

# Statistical Analysis of Dummy Variable Models and Testing for Seasonal Fluctuations

## Part-4

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Lecture 41  
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$$Y_t = \alpha + \beta_1 x_t + \beta_2 D_t + \beta_3 (x_t \times D_t) + u_t$$

$$E(Y_t | D_t=1, x_t) = (\alpha + \beta_2) + (\beta_1 + \beta_3) x_t \rightarrow 1982-1995$$

$$E(Y_t | D_t=0, x_t) = \alpha + \beta_1 x_t \rightarrow 1970-1981$$

$$\beta_2 : \{ (\alpha + \beta_2) - \alpha \} \Rightarrow \text{differential intercept}$$

$$\beta_3 : \{ (\beta_1 + \beta_3) - \beta_1 \} \Rightarrow \text{differential slope}$$

$$\left\{ \begin{array}{l} \beta_2 \text{ is sig: Post 1982 there is change in intercept} \\ \beta_3 \text{ is sig: " " " " " slope} \\ \beta_2 \text{ is sig: " " " " " slope} \\ \beta_3 \text{ is sig: " " " " " intercept} \\ \beta_2 \ \& \ \beta_3 \text{ are both sig: Post 1982 there is change in both slope as well as intercept} \end{array} \right.$$

So, from this what you can understand that, what you can understand that beta 2 that how will you derive the, now you have to, what you have to do? Again, you have to derive the interpretation beta 2. How will you derive beta 2? Beta 2 is nothing but alpha plus beta 2 minus alpha. And what is alpha plus beta 2? That is the intercept of the post-recession period, an alpha is the intercept of the pre-recession period. So that means beta 2 indicates differential intercept.

Since we have one more covariate  $x_t$  present in this model, we cannot say that alpha is basically the value of the dependent variable in the base period. We can only tell that alpha is basically intercept, because this is an ANCOVA model. Apart from the dummy we have  $x_t$  also present in the model. So that is why beta 2 also is defined as differential intercept.

And how do you define beta 3? Beta 3 is defined as beta 1 plus beta 3 minus beta 1. And if you define in this way what is beta 1 plus beta 3? Look at here beta 1 plus beta 3 is the slope of the saving income relationship post 1982 period, that means post-recession beta 1 is the slope for the pre-recession period. So that means beta 3 indicates differential slope, differential slope. So, you

now understood how the coefficients are interpreted when there is a dummy variable involved. So, you have to basically derive it.

So, depending on the significance of beta 2 and beta 3 we will conclude whether post-recession period there is a change in the intercept or change in the slope or both. So, when beta 2 is significant that means we will say that post 1982 there is change in intercept. When beta 3 is significant we will say that post 1982 there is change in slope.

When beta 2 and beta 3 both are significant, so post 1982 there is change in both slope as well as intercept. This is how you have to fix the model and then you have to estimate and this is how you can interpret. So, this would be the post estimation interpretation beta 2, beta 3 and significance of beta 2 and beta 3. So now what we will do? We will just estimate the model, I will show you the data set.

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The screenshot shows a software interface for data editing. The main window displays a table with the following data:

year	yt	xt	dt	ddat
1972	63.4	455.3	0	0
1973	69.4	565	0	0
1974	97.4	1054.2	0	0
1975	104.4	1159.2	0	0
1976	96.4	1273	0	0
1977	92.5	1401.4	0	0
1978	112.4	1500.1	0	0
1979	130.1	1709.5	0	0
1980	161.8	1973.3	0	0
1981	199.1	2288.2	0	0
1982	205.5	2347.3	1	2347.3
1983	167	2521.4	1	2521.4
1984	235.7	2810	1	2810
1985	286.2	3082	1	3082
1986	196.5	3187.6	1	3187.6
1987	168.4	3363.1	1	3363.1
1988	189.1	3648.8	1	3648.8
1989	187.8	3894.5	1	3894.5
1990	208.7	4166.8	1	4166.8
1991	246.4	4343.7	1	4343.7
1992	272.4	4613.7	1	4613.7
1993	214.4	4798.2	1	4798.2
1994	188.4	5011.7	1	5011.7
1995	189.3	5138.8	1	5138.8

The interface includes a menu bar (File, Edit, View, Data, Tools), a toolbar, and a 'Variables' panel on the right. The 'Variables' panel lists the following variables:

Name	Label	Type	Format	Value label
year	YEAR	int	%d.0g	
yt	Yt	float	%d.0g	
xt	Xt	float	%d.0g	
dt	Dt	byte	%d.0g	
ddat	Ddat	float	%d.0g	

The 'Properties' panel shows the following settings for the selected variable:

Property	Value
Name	ddat
Label	Ddat
Type	float
Format	%d.0g
Value label	
Notes	

A person is visible in the bottom left corner of the slide, sitting at a desk.

Look at the data set first see this is 1970 to 1995 data yt indicates saving, xt indicates income and this dt is basically the coefficient of the dummy and Dt xt is the interaction between the dummy and the quantitative covariate xt. So, we have assigned 0 value see, 0 starts from 1970 and it goes up to 1981. So, 1970 to 1981 it takes 0 value as we have defined earlier. This is the pre-recession period.

And starting from 1982 to 1995 in all these years this dummy variable takes the value 1 indicating post-recession starting from 1982 because that was the structural break point we have assumed. And this particular cell is simply created by the introduction that means the multiplication. So, this is how you have to create the variable.

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The screenshot displays the Stata 16.0 interface with the following data and results:

Model	1	2
Model	18456.2587	1 10456.2587
Residual	1785.83254	10 178.583254
Total	18241.2912	11 1658.2992

Source	SS	df	MS	Number of obs	F(1, 12)	Prob > F
Model	2614.39647	1	2614.39647	14	3.14	0.0800
Residual	10005.2214	12	833.768451			0.2872
Total	12619.6179	13	970.739837			

Variable	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
xt	.0093119	.0093665	0.68	0.000	-.0616901 .0997377
_cons	1.016115	11.63771	0.09	0.932	-24.91432 26.94655

Variable	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
xt	-.0148624	.0083932	-1.77	0.102	-.0334248 .0331496
cons	153.49437	17.71777	4.69	0.001	87.73875 274.7686

Command: `reg yt xt dt dxt`



1981. So that means from this result what is your alpha? Alpha is this 1.0161. And what is your  $x_t$ ?  $x_t$  is this.

So that means pre 1982 period what you have to do? That  $\hat{y}_t$  equals to what is the intercept 1.0161 plus what is the value of the slope? Slope coefficient is 0.080. So that means this would become 0.080  $x_t$ . So, this is your equation for the 1970 to 1981 pre-recession period. And what is the equation for the post-recession period? That would become  $\hat{y}_t$  equals to post-recession period 1992 to 1975 alpha plus beta 2 plus beta 1 plus beta 2 beta 3  $x_t$ .

So now from the result alpha is 1.01. And what is your differential intercept? Differential intercept is 152. So that means this would become 1.0161 plus 152 point and the value is 152.47. So, this is now your alpha plus beta 2. This is actually your alpha plus beta 2 plus, what is your beta 1 plus beta 3?

So that means you already know if you look at the beta 1 is actually is 0.080 minus. What is the differential slope? Differential slope here is 0.0654. So, 0.0654 minus 0.0654. So, this is actually your beta 1 plus beta 3 and  $x_t$ . So, this is how you will get the regression equation, saving income relationship for the pre-recession period. This is for the post-recession period that means 1982 to 1995, 1995.

Now look at the level of significance. How do you know the differential slope?  $dt$   $x_t$  the coefficient of  $dt$   $x_t$  is basically differential slope that is minus 0.0654 is that significant? Look at the p value is 0.000. If you multiply that it is again 0.000. So that means differential slope is highly significant.

So post-recession period the differential slope is highly significant that means post-recession period slope is significantly different from the pre-recession period. What about differential intercept? Value is 152.47 look at the level of significance that is also 0.000. If you multiply that by 100 that is again highly significant. So that means post-recession period both are significant, both are significant.

Now one question that I would like to raise here. Since, since both  $x_t$  and  $dt$   $x_t$  are significant by individual, individual t statistic, t statistic can we say that they are jointly significant. So that means my question is can we use this individual t statistic to say that they are jointly significant without conducting any further test.

Individual test statistic shows that post-recession period slope is significantly different from the pre-recession period. Intercept is different from significantly different from pre-recession period. That means can we say that slope as well intercept they are jointly significantly different from the pre pre-recession period slope and intercept, can you say that?

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NPTEL

$t_1$   $t_2$

$t_1$  &  $t_2$  can't indicate towards joint significance

— We need to conduct F test for that

$$y_t = \alpha + \beta_1 x_t + \beta_2 d_t + \beta_3 (x_t * d_t) + u_t$$

$\beta_2 = \beta_3 = 0 \rightarrow$  F statistic

Both intercept & slope are different in post 1982 period.

check  $H_0: \beta_2 = \beta_3 = 0$

If you recall, we discussed previously that there is no one to one mapping between t and F when you have multiple linear regression model. Why this is so? I quickly say that you are collecting the data from the same sample so that is why if this is your  $t_1$  distribution and this is your  $t_2$  distribution they are actually not independent, there is some overlap. And because of this individual significance does not indicate the joints significance. So, t statistic  $t_1$  and  $t_2$  cannot, cannot indicate towards joint significance.

Then what we need? we need to conduct F test for that because F test basically shows the joint significance of the model. So that means in terms of this model when we are saying  $y_t$  equals to  $\alpha$  plus  $\beta_1 x_t$  plus  $\beta_2 d_t$  plus  $\beta_3 x_t$  multiplied by  $d_t$  plus  $u_t$  in this model what we are basically interested to check  $\beta_2$  equals to  $\beta_3$  equals to 0 or not. This is basically we are going to test using F statistic. And this would imply that both intercept and slope are different in post-1982 period. This we have test.

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Stata 16.0

```

1  reg yt xt 1/12
2  reg yt xt 1/126
3  reg yt xt
4  reg yt xt dt dtxt

```

	yt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
xt		.0376791	.0042366	8.89	0.000	.0289353 .046423
_cons		62.42267	12.70075	4.89	0.000	36.05578 88.79257

Source	SS	df	MS	Number of obs =	F(3, 22) =
Model	88079.8337	3	29359.9442	54.78	0.0000
Residual	11790.2539	22	535.92034	R-squared =	0.8819
Total	99870.0867	25	3994.80347	Adj R-squared =	0.8658
				Root MSE =	23.15

	yt	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
xt		.0803159	.0144968	5.54	0.000	.0503673 .1101964
dt		152.4786	33.88237	4.61	0.000	83.86992 221.0872
dtxt		-.0654694	.0159824	-4.10	0.000	-.098615 -.0323239
_cons		1.016115	20.16483	0.05	0.969	-40.80319 42.83542

Command: test[dt-dtxt=0]

Stata 16.0

```

1  reg yt xt 1/12
2  reg yt xt 1/126
3  reg yt xt
4  reg yt xt dt dtxt
5  test dt-dtxt=0

```

Source	SS	df	MS	Number of obs =	F(3, 22) =
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dt		152.4786	33.88237	4.61	0.000	83.86992 221.0872
dtxt		-.0654694	.0159824	-4.10	0.000	-.098615 -.0323239
_cons		1.016115	20.16483	0.05	0.969	-40.80319 42.83542

```

. test dt-dtxt=0
(1) dt - dtxt = 0
(2) dt = 0

F( 2, 22) = 10.69
Prob > F = 0.0006

```

Command: test dt-dtxt=0

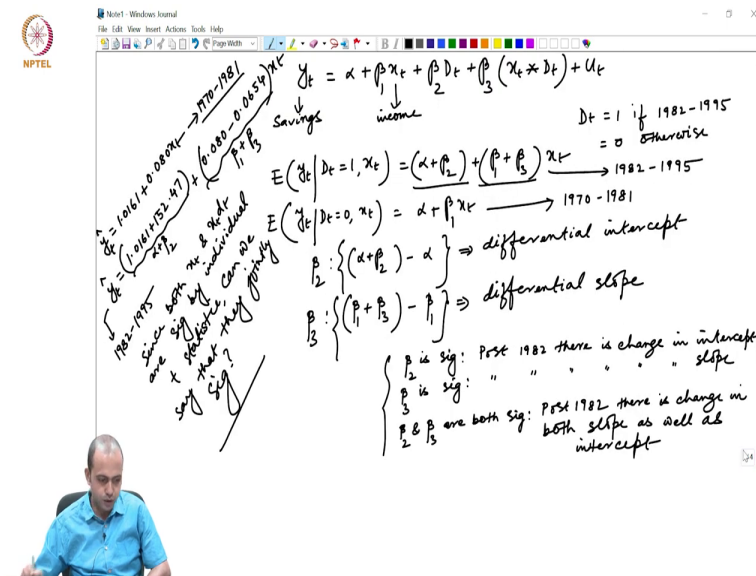
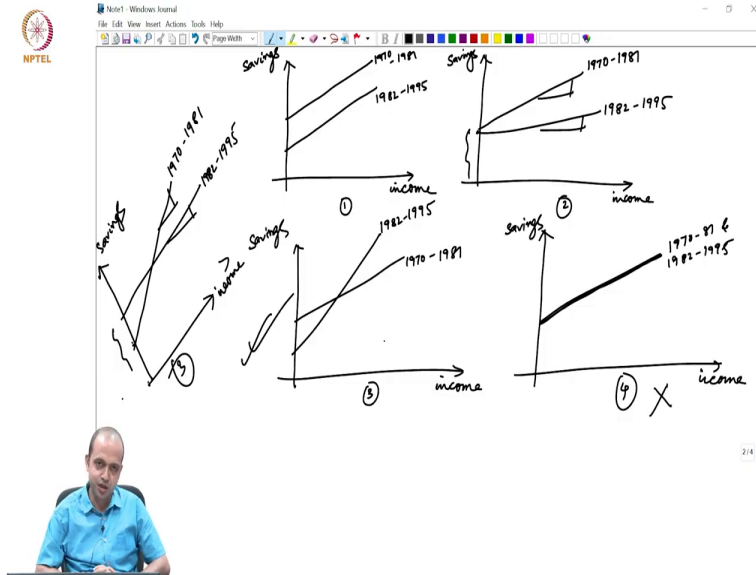
Now if you go back and as we said there is a specific command which says that you have to put the test command test you have to put dt and then space dt xt equals to 0. This is the test command in Stata. So, after estimating the model immediately after estimating the model, This is the full model. And now I want to check whether the differential intercept and differential slope they are jointly significant or not. And for that test dt equals to dt xt equals to 0 or not. F statistic is 10.69 which is exactly matching with the F statistic that we have calculated in the context of Chow test.

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So that is how the dummy variable model overcomes the limitations of Chow test. Now the dummy variable is saying that yes out of this three actually this is the case in our present context where both slope as well intercept are different, both slope as well as intercept are different.

So, we have to see carefully the diagram whether the slope is more or less depending on the sign here I have just arbitrarily drawn the diagram. So, depending on if you go to the previous one the value is actually, so, look at. So post-recession period your intercept actually is higher but slope is lower, intercept is higher but slope is lower. So, in this case intercept should be higher but slope is lower.

So, we have drawn it in a differently, so this three actually should be drawn in this way. This is income, this is saving. So, your intercept is higher but the slope is lower. So that means what you have to do? If this is your 1970 to 1981 your post-recession period should be this, intercept is higher. This is 1982 to 1995, intercept higher but the slope is lower, here the slope is small.

So that means marginal propensity to save actually came down in the post-recession period which is quite understandable, post-recession period people are not saving that much for every additional unit of income like they were doing earlier. So, this is actually the accurate case 3 instead of this. This I was drawing randomly. So, this is the case.

So, with this we have now learned how to do the structural break analysis using the Chow and using the dummy. But we do not conduct Chow test for conducting structural break analysis because that is time consuming as well as it has limitation. But I specifically and intentionally showed you both. So that you can understand the advantages of dummy variable over the Chow test in conducting the structural break analysis and you appreciate more about the dummy variable model.

That was the objective I had in my mind, that is the reason I conducted and I demonstrated both the tests Chow as well as dummy. So, with this we are closing our discussion today and in our next day we will see one more application of dummy variable model. We will see another application. And our discussion today is closed here onwards. Thank you very much.