of capital stock, to achieve a certain targeted level of growth, whatever you may think of, at the end of five years.

So let us assume in a simple diagram, this is, this entire period is 5 years. This is the duration. This is 5 years. So these two vertical lines, they indicate state of the economy. So you may think of, this is initial state and this is terminal state. Initial mean to start with and this is the terminal one. So this five year is divided into, let us say, 1, 2, 3, 4. So this is one year, two year, three year, four year and five year. So this is first plan period, let us say, this is second, this is third, this is fourth, and this is the fifth. So I have divided these five years into small-small, one year periods. This is called initial period.

So let us say that to start with economy is here, which is denoted by 0 A. This is called initial state. 0 indicates 0th period that means first period and A indicates the initial capital stock. So that means we can say that 0 A basically indicates initial state of the economy and let us say that economy is trying to achieve here denoted by T Z, so that means T Z indicates terminal state. So T indicates time and Z indicates capital stock at terminal state.

So economy started with A amount of capital, at the initial state and then economy is achieving Z amount of capital stock, at the terminal state and after T th period. So 0 A and T

Z they are indicating initial state and terminal state of the economy. Let us also assume, so that means what we are saying suppose, the economy starts with 0 A, and the economy wants to reach at T Z.

And what is our decision variable? The decision variable here is investment, investment at each period. Now there are many ways by which we can actually reach this T Z, for example, let us say, this is one path, and let us call this path as, let us say, A B C D E and then this is let us say, Z. There is one more path, another path, let us say, this is another path, and let us say, this path, is named as A P then Q then R then S and then T.

There might be another path here, and there might be another path like this, let us say, that this is M N O P and T. This is X Y and then we can say that X Y W V and T. So basically that means depending on, how much amount you invest at each and every period, you will get any one of this path, so that means, how will you name this path? So there are different paths to reach the terminal state from the initial state. And what are those paths?

These paths are named as, let us say, the paths are, let us say, A B C D E and let us say T. Or let us say this is instead of T this is Z. This is where we want to reach, this is one path. Second path is, let us say, A P Q R S and Z. Then 3 is A X Y W V and Z. And path 4 is A M N O P and Z. These are different paths.

Now the question is then, which path to select? This is the question. So each period, the economy is deciding about optimum investment, and that investment involves some amount of cost. Let us say that the initial cost for this path is rupees 50. This is 100. This is 60. And this is let us say 70. So which path the economy will select?

Apparently, it may look like the economy will select this path A M that means path 4. Because the cost from travelling to A to M is rupees 50. But our objective is not to take the path based on minimum cost of one destination to another destination, its immediate destination, rather, what we need to do is, even though this path involves minimum cost initially, it may so happen that for next periods the costs are too high.

So when you add the total cost of traveling A to Z that is the highest even though initially it involved 50 rupees. So the economy, the social planner, is having a broader vision, rather than the myopic vision here. So instead of selecting the path based on a particular cost, what

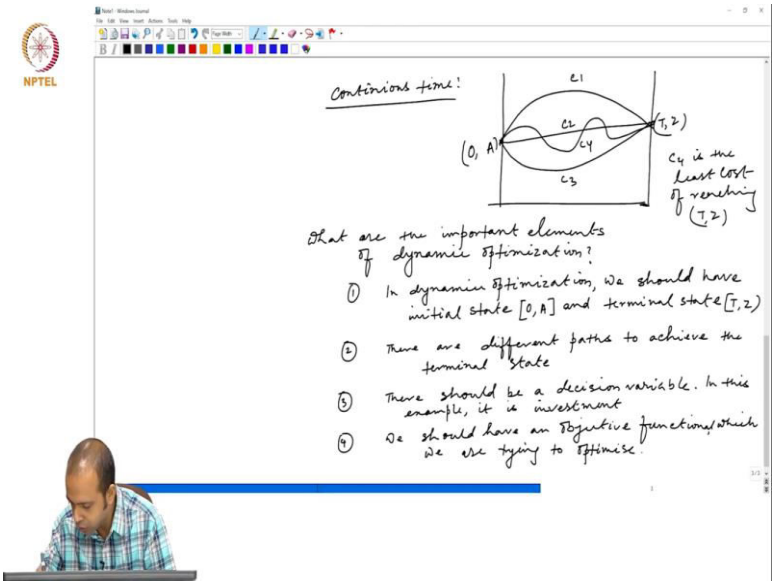
we need to select is, what we need to decide, we will select the path, which minimizes the total cost of traveling from A to Z.

So in this case, even though this is 100, let us say, the other components are very minimum. So in that case this A B C D, this component, might give the minimum cost of travel from A to Z. So we will select that particular path that minimizes total cost of investment to reach Z from A. This should be clear. This is the idea of dynamic optimization.

So at each period, we need to select optimum investment, so that at the end of the day, when we reach our T Z that means economy achieves Z amount of capital stock, my total cost of investment is minimum. I am not guided by a myopic vision. If you are guided by myopic vision, so obviously, you will select this path A M N O P and Z. Because here the cost of travel from A to M is minimum.

But I am not only travelling from A to M, I have to reach to A to Z. That is why total cost should be minimum. That is the idea. So here time period is considered as discrete. This is a discrete time period. If you assume time to be continuous, then what will happen?

(Refer Slide Time: 22:33)



In case of continuous time, the diagram will look like this. So you are here. So 0, A, and you will reach here T Z. So either you can travel directly from here or you can travel directly from here, this is here or let us say here. So let us say this is C 1, this is C 2, this is C 3, and this is C 4. So obviously the C 4 is the least cost of achieving T Z. C 4 is the least cost of reaching T Z. Because that is the straight line.

So obviously a line, straight line, which connects between A and Z, the distance should be minimum than the other paths. So this is what, is the case in case of continuous time period. So that means in dynamic optimization, what are the important elements? What are the important elements of a dynamic optimization then from this discussion? What are the important elements of dynamic optimization?

So firstly in dynamic optimization, we should have initial state which is denoted by 0 A, and terminal state denoted by T Z. Then there are different paths to achieve the terminal state. Then there should be a decision variable. In this example, it is investment. Then we should have an objective functional which we are trying to optimize. Now look at here we are saying objective functional this is called functional.