Environmental and Resource Economics
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Effectiveness if Incentive design and Economic valuation of Environmental goods and services Part: 3

For that, I will take you to some other example simple objective function to understand the Lagrangian multiplier.

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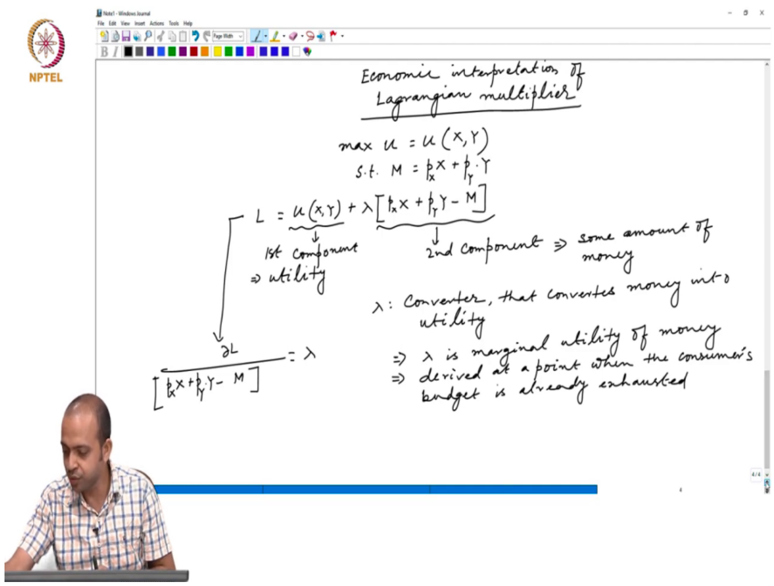

This is economic interpretation of Lagrangian multiplier. In economics we all have studied utility maximization in our micro economic theory. Let us assume that an individual is trying to maximize utility which is a function of two commodities X and Y and the individual is subjected to a budget constraint which is given by M equals to px into X plus py into Y.

So, total money or income is M that is spent on two commodities X and Y and px is the price per unit of X, py is the price per unit of Y this is the utility maximization problem that a consumer is facing. So, what will do will try to maximize this utility? This is a constrained optimization because the consumer is facing a budget constraint and constraint optimization we will try to solve by the Lagrangian multiplier method.

So, what would be the Lagrangian function? The Lagrangian function would be the utility function. So, first we have to write the objective function plus a Lagrangian multiplier lambda into what I will say that px into X plus py into Y minus M or you can write M minus this minus this that is also fine. Is that clear?

And what are the control variables. Control variable is only X and Y individuals can choose only the amount of X and Y not px and py because those two are the prices given in the market. So, if I differentiate these now, to understand the meaning of this. So, this Lagrangian function has two component let us say that this is the first component and this entire thing is second component.

Now, if you look at the first component of this Lagrangian function is nothing but some amount of utility and what is this second component for the timing just keep the lambda part a side what is the bracketed term px into X plus py into Y minus M what is that? What is that? The second component can you think of in this Lagrangian multiplier function?

What is the nature that means physical interpretation of the second component. I am talking about particularly the bracketed term. The bracketed term since everything is in monetary units. So, that means, px into X plus py into Y minus M. It is nothing but some amount of money. This is nothing but some amount of money. That means in this Lagrangian function what we are trying to do is we are trying to add utility with money.

We are trying to add utility with money. Is that possible? Can we add utility with money can we add apple with the oranges. No. You cannot add apple with oranges if you want to add apple with oranges, then what we need to do you need to have a converter that will convert either apple into orange or orange into apple same logic if you apply here you will never forget the economic interpretation of this Lagrangian multiplier.

In this Lagrangian multiplier we are adding money with utility which is impossible that is why we are using it convertor which is lambda. So, lambda is a converter that converts money into utility. So, that means, we can say lambda is nothing but marginal utility of money. So, what is the utility that we derive from one unit of money that is the interpretation. So, in your lifetime, whenever you set a Lagrangian function for this type of optimization, if you want to understand the meaning of the multiplier.

You just decompose the Lagrangian multiplier into two components look at what is the physical interpretation of the first component. What is the physical interpretation of the second component always you will see that in the Lagrangian function. We are trying to add two things whose physical interpretation are different in this case, it is utility with money in the case where we are talking about may be different.

But ultimately we are trying to add two different things and the Lagrangian multipliers rule is only to convert the second element into the first one. So, that we can add these two different things here the interpretation of the lambda is marginal utility of money we are converting money into utility. Now, the moment I say marginal utility of money, marginal utility of money is not constant it varies from context to context.

Now, when I am saying marginal utility of money derived at what point of time look at the nature here px into X plus py into Y minus M. what is the value of this if you look at the budget constraint, this is actually 0. If M equals to this, so obviously, px into X plus py into Y minus M equals to 0. That means, are we multiplying lambda with 0 and if that is the case is there any sense. Am I making any sense by multiplying 0 with lambda apparently it may look like we are multiplying lambda with 0. But we are not doing that.

What we are doing actually suppose, you have M amount of money and that you already spend on buying two goods X and Y your total money is exhausted. So, when your budget is totally exhausted at that point of time if I give you additional 100 rupees. What is the utility of that 100 rupees that is what we are talking about.

Now, intuitively you can very well understand 100 rupees given at the beginning of a month when you have just received your monthly expenditure money from your parents that may not give that much of utility. Because that time you are already having that money. But that same 100 rupees given towards the end when your monthly expenditure money is almost exhausted, definitely will give you a higher utility than what you would have derived at the beginning.

That is why when I am saying lambda is marginal utility of money derived at a point when the consumers budget is already exhausted. So, that means how will you get that interpretation, you differentiate this Lagrangian with respect to the entire budget px into X plus py into Y minus M you will get lambda. So, that means, what I am doing, change in the Lagrangian multiplier with respect to the change in the entire budget that means what, by giving additional money, I am relaxing the budget constraint for you.

And if I relax the budget constraint by one unit, what is the additional utility that we are going to get? And that is nothing but the lambda or Lagrangian multiplier, that means additional utility what do we get when your total money is exhausted? The question of relaxing the budget is valid only when your budget is exhausted, is not it? When your budget is exhausted, you are constrained. Now, I am relaxing the budget for you by supplying additional money.

And I am trying to get calculate the additional utility what you get out of this additional money, what I am supplying to you and relaxing the budget constraint. So, that is why the interpretation of the Lagrangian multiplier in this context is marginal utility of money derived at a point when the consumers budget is already exhausted.

Now, if you keep this type of intuition in mind, in your lifetime, you will never have any problem in understanding the meaning of Lagrangian multipliers. So, now we will go back to our initial situation and we will see how to interpret those two Lagrangian multiplayer.

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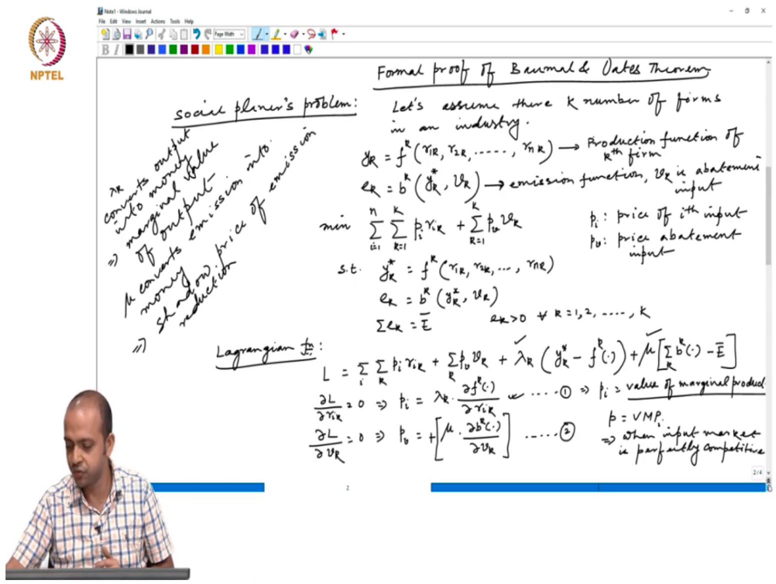

Come here. We have two Lagrangian multipliers here first of all. In this Lagrangian function, what is the nature of the first component that means pi into rik plus pv into vk that is also nothing but summation of input cost and abatement cost and if that is the case that is nothing but some amount of cost which again is measured by money but what is this other two components look at the first component yk minus fk which is nothing but some amount of output. What is this bk minus e bar which is nothing but some amount of emission.

So, that means, in this process lambda k converts output into money and Mu converts emission into money. Now, take the first component when you convert output into money. So, that means lambda k converts output into money. Then lambda could be marginal value of output that you if you the marginal value that means in monetary units, then you can get this is also in monetary units.

What is this Mu actually converts emission into money. That is why Mu is called shadow price of emission or you can say emission reduction. Because bk minus e that is nothing but we are trying to reduce the emission and if that is the case what is Mu, you need to convert that into money. So, that is shadow price off emission or emission reduction there will be always some kind of cost of pollution, but we cannot directly measure how will you measure you have to use some shadow price.

Because of this emission, I am using some resources for pollution control that, that is the price of pollution because of pollution, I am getting some health hazard and to solve that health problem, I am spending some amount of money that is price or we have to pay for additional pollution, if you reduce the pollution, you will avoid that amount of money that is why Mu is called shadow price of pollution or pollution reduction whatever you may think.

Now, if you look at the first condition pi equals to lambda k into del fk del rk. So, this condition says that means pi equals to what is del fk, del rk? This is nothing but marginal product of the I eth input and that we are multiplying with the marginal value of output. So, that means marginal value of output or input. This the marginal change marginal product. So, p equals to then we are getting value of that marginal product or VMPL del fk del rk is nothing but change in output with respect to one unit change in I eth input.

If I multiply with lambda that becomes a value of marginal output. Now, here also what we are getting pv equals to del bk into Mu. So, that means, that is also some kind of this is the change in emission multiplied by the price. That is also value of emission reduction. Now, if you look at this condition p equals to Vm Pi where is this condition satisfied price equals to value of the marginal product of the ith input.

If you recall these conditions is satisfied when input market is perfectly competitive. So, we are talking about a competitive input market that is where this entire story we can satisfy. If there is market distortion, we cannot get this resource cost minimization to achieve the social planners objective of the targeted emission reduction. So, that means, this entire story explains how it tax rate set at a level which achieves targeted level of emission reduction at minimum cost. That is what Baurmel and Oates said long back that if t equals two t star at which both the forms or each form in an industry equating their marginal cost of abutment t equals to MAC A equals to MAC B equals to MAC C dot dot dot MAC N.

That is the cost minimizing condition and this is the formal proof of that we have set one social planner objective we have set one private forms objective and by equating social planners objective with the private firms objective we derived the condition that at optimality t star should be the marginal abatement cost of that k eth firm. And since k is a representative firm as I said that is true for each and every firm in that industry.

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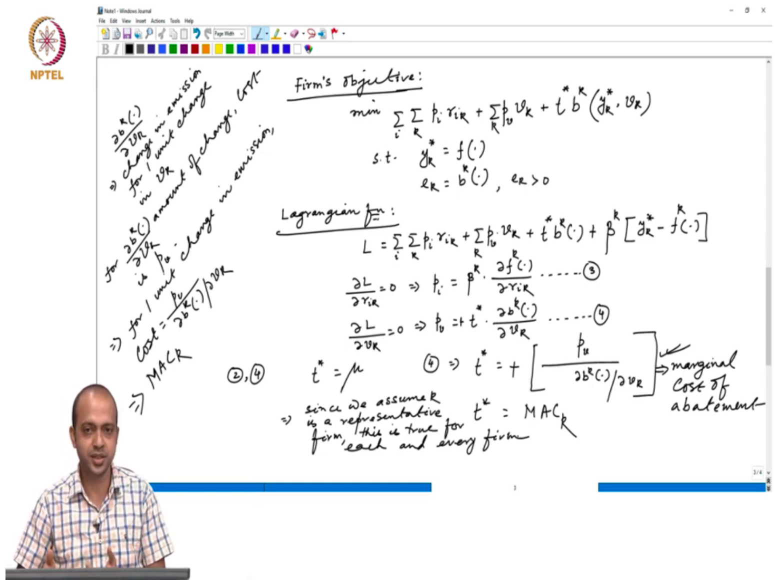

So, this fantastic story explains what is exactly the cost we are talking about. So, now we understood what is the cost for the society. What is the cost of the firm? Firm cost function is same as societies cost function. Additionally the tax amount the firm is adding that is all. That is why we say that the firms reaction to the taxes helps coinciding firms subjective with the social planners objective. And we have also understood the meaning of Lagrangian multiplier with an example of consumers utility maximization, you should never forget this interpretation of Lagrangian multiplier in your lifetime whenever you get this type of langrangian function immediately you should understand. So, with this, we are closing our discussion today, and we will again discuss this incentive design in our next session. Thank you very much.