

**Statistical Analysis of Dummy Variable Models and Testing for Seasonal Fluctuations**  
**Part-3**  
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**Lecture 40**  
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Welcome to the class of discussion of Dummy Variable Model once again. So today what we will do? We will take another dummy variable model that actually we have discussed earlier in the context of Chow test. Do you remember that we were discussing structural break analysis in time series data particularly the savings income example in the context of U.S.? The same example, same data set will take here and what we will do?

We will do the same structural break analysis using dummy variable model. Why this is so? Because dummy variable model will overcome the short comings of Chow test. And what were the short comings of Chow test? Because Chow test cannot tell us if there is any structural break and what is the source of that.

So, what we will do? We will quickly recap the structural break using Chow test and then we will come back to the dummy variable model and we will see how to estimate the same thing using a dummy.

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The slide content includes the following handwritten notes:

Structural break analysis

Using Chow test:  $y_t = \alpha + \beta x_t + u_t \rightarrow 1970-1981$       1970  
 $y_t = \alpha_0 + \beta_1 x_t + u_{2t} \rightarrow 1982-1995$        $\vdots$  1982  
 $y_t = \lambda_0 + \lambda_1 x_t + u_t \rightarrow 1970-1995$       1995

$$F_{stat} \leftarrow \begin{matrix} \alpha = \alpha_0 = \lambda_0 \\ \beta = \beta_1 = \lambda_1 \end{matrix}$$

$$\frac{(RSS_R - RSS_{UR})/df = R}{RSS_{UR}/df = n_1 + n_2 - 2R}$$

$\sim F_{R, n_1 + n_2 - 2R}$   
 $\sim F_{2, 22}$

On the left side, there is a vertical derivation:
 
$$\begin{aligned} & \frac{RSS_R - RSS_{UR}}{n_1 + n_2 - 2R} \\ &= \frac{RSS_R - RSS_{UR}}{n_1 + n_2 - 2R} \\ &= R \end{aligned}$$

The slide also features a small video inset at the bottom left showing a man in a blue shirt sitting at a desk.

So, this is structural break analysis once again. So, what we are doing? So, we had some 1970 to 1995 data on saving and income and then in between we assume that in 1982 there was a structural break because that was the time U.S. economy was suffering from the peacetime recession.

So, what we did actually? We first ran three regressions one for the time period 1970 to 1981. So that was  $y_t = \alpha + \beta x_t + u_{1t}$ , And then another one for  $y_t$  equals to let us say some  $\alpha_0 + \beta_1 x_t + u_{2t}$  which is for 1982 to 1995. So, these are the unrestricted models and the restricted model was  $y_t = \lambda_0 + \lambda_1 x_t + u_t$ , this is for the entire period 1970 to 1995.

So, one for the pre pre-recession and another for post-recession and the third regression is for the entire period. And we were checking the stability analysis whether the parameters  $\alpha$  equals to  $\alpha_0$  equals to  $\lambda_0$  or  $\beta$  equals to  $\beta_1$  equals to  $\lambda_1$ , that is what we were testing.

Now these notations might be different earlier might you some other notations. So, please be careful about the notation, do not get misguided. So, I might be using different notation at different point of time. So basically, the idea is same, what we are doing, that we are testing  $\alpha$  equals to  $\alpha_0$  plus equals to  $\lambda_0$  and then  $\beta$  equals to  $\beta_1$  equals to  $\lambda_1$ , this is the restriction what we are testing.

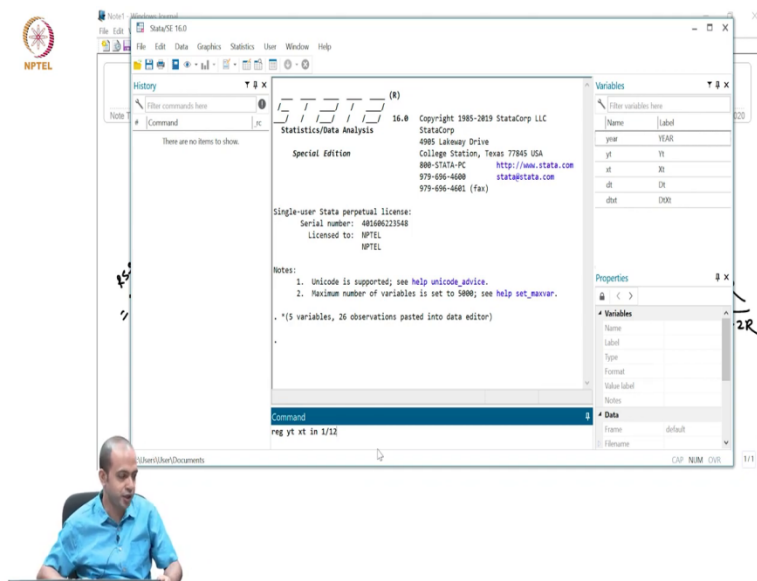
And we are using F statistic. How that is defined? What is the restricted model? This one is the restricted model. When I am specifying one single equation for the entire period, that is our restricted model. And then  $RSS_{ur}$  divided by its degrees of freedom.


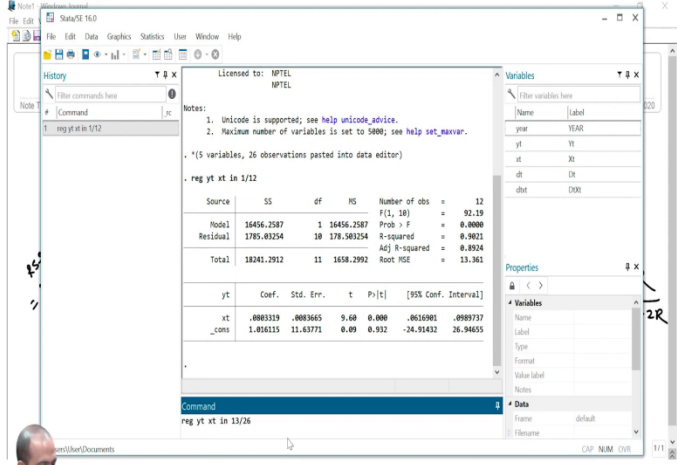
And if you recall what is a degrees of freedom for  $RSS_r$  minus  $RSS_{ur}$ ? Because that is  $RSS_r$  what is the degrees of freedom?  $n$  minus  $k$ . And  $RSS_{ur}$  how you are getting  $RSS_{ur}$ ?  $RSS_{ur}$  was  $n_1$  minus  $k$  plus  $n_2$  minus  $k$ . So it was  $n_1$  plus  $n_2$  minus  $2k$ . And this is  $n$  minus  $k$ . So this is basically  $n_1$  plus  $n_2$  equals to  $n$ . So I can write  $n$  minus  $2k$ .

So  $RSS_r$  minus  $RSS_{ur}$  equals to  $n$  minus  $k$  minus  $n$  plus  $2k$  equals to  $k$ . So, degrees of freedom for the numerator is equal to  $k$ . And what is the degrees of freedom for this  $RSS_{ur}$ ? Which is  $n_1$  plus  $n_2$  minus  $2k$ ,  $n_1$  plus  $n_2$  minus  $2k$ . Because there are two unrestricted models. So now what you can do? This follows the F statistic with degrees of freedom  $k$  for the numerator and  $n_1$  plus  $n_2$  minus  $2k$  for the denominator.

So that means here it is 2 and 22,  $n_1$  plus  $n_2$  equals to 26. There are total 26 periods of time and  $2k$  means 2 into 2, 4. So 26 minus 4, 22. This follows F distribution with 2 and 22 degrees of freedom. So, this is the Chow test what we have discussed already. So we will quickly estimate the two models and we will just go to the data set once again.

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Stata/SE 16.0

History

```

1 reg yt xt in 1/12

```

Command

```

reg yt xt in 1/12

```

Notes:

- Unicode is supported; see help unicode\_advice.
- Maximum number of variables is set to 5000; see help set\_maxvar.

\*(5 variables, 26 observations pasted into data editor)

Source

Source	SS	df	MS	F(2, 10)	Prob > F
Model	16456.2587	1	16456.2587	10	0.0000
Residual	1785.03254	10	178.503254		0.3924
Total	18241.2912	11	1658.2992		13.361

Number of obs = 12

Root MSE = 13.361

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
yt	.088319	.0083665	9.60	0.000	.0616981 .0989737
_cons	1.016115	11.63771	0.09	0.932	-24.91432 26.94655

Variables


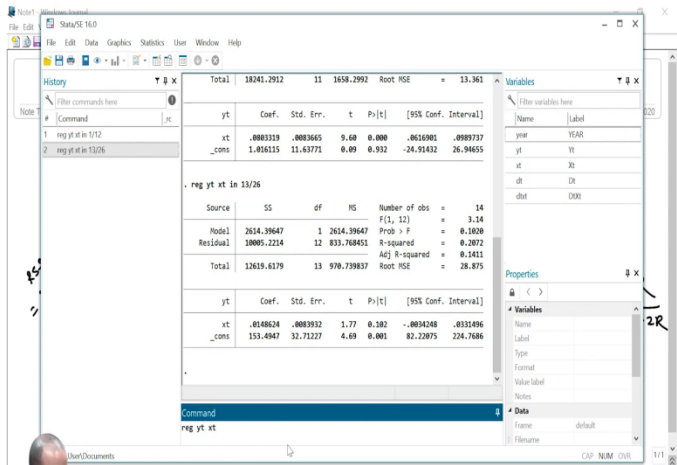
Name	Label
year	YEAR
yt	Yt
xt	Xt
dt	Dt
cbst	DKB

Properties

Variables

Data

1/1

Stata/SE 16.0

History

```

1 reg yt xt in 1/12
2 reg yt xt in 13/26

```

Command

```

reg yt xt

```

Source

Source	SS	df	MS	F(2, 12)	Prob > F
Model	2614.39647	1	2614.39647	12	0.1828
Residual	18095.2214	12	833.768451		0.2072
Total	12619.6179	13	978.798837		28.875

Number of obs = 14

Root MSE = 28.875

	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
yt	.0148624	.0083932	1.77	0.102	-.0034248 .0331496
_cons	153.4947	32.71227	4.69	0.001	82.22075 224.7686

Variables

Name	Label
year	YEAR
yt	Yt
xt	Xt
dt	Dt
cbst	DKB

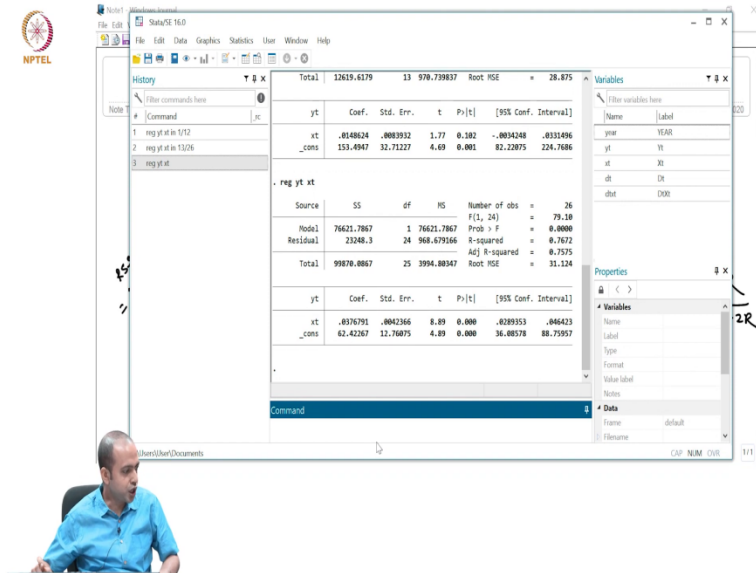
Properties

Variables

Data

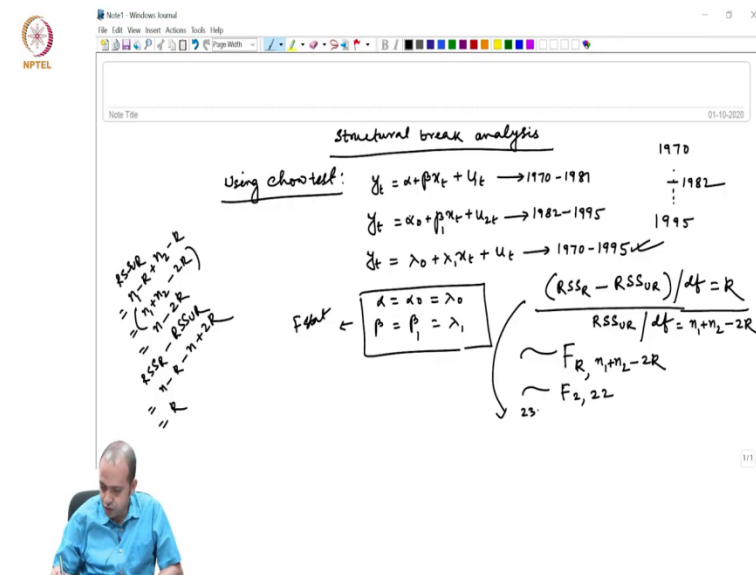
1/1





Now what we will do? We will estimate `reg yt` and then `xt` and we will first estimate for the first 12 observations 1970 to 1981, there are total 12 observations. And the second regression is `reg yt` then `xt` in 13 by 26, put enter. And then another for the entire period `yt xt`. This is what, so now you have to collect the RSS from all these models and calculate the F statistic. So, from the restricted model RSS is 23,248.3.

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The screenshot displays the Stata 16.0 interface with the following data:

Model	SS	df	MS	Number of obs	F(1, 12)	Prob > F	R-squared	Adj R-squared	Root HSE
Total	18241.2912	11	1658.2992	14	3.14	0.0900	0.3021	0.8724	13.363
yt	1456.2587	1	1456.2587	14	3.14	0.0900	0.3021	0.8724	13.363
Residual	1785.03254	10	178.503254	14	3.14	0.0900	0.3021	0.8724	13.363

Model	SS	df	MS	Number of obs	F(1, 12)	Prob > F	R-squared	Adj R-squared	Root HSE
Total	12619.6179	13	970.739837	14	3.14	0.1020	0.2072	0.2412	28.875
yt	2614.39647	1	2614.39647	14	3.14	0.1020	0.2072	0.2412	28.875
Residual	10005.2214	12	833.768451	14	3.14	0.1020	0.2072	0.2412	28.875

So, if you look at this formula RSSr is 20. So, from this what I can say that this is 23,248.3 minus, minus 23,248.3 minus what you have to do you have to take these two RSS, these two RSS 10,005. first one is this 1,785, 1,785. So basically, so I will calculate these in an excel sheet that would be easier, that would be easier for you to understand. So what I will do? I will just, I will just close this, close this and I will open an excel sheet.

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The screenshot shows the Stata 16.0 interface with an Excel spreadsheet overlaid. The spreadsheet contains the following data:

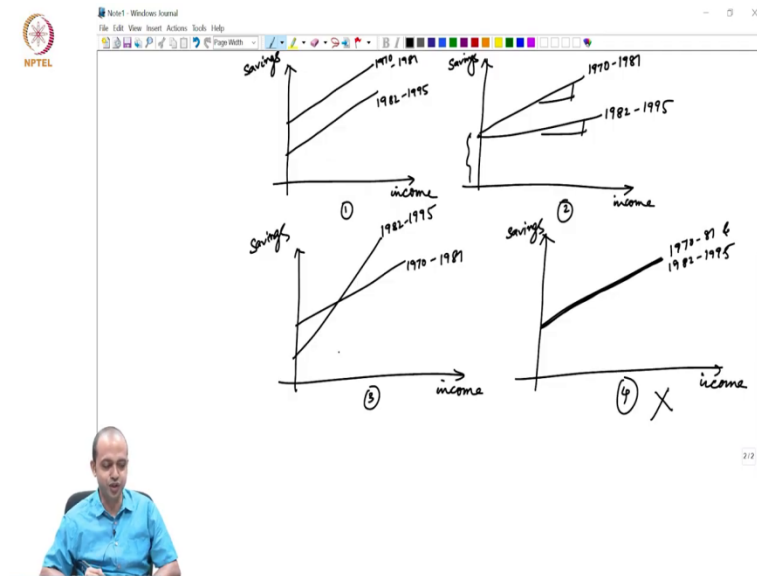
Cell	Value
C4	11458.05
D2	11790.25
D3	5729.024
D4	535.9205
D5	10.69005

This equals to 23,248 I think 23,248.3 minus minus you have 10,005, 10,005.2214 plus, plus the other one, other one is 1,785, 1,785.03. This is your RSSr minus RSSur. And this you have to



are satisfied that means there is no structural break. Pre and post finance recession period can be modeled by one single equation. So, both the intercept and the slope coefficients are same. So, no structural break is actually rejected.

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And what we said when this is rejected that means Chow test is indicating three situations. So here it is income and here it is saving. So, this is let us say 1970 to 1981 and this is 1982 to 1995, this is case 1. So, in this case post-recession period, the slope is same but the intercept is different.

In this case income-saving, so this is 1970 to 1981 this is 1982 to 1995. So intercept is same but the slope is different which is case 2. And this is case 3, income and saving, so it is 1970 to 1981, this is 1982 to 1995. So post-recession period both slope and intercept are different, this is case 3.

And this is actually case 4 saving and this is income. So this basically indicate 1970 to 81 also 1982 to 1995 for both the period neither the slope nor the intercept is different. So that means this is called concurrent relationship. So, both the time periods can be specified by one single equation.

So, Chow test can tell you that this case 4 is actually rejected. So, we are eliminating case 4 there are differences in the two time periods saving income relationship but we do not know which among this three are actually the relevant for this particular data set. So that means the difference



in saving income relationship is coming due to intercept or slope or both that we do not know. And here comes the dummy variable model handy. Now we will see how to do the same analysis using a dummy variable model.

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The slide displays the following content:

$$y_t = \alpha + \beta_1 x_t + \beta_2 D_t + \beta_3 (x_t * D_t) + u_t$$

Annotations on the slide:

- $y_t$  is labeled "Savings"
- $x_t$  is labeled "income"
- $D_t = 1$  if 1982-1995,  $= 0$  otherwise
- Conditional expectation for  $D_t = 1$ :  $E(y_t | D_t = 1, x_t) = (\alpha + \beta_2) + (\beta_1 + \beta_3) x_t$  (1982-1995)
- Conditional expectation for  $D_t = 0$ :  $E(y_t | D_t = 0, x_t) = \alpha + \beta_1 x_t$  (1970-1981)

So, let us first try to construct the model. Let us say that this is our  $y_t$  equals to alpha plus beta 1  $x_t$  plus beta 2  $D_t$  plus beta 3  $x_t D_t$  plus  $u_t$  this is our model. And how we have defined  $D_t$ ?  $D_t$  equals to 1 if the time period is 1982 to 1995 and 0 otherwise.

Because when we say that the crisis or recession so presence of recessions in when the impact of recession is present that we are denoting by 1. Because in the dummy variable generally we say that presence of the attribute is 1 and absence is 0. So post-recession period that is 1 and 0 otherwise. So, this is how we have defined the dummy variable.

So now again here what is  $y_t$ ?  $y_t$  is your actually savings and  $x_t$  is actually income and if you take expectation, expectation of  $y_t$  given  $D_t$  equals to 1 and  $x_t$  equals to alpha plus beta 2 and then plus you have beta 1  $x_t$  here also you will get beta 3  $x_t$ . So, I am adding the two coefficients and then I am writing  $x_t$ . So that means for the period 1982 to 1985 this is my saving income relationship with slope intercept equals to this and slope equals to this, this is for the period 1982 to 1995.

Post-recession period saving income relationship is defined as expectation of  $y_t$  given  $D_t$  equals to 1 equals to alpha plus beta 2 plus beta 1 plus beta 3  $x_t$ . So that means this is the intercept and

this is the slope. And pre-recession period how it is defined? Expectation of  $y_t$  given  $D_t$  equals to  $0$   $x_t$  equals to  $\alpha$ . So, this will vanish  $\alpha + \beta_1 x_t$ ,  $\beta_1 x_t$ . So, this is for the period 1970 to 1981.