Environmental and Resource Economics
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Lecture 42
Qualitative Response Models - Linear Probability Model, Logit and Probit Models Part - 3

So, welcome once again to our discussion of qualitative response model, that we were discussing in our last class. So, we will continue again, the qualitative response models today also. So, in our last class if you recall, we discussed basically, we started with our discussion with the linear response, linear probability model.

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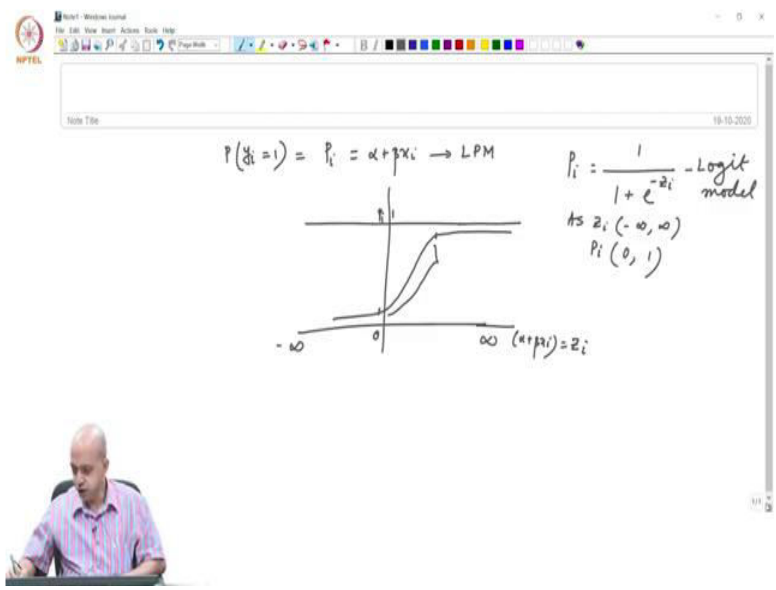

And we said that, the linear probability model, that takes this form Pi equals to alpha plus beta xi. So, this is, what is this Pi? Pi is basically probability that yi equals to 1, that is Pi. And then we said that, this linear probability model, or in short LPM, what is the major limitation of this? Here the probability is modeled as a linear function of x, linear function of x.

So, that means if you think about the house ownership problem, that we are discussing in our previous class. So, what happens actually, when the individual's income is very low, in that range almost all the people they do not have a house actually. So, at lower income, people do not have house, almost all of them. And at a higher level of income, they will, almost all of them will have a house.

But then once you achieve that level of income, then probability of owning a house, that does not change actually. For example, when your income is 1.5 lakhs per month, then you have a house.

And that probability of owning a house at that income range is almost 1. But suppose now income is increasing from 1.5 lakhs to 2 lakhs, then once you have the house, then you cannot buy that you, that particular individual would not buy any new house.

So, that means basically it says, once you achieve that level of income, probability does not change, it will almost 1, and at lower level of income nobody is having a house and at the lower level of income, when your income is let us say 5000 per month to 5500, 5600 like that probability does not change that much.

So, at the lower end and at the higher end, it is constant and it changes in between. So, that means a linear characterization, linear characterization of probability is a much problematic thing in this context. So, what we actually want, if you plot your probability in this way, let us say, this is 0 and this is Pi, this is let us say minus infinity, this is plus infinity.

So, what, our probability should, should be like this, it should behave in this way. And this is what is called a sigmoid S curve type relationship, this is a sigmoid S and to capture this type of non-linearity, non-linearity, so that means in this axis I am measuring let us say alpha plus beta xi, alpha plus beta xi. It ranges from this to this, and and this is equals to Zi, this is equals to Zi.

So, Zi, basically ranges from minus infinity to plus infinity. So, this is Zi and what do you want is the relationship of Zi and Pi, like this, at the lower end it will almost 0, but it will never touch 0. Here it is 1 actually, it will approach towards 1 at a higher level of income, but it will never touch 1. So, basically it asymptotically approaches 1 and 0.

And after that suppose from this portion it almost constant, here also once you achieve here, it almost constant, it is not changing. And it is changing in this particular this region, like this region. So, to overcome the problem of linear characterization of probability with Zi in logit model, what we assume that Pi equals to 1 by 1 plus e to the power minus Zi, Zi.

And from here you can understand as Zi, as this model ensured as Zi, ranges from minus infinity to plus infinity, then your Pi will become 0 to 1, that is the advantage of this model, that is the advantage of this logit model, logit model. Is it clear? So, I will repeat once again, this linear probability model, it assumes probability, is a linear function of x, here x is income, linear function of x, or you can consider alpha plus beta xi, entire thing is Z.

So, it is a linear characterization between Pi and zi. But in reality, what happens is that probability does not change linearly, when income changes from 15,000 to 20,000, the change in probability is not same, when income changes from 1 lakh to 1,20,000. Probably, when income changes from 1 lakh to 1,20,000, you will observe either very, very insignificant change in probability of owning a house, or no change at all.

So, it only changes from 20,000 to 1 lakh, in that range, in this range actually probability changes, after that it constant. Similarly, at the lower end and to overcome that problem we hypothesize a non-linear characterization of probability of owning a house Pi, with the income xi. And that is basically the logit model, which is 1 by 1 plus e to the power minus Zi, and as Zi, ranges between from minus infinity to plus infinity Pi, will range between 0 and 1, that is how logit model overcomes, the problem, major problem of linear probability model.

But then you end up having a non-linear model Pi equals to 1 by 1 plus e to the power Zi, you cannot estimate directly this model applying the linear technique and that is the reason, we characterized that means we transformed the apparently looking non-linear model into a linear model by taking log and then we discussed how to estimate that model using the maximum likelihood estimates, or MLE, where OLS does not work, that is how we discussed about the linear probability model and the logit model.

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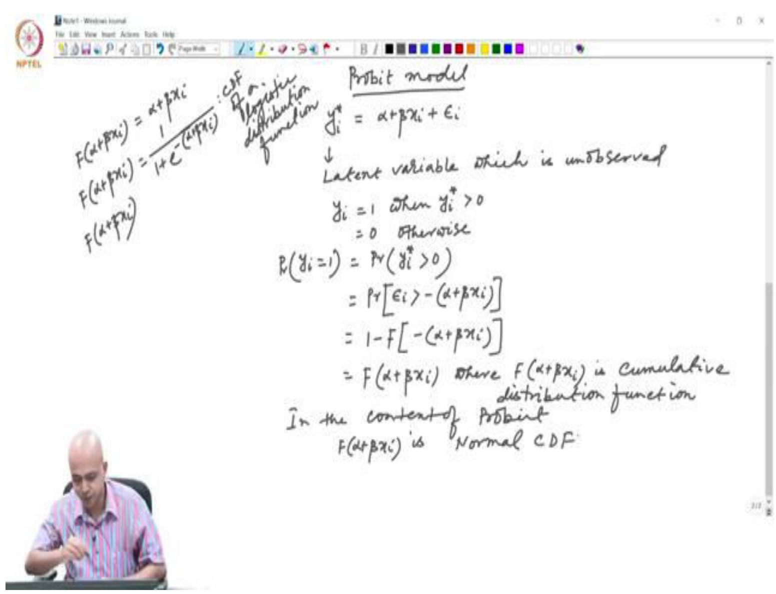

Now, today we will discuss another qualitative response model, which also characterize non-linear relationship between, between the probability and xi. And this models name as probit model, probit model. So, let us try to understand the theoretical structure of this probit model. Now, to understand the theoretical structure of this Probit model, we will introduce a variable, which is called latent variable.

Let us say yi star equals to alpha plus beta xi plus, let us say epsilon i, here yi star is called a latent variable, which is unobserved. And then there is a relationship between yi and yi star, how? yi equals to 1, when yi star greater than 0, 0 otherwise, 0 otherwise. Now, you might be thinking, what is this latent variable and how this, how can you get a relationship between yi and yi star, think about the house owning problem.

Given your income each and every individual calculate some amount of utility of satisfaction of buying a house, or buying a car, or anything. And you will observe, that individual has actually bought a house, when the individual derives a positive amount of utility, is not it, a positive amount of utility. If the utility is negative, then that means if there is dissatisfaction of owning a house at that level of income, then you will see that individual has actually not bought the house.

Now, you might be thinking what is the disutility of owning a house there, actually there is no disutility of owning a house as such, but at that level of income, when my income level is very less, let us say 10,000 and if I buy a house, how buying a house is not my priority at that level of income, because I have so many other important things to do. So, if I buy a house and then if I start giving EMI for that house, probably that will give a dissatisfaction.

So, each and every individual will calculate the utility, at that level of income of owning a house. Depending on the utility household will decide, or the individual will decide, whether to buy the house, or stay in rented apartment. But utility is something you cannot observe, what you observe is actually the decision. And what is the decision? Whether I have bought, or not, that is the realization.

So, that is why you cannot observe the utility, but you can observe the decision. Here yi is basically the decision, the ultimate realization, whether the event has happened, or not. But in between how and what amount of utility the individual has derived, that you cannot observe, and that unobserved utility, let us say we defined as, as yi star, yi star, it depends on your income, but then there is some amount of error term also, which makes the utility unpredictable, unobserved.

So, when y star is greater than 0, you derive a positive amount of utility and then yi equals to 1, 0 otherwise, 0 otherwise. This is the structure of the probit model, that yi is related to an unobserved variable yi star, which is called latent variable. Now, once you, once you hypothesize that type of relationship between yi and yi star, then what you have to do?

Basically, when you are calculating probability, probability yi equals to 1, that means you are saying in turn it is nothing but probability yi star greater than 0, because then only yi equals to 1. Now, from the relationship, from this relationship you can easily understand, when can you get yi start greater than 0. So, from this relationship, I can easily understand that yi star will become 0, greater positive when, when your epsilon i, is actually greater than negative of this alpha plus beta xi.

From this relationship it is very easy to understand yi star will become greater than 0, when epsilon i is actually greater than minus alpha plus beta xi. And if you recall the definition of probability density function from the properties of probability density function, we can write when epsilon i is actually, when epsilon i is actually a random variable and this is less than which is greater than some alpha plus beta xi, then we can say that this is nothing but 1 minus F of minus alpha plus beta xi.

Which is nothing but F of alpha plus beta xi, F of alpha plus beta xi, that is how you can, that is how you can derive this one. So, this Fi, what is this F of alpha plus beta xi? This is actually, I will say that, this is actually where F of alpha plus beta xi is cumulative distribution function.

Now, what type of depending on, what type of specific cumulative distribution function this F of alpha plus beta xi will take, you will get either linear probability model, logit model, or probit model. What I am saying, this F of alpha plus beta xi, can take three different values, it can be a cumulative, linear distribution function, which is that means I can say that F of alpha plus beta xi can be simply alpha plus beta xi, or F of alpha plus beta xi can be 1 by 1 plus e to the power minus alpha plus beta xi.

And then you will get the logit model, and in the context of probit, this F of alpha plus beta xi, alpha plus beta xi takes this type of form equals to and this is called, this is actually cumulative

CDF of a logistic distribution function. So, this is basically, this is actually F of alpha plus beta xi, I will say that cumulative distribution function, or CDF, CDF.

So, in the context of logit, this is CDF of a logistic distribution function, distribution function. And in the context of probit, in the context of probit, this cumulative distribution function in the context of prohibit, in the context of probit, this F of alpha plus beta xi is actually the cumulative distribution function of a normal distribution. So, that means this is normal CDF.

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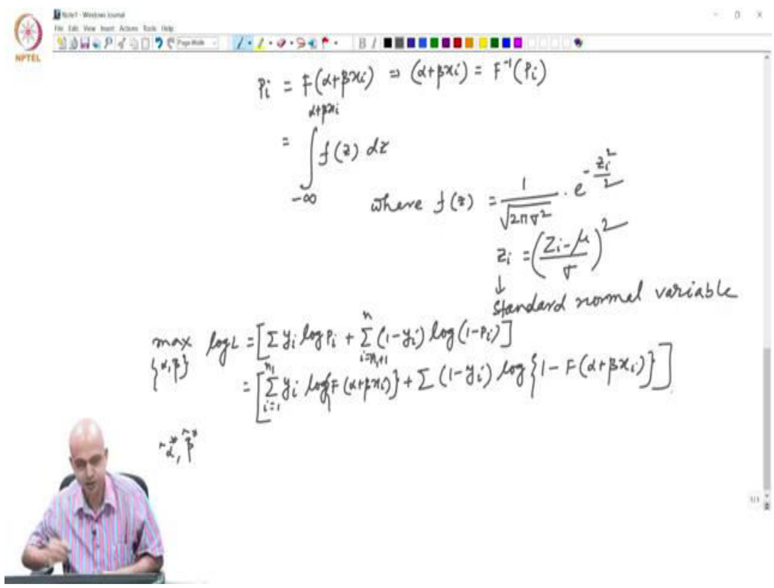

That means in the context of probit, what I can write that, this Pi equals to F of alpha plus beta xi equals to, I can write integration minus infinity to alpha plus beta xi, fz dz. And what is this fz? fz is basically a normal probability density function and I can write that, where fz equals to 1 by root 2 over 2 pi into sigma square into e to the power minus zi square by 2.

And what is zi? zi minus mu divided by sigma whole square, which is nothing but a standard normal variable, is this clear. So, that means here in the context of probit only difference, that it makes is F of alpha plus beta xi, takes the cumulative, since I am taking the integration of this fz, which is basically a normal distribution function.

When I am taking integration that becomes the cumulative density function of CDF. So, this is the CDF of a normal distribution function, where fz is root over 2 pi sigma square into e to the power minus zi square by 2. And how zi is defined? zi is defined as, zi, small zi by minus mu divided by sigma whole square, that means zi is basically a standard normal variable, standard normal variable.

So, if Pi equals to this, then from here you can say that, that means alpha plus beta xi equals to F of inverse Pi, that is how you can get. Now, if you will recall the log likelihood function, what we got in the context of logit, same type of log likelihood function you will get in the context of probit also, that means your log L, log L would become summation yi into log Pi plus summation 1 minus yi into log of 1 minus Pi, 1 minus Pi. And that you are trying to maximize with respect to alpha and beta. And this Pi, what is this Pi?

Pi equals to summation yi, i running from 1 to n1, here i running from 1 to sorry, n1 plus 1 to n and then this is log of what is Pi, Pi, is basically F of alpha plus beta xi, plus summation 1 minus yi 1 minus yi log of 1 minus, 1 minus this I will write log of 1 minus F of alpha plus beta xi. So, this is your log likelihood function in the context of probit, in the context of probit. And that you maximize once again with respect to alpha and beta. And then you will get your alpha star and beta star, you will get alpha star and beta star by maximizing this.