Environmental and Resource Economics
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Effectiveness if Incentive design and Economic valuation of Environmental goods and service Part: 2

Welcome to our discussion on incentive design once again. So, in our last class we study we are talking about the efficiency, effectiveness and equity properties of these two main instruments market based instruments particularly emission charge and tradable pollution permit. And then we said that depending on the social planners or policymakers objective, we have to carefully select either a emission charge or tradable pollution permit.

We said that, when the policymakers objective is to maintain certainty over cost of pollution control, then emission charges preferred to market based instrument while tradable pollution permit is preferred when the policymakers objective is to get a fixed amount of pollution in a given period of time. And we have also discussed about the efficiency how the emission charge is efficient compared to uniform standard.

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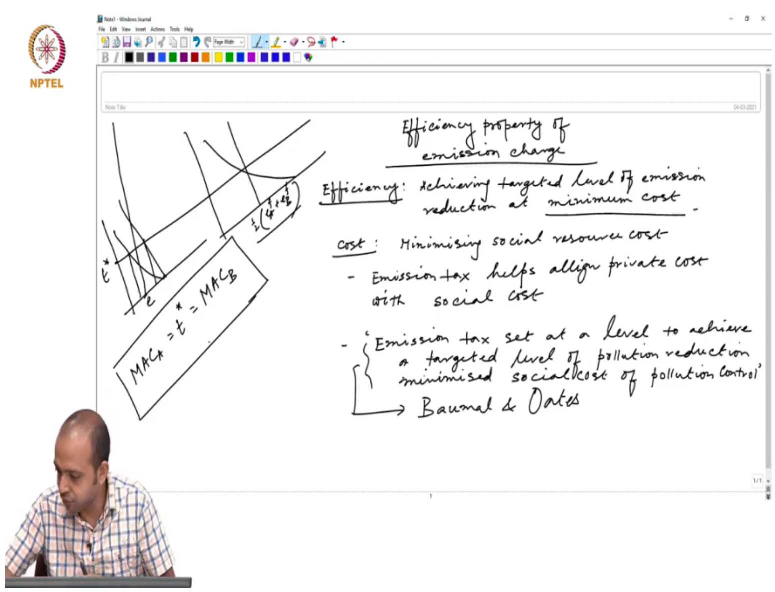

Now, what we will do today. So, this is we are talking about in detail a little more about the efficiency property emission charge why once again we are discussing the efficiency property I will discuss and I will explain to you in detail. So, this is efficiency property of a emission charge. So, by efficiency what do you mean we say that efficiency is achieving targeted level of emission reduction at minimum cost.

Now, when you are saying achieving targeted level of emission reduction at minimum cost, what is exactly the cost we are talking about here by cost we mean, actually what we are minimizing social resource cost and the interesting point here is to mention why the society is bothered about what is the resource cost here. Resource cost involves some amount of resources which are diverted for pollution control which could otherwise when used for making goods and services, is this clear?

So, that means, the moment society wants to achieve pollution reduction. So, that means society is imposing some regulation let us say that in this case in the form of emission tax and the firms are they are doing the firms are diverting some of their resources for pollution control. So, that means the inputs which were earlier used for let us say there is one input, let us say labor and capital.

Those are used earlier they were used for making goods and services. Now, some part of your labor some part of your capital is now diverted for pollution control. Similarly, the firms are also buying some specific inputs let us say some end of pipe technology for pollution control and that also involves some amount of cost and society wants to ensure that this cost this resource cost for pollution abatement should be minimum.

Why the society wants to achieve that because higher is the cost of abatement higher with the price of the product and the private players will try to bypass some amount of cost to the consumers. So, consumers they have to pay a higher price for their product the moment regulation is imposed. So, that means there is a private cost there is a social cost private players the firms.

So, that means, the moment tax is imposed, what is happening here the emission tax helps align private cost with social cost that means, the reaction of private players the reaction all these polluting firms when the tax is imposed coincides with the social planets objectives, that is why we say that these emission tax helps aligning private cost with the social cost.

So, that means private objective with the social objective that is what happens in the form of emission tax. So, what we believe that what we believe a tax emission tax set at a level to achieve a targeted level of pollution control pollution reduction minimizes social cost of pollution control. So, this is a theorem given by famous economist Baurmal and Oates.

So, in terms of a diagram if you look at the diagram what we have discussed that this is the industry level and society wants to achieve this much of pollution control. So, here what we have to do so, that means when this t star is fixed at A level. So, this is MAC B, this is MAC A. So, A will reduce this much pollution will reduce this much of pollution and that is better than the uniform standard e-bar. And this is let us say half of e f A plus e f B.

So, a tax rate which is set at a level t star that achieves the targeted level operation control which is half of e f A plus e f B minimizes the social cost of pollution control that is a theorem Baurmal and Oates set long back and what is the social cost operation control we are talking about minimizing the resource cost minimizing the resource cost.

So, in this case what is the assumption that we are making the assumption or to make here is that in the process of pollution control we use some inputs which again does not generate any other form of emission. For example, let us say we are generating some garbage and there is a garbage incinerator used to recycle that waste and the garbage incinerator again is not discharging some other form of emission.

If that is the case, then this theorem is not valid. So, that means, we are ruling out the possibility of any other form of emission again in the form of pollution control. If that assumption is valid, if there is no other emission, then the reaction of the private player to this tax emission tax t star which is imposed by the social planner on the policymaker that achieves the targeted level of emission reduction which is half of e f A plus e f B at minimum cost.

Now, how is it happening because every firm is equating their marginal cost of abatement with the tax rate. Now, we got a graphical and intuitive explanation of this particular theorem that when this condition is satisfied, that means, when t equals to MAC A equals to MAC B when the both the firms are equating their marginal cost of apartment with the impose tax rate, then social cost of pollution control is minimum.

But we have not yet got any formal proof of this theorem. So, what we need to do now, we need to have a formal proof of this theorem why we are interested in that formal proof because then only we will understand what the social planners objective in the process of pollution control, what is the objective of the private firms and how this tax rate is helps coinciding the private firms objective or objective function with that of the social planners.

So, social planner is also interested in some kind of cost minimization private firms is also interested in some kind of cost minimization both trying to minimize their resource cost and tax rate is helping them aligning private costs with the source alone. How is that happening that we will now try to understand so that is a formal proof of this Baurmel and Oates theorem formal proof.

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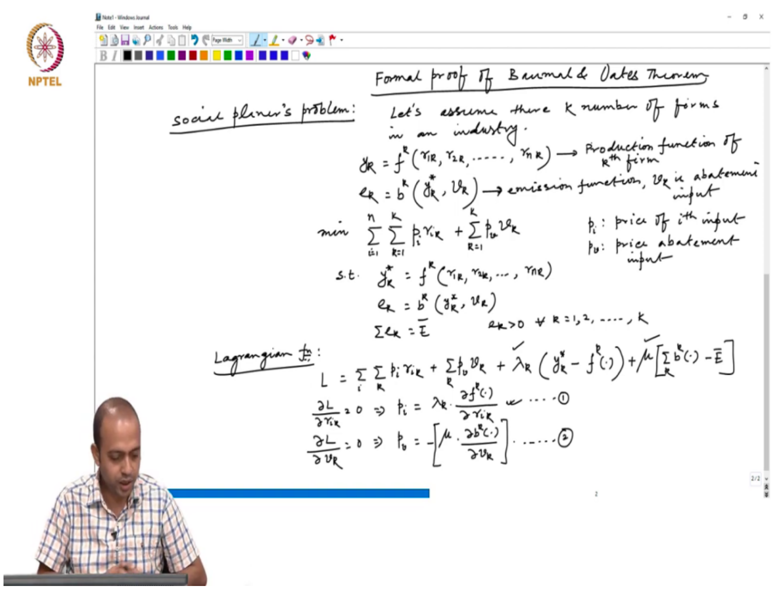

So, first we will discuss about what is a social planners problem. So, we assume, let us assume there are key number of firms in an industry. Each firm is producing some output and the production function is given by production function of the kth firm is given by let us say production function is given by r1 k, r2k r let us say nk.

So, that means what is r1k, r1k is the first input used by the kth firm, r2k is a second input used by the kth firm and rnk is the n eth input used by the k eth firm in this production process. The production function and then there is some kind of emission function also each firm is generating some kind of emission and that emission function emission generating function is emission is a function of what is your output at optimum plus some abatement input. So, this is production function of kth firm and this is emission function and this vk is an abatement input ,let us say the kth firm has installed some kind of indo pipe treatment technology and vk is the abatement input.

Now, what the social planner will try to minimize then, the social planners objective function is to minimize sum of production costs that means pi rik plus summation pv into vk and this is i and this is k. So, if you look at, so pi is price of ith input and p v price of a abatement input. So, that means in the social plans objective function what we are seeing that this pi plus rik that means, there are k number of firms and each form they are using n number of inputs.

So, when I am taking double summation, first summation is summing over input second summation is the summing over firms. So, if I take double summation pi ri k that captures all the firms input cost and here this is only summing over firms that means each firm is using only one abetment input that is v and price of this is pv.

So, this captures the abatement cost of all the firms. So, full planner is trying to minimize that means cost of production plus cost of abatement. This is the objective function and what is the condition here subject to social planner once there is some kind of optimum input for the kth firm that means. This is let us r1k r2k and then e k equals to the emission function which is again and total emissions should not exceed sum of ek this.

So, social planner wants to minimize total cost of production plus total cost of abatement subject to these are the restriction. That kth firm should produce a given level of output, this is the emission function and total emissions should not exceed some e bar amount and here. What do we assume that ek is positive greater than 0 for all k.

So, this is the social planers problem and what we need to do we need to set a lagrangian function. In the lagrangian function for cost minimization what is the lagrangian function for cost minimization let us say this is L is the lagrangian function equals to first we have to write the objective function double summation pi rik this is i and this is k plus pv into vk summing over k plus some lambda k into yk star minus f plus another lagrangian multiplier which is let us say Mu then some of these bk minus e bar.

So, there are two lagrangian multipliers here one is lambda k and another one is Mu. So, first will try to derive the condition mechanically and then will try to understand what is this lambda k and what is this Mu. What is the interpretation of these two lagrangian multipliers. This is summing over k which is coming from this function it should be sum of ek minus this and eks is given by this function that is why we have written some of v function this.

And what is are control variable here. The control variable here is rik that means, amount of input which is used for production and amount of input which is used for abatement rik and vk. These are the two control variables so, that means the minimization requires that differentiating this lagrangian function with respect to del rik and set it equal to 0 and that implies that pi minus this lambda k into del fk del rik equals to 0.

So, that means, I can write pi equals to lambda k into del f k into del rik this is one condition and second condition is differentiating this lagrangian with respect to vk equals to 0 and if you do that, then you will get pv equals to minus of Mu into del bk del vk. Now, here since this is there is a negative sign here.

And if you think about this, what is del bk del vk that means change in emission for a change in abatement input and when you increase the abatement input emission goes down that is why this also negative these negative and this negative will turn out to be positive. So, that means pv equals to this. Let us say this is equation 1 and let us say this is equation 2. So, these are the two conditions we derived from the social planners objective.

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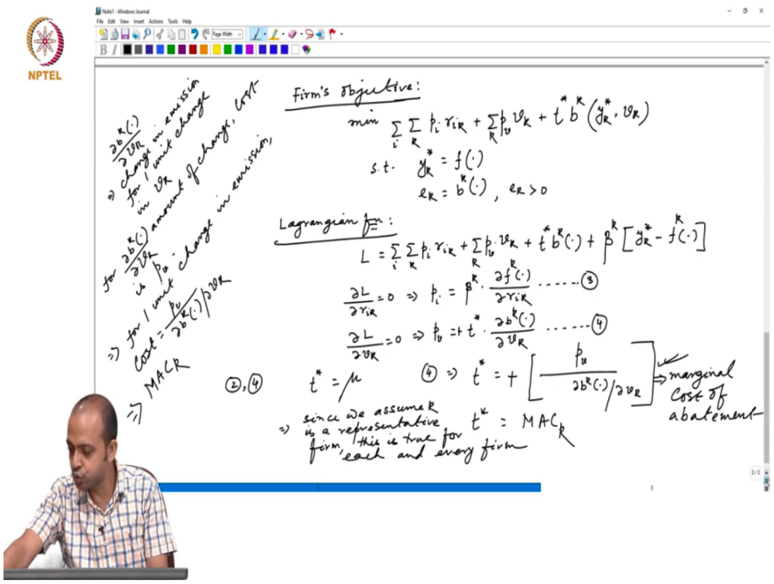

Now, we have to see what the private firm is trying to firm objective. So, we said that the moment tax is imposed private firms objective function will coincide with the social planner objective. So, that means in the firm objective also we will get the same component of same objective function which we can borrow firm the social planner one. Additionally, the private firm additionally what the private firm will try to incorporate in that objective function is the tax amount. Because anyway the resource cost is from derived from the firms point of view and for that in that process the firm will include the tax amount also to get total costs.

So, that means the firms objective would be minimize sum of double summation i and k, then I will write pi rik plus pv into vk this is common last the tax rate which is imposed at t star level to achieve that targeted level of emission and how much is the tax the amount is decided by your optimum amount of production and the abatement input and what is the condition here the same condition yk star is given by the production function and then ek is bk function and ek also greater than 0.

So, this is firms objective the first two components were there in social planner objective also these additional component This is the tax t star multiplied by amount of emission which is decided by your optimum level of output and the abatement input that we you are going to use. So, again the lagrangian function for cost minimization L equals to will write double summation pi rik, this is i this is k plus summation pv into vk plus t star into bk plus the lagrangian multiplier let us say this is the lagrangian multiplier which is this yk star minus f of this, this is the Lagrangian multiplier.

And again if you differentiate this with respect to del rik and set equals to 0. They not will get that he will give you pi equals to, so this r is here only this function so, that means equals to beta k into del fk del rik. Let us say this is equation 3 and del L del vk if you set it equals to 0 that will give you where is v v is only here in this production function. So, that means pv would be equals to t star pv equals to that means minus t star into del bk into del vk again since this is negative this is negative this would become positive this is 4.

Now, since the tax rate helps coinciding firms objective with the social planners one. Now, by equating equation 1 and 2 with 3 and 4 what will get. So, that means if we say that these 3 is equivalent to 1 these 1 and 2 these 2 are equivalent to 3 and 4 look at the condition we are getting pv equals to this. So, this is also become plus. So, that means, we will say that here we are getting pv equals to this and Mu into del bk del v k and here we are getting p v equals to t star into del bk del vk.

So, from these two what we can write from 2 and 4 what we can write that t star equals to and what is t star from here from 4 what you can get 4 says that t star is equals to minus of pv divided by del bk del vk since this is negative this will become positive. So, what is the meaning of this pv divided by this? So, this particular mathematical expression it has some meaning what is that meaning we will try to understand see first of all del bk del vk is indicates change in emission, for one unit change in vk that is what it means del bk del vk.

So, change in emission because the numerator is emission function. So, the moment you differentiate that with respect to vk, what you can get that this is nothing but change in emission for one unit change in vk. Suppose I am increasing the abatement input. If I buy one additional abatement input, what is my cost? My cost is pv because that is the price of abatement input.

So, that means, indirectly I can say that to get these amount of change in emission, my cost is pv amount I am repeating this once again, this indicates the change in emission for one unit change in vk and one unit change in vk costs pv amount that is why I can say for del bk del vk amount of change cost is pv for one unit. So, for one unit change in emission cost is pv divided by this del b k and that is nothing but for one unit change in emission the cost what I incurred is nothing but marginal cost of abatement for the kth firm.

So, that means this is nothing but t star equals to MAC marginal cost of abatement for the kth firm. This is nothing but marginal cost of abatement. Since we assume k is a representative firm this is true for each and every firm. So, that means the condition that we have derived that cost minimization requires equating marginal abatement cost with the tax rate that is true for each and every firm and that is what we are talking about earlier that t equals to MAC A equals to MAC B.

If that condition is satisfied, that means at that condition only society can achieve targeted level of pollution reduction at minimum cost. This is the formula because this condition is derived from cost minimization only additionally what we have assumed that a firms reaction to the emission tax helps coinciding the social planners objective with the private firms objective, is that fine? That means at optimality each firm will equate the impose tax with a marginal abatement cost and that will help achieving the target at level of emission tax at minimum resource cost.

Now what I said earlier that this is the mecahnical derivation of the condition. But since this process of minimization requires Lagrange multiplier we have not understood two things. When I said that this is the lagrangian case. So, what is the meaning of this lambda k and what is the meaning of this Mu, these are the interpretation of lambda k and Mu these are the two lagrangian multiplier we have use we need to learn the economic interpretation of these to langrangian multiplier.