Introduction to Econometrics
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Qualitative Response Models-Linear Probability Model,
Logit and Probit Models Part – 3

So, welcome once again to our discussion of Qualitative response model that we are discussing in our last class. So, will continue again the Qualitative response model from today also, so in our last class if you recall we stared with our discussion with linear probability model.

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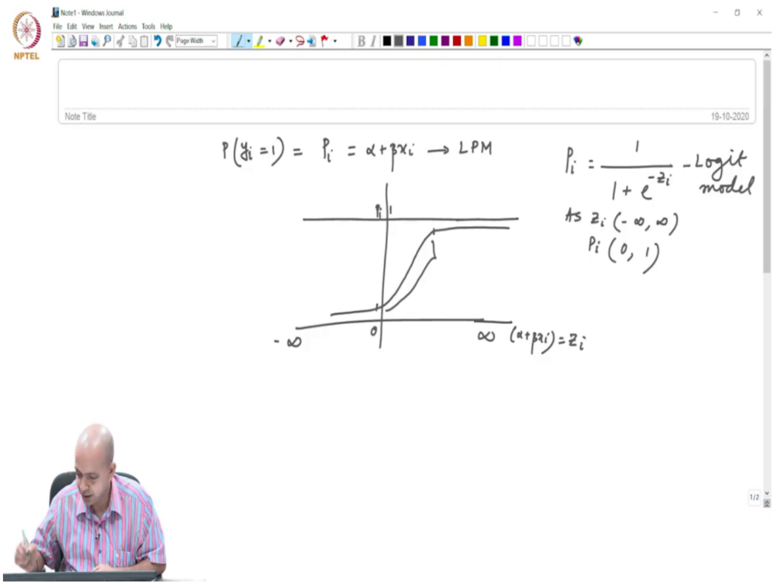

And we said that the linear probability model that takes this form pi equals to alpha plus beta xi, so pi is basically probability that yi equals to 1 and then we said that this linear probability model or in short LPM, what is the major limitation of this here the probability is modelled as a linear function of x. So, that means if you think about the house ownership problem that we are discussing in our previous class, so what happens actually, when the individual's income is very low in that range, almost all the people they do not have a house actually.

So, at lower income people do not have house almost all of them and at a higher level of income, they will almost all of them will have a house, but then once you achieve that level of income, then probability of owning a house that does not change actually for example, when your income is 1.5 lakhs per month, then you have a house and that probability of owning a house at that income range is almost 1.

But, suppose now income is increasing from 1.5 lakhs to 2 lakhs, then once you have the house, then you cannot buy that you that particular individual would not buy any new house, so that means, basically it says once you achieve that level of income, the probability does not change, it will almost 1 and at lower level of income, nobody is having a house and at the lower level of income when your income is let us say 5000 per month to 5500, 5600 like that, probability does not change that much, so at the lower end and at the higher end it is constant and it changes in between.

So, that means, a linear characterization of probability is a much problematic thing in this context, so what we actually want, if you plot your probability in this way, let us say this is 0 and this is pi this is let us say minus infinity this is plus infinity, so what our probability should be like this, it should behave in this way and this is what is called a sigmoid S curve type relationship. This is a sigmoid S and to capture this type of non linearity, so that means, in this axis I am assuming let us say alpha plus beta xi plus beta xi it ranges from this to this and this is equals to zi, so zi basically ranges from minus infinity to plus infinity.

So, this is zi and what do you want is the relationship of zi and pi like this, at the lower end, it will almost 0, but it will never touch 0 here it is 1 actually, it will approach towards 1 at higher level of income but, it will never touch 1. So, basically it asymptotically approaches 1 and 0 and after that suppose, from this person it almost constant here also once you achieve here it almost constant it is not changing and it is changing in this particular this range like this range.

So, to overcome the problem of linear characterization of probability with zi in Logit model what we assume that pi equals to 1 by 1 plus e to the power minus zi and from here you can understand as zi as this model ensured as zi 10 ranges from minus infinity to plus infinity then your pi will become 0 to 1 that is the advantage of this model that is the advantage of this Logit model, is it clear?

So, I will repeat once again this linear probability model it assumes probability is a linear function of x here x is income linear function of x or you can consider alpha plus beta xi entire thing is z, so it is a linear characterization between pi and zi. But, in reality what happens is that probability does not change linearly when income changes from 15,000 to 20,000 the change probability is not same, when income changes from 1lakh to 1lakh 20,000.

Probably, when income changes from 1lakh to 1,20,000 you will observe either very insignificant change in probability of owning a house or no change at all, so it only changes from 20,000 to 1lakh in that range in this range actually, probability changes after that it constant.

Similarly, at the lower end and to overcome that problem we hypothesise a nonlinear characterization of probability of owning a house pi with the income xi and that is basically the Logit model which is 1 by 1 plus e to the power minus zi. And as zi ranges between from minus infinity to plus infinity pi will range between 0 and 1 that is how Logit model overcomes the major problem of linear probability model.

But, then you end up having a nonlinear model pi equals 1 by 1 plus e to the power zi you cannot estimate directly this model applying the linear technique and that is the reason we characterised that means, we transform the apparently looking nonlinear model into a linear model by taking log and then we discussed how to estimate that model using the maximum likelihood estimates or MLE where does not work, that is how we discussed about the linear probability model and the Logit model.

Now, today we will discuss another Qualitative response model, which also characterise nonlinear relationship between the probability and xi and this model is named as probit model.

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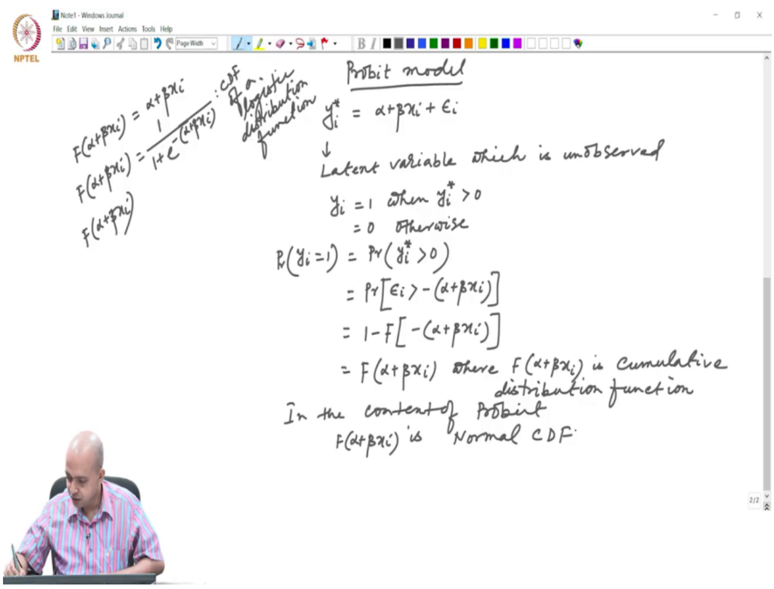

So, let us try to understand the theoretical structure of this probit model now to understand the theoretical structure of this probit model, we will introduce a variable which is called latent variable. Let us say yi star equals to alpha plus beta xi plus let us say epsilon i here y star is called a latent variable, which is unobserved and then there is a relationship between yi and yi star, how? Yi equals to 1 when yi star greater than 0 otherwise.

Now, you might be thinking what is this latent variable and the how can you get a relationship between yi and yi star think about the house owning problem given your income, each and every individual calculate some amount of utility of satisfaction of buying a house or buying a car or anything and you will observe that individual has actually bought a house when the individual derives a positive amount of utility, is not it? If the utility is negative then that means, if there is dissatisfaction of owning a house, at that level of income, then you will see that individual has actually not bought the house.

Now, you might be thinking, what is the disutility of owning your house, the actually there is no disutility of owning a house as such, but at that level of income, when my income level is very less, let us attend 10000 and if I buy a house, how buying a house is not my priority at that level of income, because I have so many other important things to do. So, if I buy a house and then if I start giving EMI for that house probably that will give a dissatisfaction.

So, each and every individual will calculate the utility at that level of income of owning a house depending on the utility household will decide or the individual will decide whether to buy the house or stay in rented apartment, but utility is something you cannot observe, what you observe is actually the decision and what is the decision? Whether I have bought or not, that is the realisation. So, that is why you cannot observe the utility, but you can observe the decision here yi is basically the decision the ultimate realisation whether, the event has happened or not.

But in between how and what amount of utility the individual has derived that you cannot observe and that unobserved utility, let us say we defined as yi star it depends on your income, but then there is some amount of error term, which makes the utility unpredictable on unobserved. So, when y star is greater than 0 you derive a positive amount of utility and then yi equals to 10 otherwise. This is the structure of the probit model that yi is related to an unobserved variable yi star, which is called latent variable.

Now, once you hypothesise that type of relationship between yi and yi star then what you have to do basically, when you are calculating probability yi equals to 1 that means you are saying in turn it is nothing but probability yi star greater than 0 because, then only yi equals to 1 now, from this relationship you can easily understand when can you get yi star greater than 0.

So, from this relationship, I can easily understand that yi star will become 0 or greater positive when your epsilon i is actually greater than minus alpha plus beta xi and if you recall the definition of probability density function, from the properties of probability density function, we can write when epsilon i is actually a random variable and this is less than which is greater than some alpha plus beta xi then we can say that this is nothing but 1 minus f of minus alpha plus beta xi, which is nothing but af of alpha plus beta xi that is how you can derive this one.

So, these fi, what is this f of alpha plus beta xi this is actually I will say that this is actually where f of alpha plus beta xi is cumulative distribution function. Now, what type of specific cumulative distribution function this f of alpha plus beta xi will take you will get either linear probability model, Logit model or probit model, what I am saying this f of alpha plus beta xi can take 3 different values it can be a cumulative linear distribution function which is that means, I can say that f of alpha plus beta xi f of alpha plus beta xi can be simply alpha plus beta xi or f of alpha plus beta xi can be 1 by 1 plus e to the power minus alpha plus beta xi.

And then you will get the Logit model and in the context of probit this f of alpha plus beta xi take this type of form equals to and this is called this is actually cumulative, CDF or logistic distribution function, so this is basically this is actually f of alpha plus beta xi I will say that cumulative distribution function or CDF. So, in the context of Logit, this is CDF of a logistic distribution function in the context of probit this f of alpha plus beta xi is actually the cumulative distribution function of a normal distribution. So, that means, this is normal CDF that means, in the context of probit what I can write.

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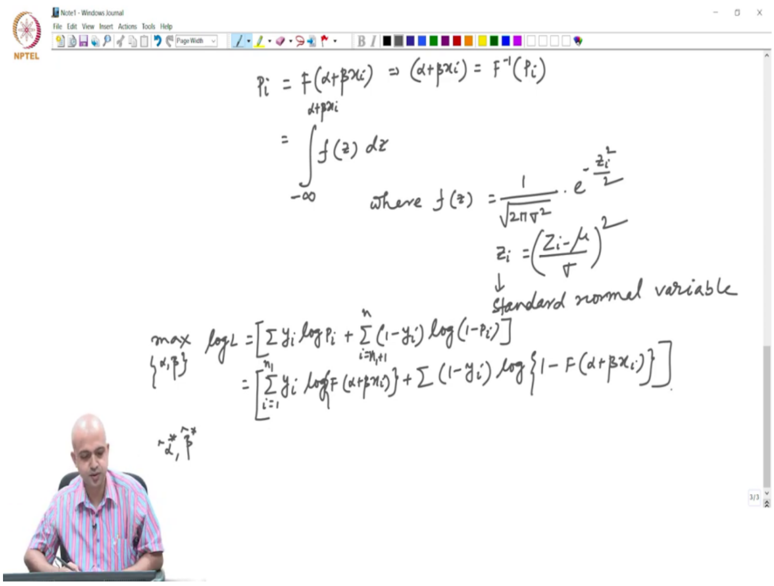

That this pi equals to f of alpha plus beta xi equals to I can write integration minus infinity to alpha plus beta xi fz dz and what is this fz, fz is basically a normal probability density function and I can write that where, fz is basically, where fz equals to 1 by root 2 over 2 Pi into sigma square into e to the power minus zi square y2 and what is zi, zi is actually, zi how it is defined, zi is defined in this way zi minus mu divided by sigma whole square, which is nothing but a standard normal variable, is this clear?

So, that means here in the context of probit, only difference that it makes is f of alpha plus beta xi takes the cumulative. Since I am taking the integration of this fz which is basically a normal distribution function I am taking, when I am taking integration that becomes the cumulative density function or CDF. So, this is a CDF of a normal distribution function were fz is root over 2 Pi sigma square into e to the power minus zi squared by 2 and how zi is defined zi is defined as zi small zi minus mu divided by sigma whole squared that means, zi is basically a standard normal variable. So, if pi equals to this, then from here you can say that, that means, alpha plus beta xi equals to f inverse pi that is how you can get.

Now, you will recall the log likelihood function, what we got in the context of Logit same type of log likelihood function you will get in the context of probit also that means, your log L would become summation yi into log pi plus summation 1 minus yi into log of 1 minus pi and that you are trying to maximise with respect to alpha and beta and this pi? Pi equals to summation yi i running from 1 to n1 here, i running from n1 plus 1 to n and then this is log of what is pi, pi is basically of alpha plus beta xi plus summation 1 minus yi log of 1 minus this I will write log of 1 minus of alpha plus beta xi.

So, this is your log likelihood function in the context of probit and that you maximise once again with respect to alpha and beta and then you will get your alpha star and beta star; we will get alpha star and beta star by maximising this.