

**Introduction to Econometrics**  
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**Lecture 32**  
**Structural break analysis using Chow Test Part - 5**

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The screenshot shows a Windows Journal window with the following content:

**Savings** =  $\alpha + \beta \text{income}_t + U_t$

$y_t = \alpha_0 + \alpha_1 x_t + U_t \rightarrow 1970-1981 - \text{RSS}_1$

$y_t = \lambda_0 + \lambda_1 x_t + U_t \rightarrow 1982-1995 - \text{RSS}_2$

$\alpha_0 = \lambda_0 = \alpha$   
 $\alpha_1 = \lambda_1 = \beta$

$n_1 = 12$   
 $n_2 = 14$   
 $n = 26$

$F = \frac{(RSS_R - RSS_{UR})/df}{RSS_{UR}/df}$

$= \frac{(RSS_R - RSS_{UR})/R}{RSS_{UR}/(n_1 + n_2 - 2R)}$

$\sim F(2, 22)$

Handwritten calculation:  $\frac{(23,248 - 11,740.25)/2}{10.45} = F_{tab}(2, 22)$

Table:

Savings	income
1970	1970
...	...
1981	1981
...	...
1995	1995

Annotations: "26 yrs." between 1970 and 1995; "26 yrs." between 1970 and 1995.

Formulas:

$RSS_R = (n - R)$

$RSS_{UR} = RSS_1 + RSS_2$

$= n_1 - R + n_2 - R$

$= (n_1 + n_2) - 2R$

$RSS_R - RSS_{UR} = n - R - n$

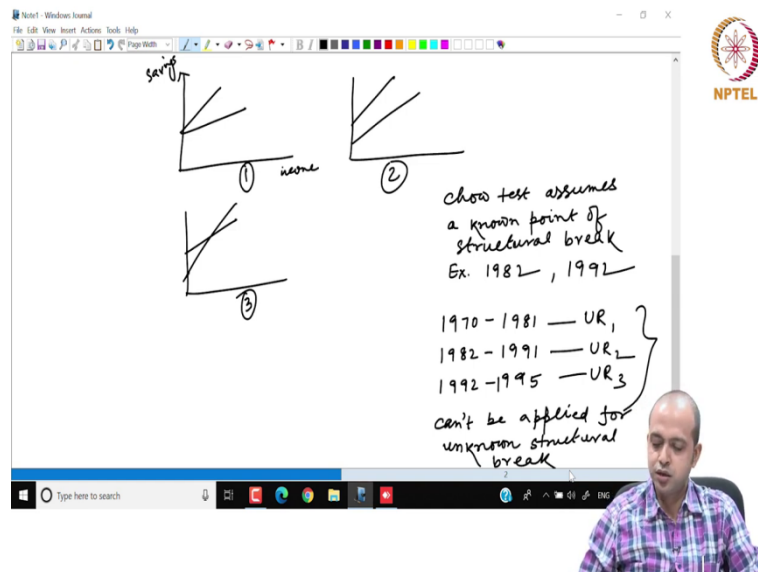
If you compare the calculated value of F with the tabulated value of F from the F table then what we have to do? You have to check the F value from the F table at 1 percent, 5 percent and 10 percent. But, since F value is more than, is almost 10 we can we without checking the table also we can say that this is highly significant. This value is highly significant so your F calculated would be F tabulated also. So, you can check this 10.45 from the table and you can confirm.

So, since this F calculated is greater than F tabulated at 1 percent and 5 percent level of significance at 1 percent as well as 5 percent. So, you can say that that means we are rejecting our null hypothesis. And what was our null hypothesis? Null hypothesis was there was no structural break. There was no structural break so that means post-recession period also there is no struck significant change in the individual's saving behaviour.

So, savings income relationship got unchanged even the post-recession period also that was our null hypothesis. Since this null hypothesis is rejected that means we can conclude that post-recession period of US economy undergone significant change in terms of the savings

and income relationship. So, that means there is a drastic structural break. So, structural break is significant that we can say. Alright. But we have to carefully keep in mind that when we say the structural break is there if you recall previous class what we were discussing yesterday.

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That when we say that there is structural break. Structural break may happen due to change in slope. Let us say this is income, if you recall, this is saving or due to intercept or due to both. So, Chow test is only informing us that there is change in the saving income relationship. So, that means we cannot fit one single equation one single model to estimate the relationship for the entire period. But we cannot say from this Chow test this change is due to change in slope or intercept or both.

So, which among these three possible cases are there, that we are not sure about. So, Chow will only tell you whether there is change or not it cannot inform you about the source of that structural change that you have to clearly keep in mind. And also, in the process we have made several assumptions. What is that assumption? The main important assumption that chow test makes is that you know the number of structural breaks and when it had happened.

For example, in this case we know that there is only one structural break and it happened in 1982. You can extend this analysis. So, chow test assumes a known point of structural break. In known point of structural break in this case here it is 1982. Let us now assume that there is

one more structural break that happened in US economy, that is in 1992, probably when George Bush, let us assume that there is there is one more structural break happen in 1980, 1992 also.

So, there are two structural breaks so you can extend this analysis in the context of two structural breaks also. In that case what you have to do? Instead of running two unrestricted models, you will have three unrestricted models. So, one for 1970 to 1981 that is unrestricted model 1. Then another for 1982 to 1991 that is unrestricted model 2 and then another from 1992 to 1995. So, this can be extended in the context of multiple structural breaks also.

But what Chow test cannot handle is, in case of unknown period of structural break, that means you have a data point but you do not know in which particular year the structural break happen. In that case the Chow test is not applicable. So, it cannot handle cannot be applied for unknown structural break. So, for that we have to apply advanced time series econometric model. Advanced time series econometric model we have to apply.

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② two subsamples are independent  
 $RSS_{UR} = RSS_1 + RSS_2$   
 -  $u_{1t}$  &  $u_{2t}$  are independent  
 $u_{1t} \sim N(0, \sigma^2)$   
 $u_{2t} \sim N(0, \sigma^2)$

Applications of F test

①  $y_i = \alpha + \beta x_i + u_i \rightarrow$  sig  $\beta$  using t test  
 $F = t^2$

②  $y_i = \alpha + \beta_1 x_i + \beta_2 x_i + u_i$   
 $H_0: \beta_1 = \beta_2 = 0 \rightarrow F = \frac{ESS/R-1}{RSS/(n-R)} = \frac{n-R}{R-1} \cdot \frac{ESS}{TSS-BSS}$   
 $= \frac{n-R}{R-1} \cdot \frac{ESS/TSS}{1 - ESS/TSS}$   
 $= \frac{n-R}{R-1} \cdot \frac{R^2}{1-R^2}$   
 $R^2 \rightarrow 1$   
 $F \rightarrow \infty$

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Another assumption, also important assumption that that Chow test makes is that we have added the two unrestricted models to arrive at the final unrestricted model. That means we have added RSS UR1 plus RSS UR2 to arrive at RSS UR. So, what is the assumption we are making? The assumption is that the two periods two sub samples are independent. And that is

the reason we are doing this  $RSS_{UR}$  equals to  $RSS_1$  plus  $RSS_2$ . And this also implies that  $u_{1t}$  and  $u_{2t}$  are independent.

And we also assume both  $u_{1t}$  and  $u_{2t}$  follows a normal distribution with 0 mean and sigma square variance. So, that means both the error terms, they are independently but identically distributed. So, both of them are having variance of sigma square. So, in case they are not independent and they are not identically distributed then we cannot apply the structural break analysis using the Chow test. Chow test cannot be applied.

So, these are the assumption you must make. You must remember the assumptions of Chow test and also the limitation. Limitation is, once again I repeat that it cannot inform you what is the source of the structural break. Is it due to the change in the slope or due to the change in intercept or due to change in both? This is the major limitation. And secondly it also assumes that these two sub periods have independent error terms that is how we are calculating  $RSS_{UR}$  using the summation of  $RSS_1$  plus  $RSS_2$ . Error terms are independently and identically distributed.

So, with this we are basically completing our discussion on hypothesis testing in the context of multiple regression model. And we have now learned basically several applications of F test and I will just quickly summarize what are the applications of F test that we have learned here. So, I will quickly summarize. Applications of F test. It is a powerful test. So, first we learn that using F test what you can do? You can check the individual significance as well as overall significance of the model.

For example, when your model is  $y_t$  equals to  $\alpha + \beta x_t + u_t$  instead of  $y_t$ . I will just write  $y_i$  in the context of cross-sectional data which is  $\alpha + \beta x_i + u_i$ . So, when you have only one explanatory variable, F test can be applied but it does not have additional significance. Because, since you have only one explanatory variable and the significance of this explanatory variable let us say it is consumption and this is income, that you can, using t test itself you can get using t test.

So, significance of x variable or significance of beta through using t test. And in that case F would be simply  $F$  equals to  $t^2$ . So, you can apply but it does not give you additional information you can simply apply t test. But when you have more number of explanatory

variable let us say  $\alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + u$ , then actually you can get additional significance for F.

What you can do? In this case you can test this type of hypothesis  $\beta_1 = \beta_2 = 0$ . That means we can test the overall significance of the model using F. So, this you can do using F. This is called overall significance of the model. And overall significance of the model how we are doing it?

If you recall, that we were deriving some kind of relationship between F and R square because initially F was defined as  $\frac{ESS}{k-1} \div \frac{RSS}{n-k}$ . So, it would become  $\frac{n-k}{k-1} \times \frac{ESS}{RSS}$  you can write keep ESS but this RSS you can change as TSS minus ESS.

So, this would become  $\frac{n-k}{k-1} \times \frac{ESS}{TSS - ESS}$  if you divide numerator and denominator by TSS, so this would become  $\frac{ESS}{TSS} \div \frac{1 - \frac{ESS}{TSS}}{1}$ . So, that means  $\frac{R^2}{1 - R^2}$ . So, from the model what you are estimating? If you put the values of R square you will get the values of F.

And in case R square tends to 1 then what I will get? F will tend to infinity, that we have already discussed. So, from this relationship we can do two things. We can check overall significance of the model and also, we can test the significance of R square itself. Higher the value of F, higher would be the probability will say that R square is significantly different from zero. That is also we have learned another application of F test.

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③ equality of two regression coeff  
 $\ln y_i = \beta_0 + \beta_1 \ln x_{1i} + \beta_2 \ln x_{2i} + u_i$   
 $H_0: \beta_1 = \beta_2$

④ validity of a linear restriction  
 $H_0: \beta_1 + \beta_2 = 1$

⑤ structural break analysis

test labor = Capital

test labor + Capital = 1

$F = \frac{(RSS - RSSR) / df}{RSSR / df}$   
 $H_0: RSSR = RSSUR$

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Then thirdly we have learnt using F test the equality between two regression coefficients. And in the context of a production function, what we are discussing? That when your model is like this  $\log y_i$  equals to  $\sum \beta_0$  plus  $\beta_1 \log x_{1i}$ , let us just forget about  $t$ , simply put  $y_i$  this is output is a function of labor and capital  $\log x_{2i}$  plus  $u_i$ .

Then you can also test whether returns from labor and capital they are same or not. So, that means what is the null hypothesis you can do?  $\beta_1$  equals to  $\beta_2$ . That also, we can test using the F statistic. This is equality between two regression coefficients. Then fourthly, we can also test the validity of a linear restriction using F stat.

So, that means in this particular context of a production function we can also test whether  $\beta_1$  plus  $\beta_2$  equals to 1 or not. So, that means we are interested in checking whether the returns to scale from this particular production function is constant or not. That also we can test using the F statistic. And what is our alternative hypothesis? The alternative hypothesis is that the production function exhibits non-constant returns to scale.

And lastly, what we have learned is the structural break analysis using F statistic. Structural break analysis using the F statistic. This is the last thing we have learned applying F test or Chow test. So, the idea is very simple that all the times you put the restriction and basically you derive the restricted model and you already have one unrestricted model. If you closely think about the basic philosophy of this F test is, what we are doing actually? We are trying to

impose the restriction in the context of this production function, we are putting this restriction  $\beta_1 + \beta_2 = 1$  and then we are trying to get a restricted version of the model and applying OLS.

If you remember, we called that model as restricted least square approach in our previous class. And then the numerator, there is  $RSS_R - RSS_{UR}$ . If you look at the, this is the model  $RSS_R - RSS_{UR}$ .

So, this is the numerator and denominator is  $RSS_{UR}$ . Then its corresponding degrees of freedom. So, in the case of both linear validity of checking the validity of linear restriction as well as structural break analysis, the F statistic formulation is same. That means what is the basic philosophy here? Basic philosophy here is the numerator is  $RSS_R - RSS_{UR}$ . So, that means greater the difference between  $RSS_R$  and  $RSS_{UR}$  higher would be the value of F.

And higher is the value of F higher would be the probability of rejecting null hypothesis. And what is the null hypothesis? Null hypothesis is that the restriction is not valid. So, that means there is no structural break or there is no constant returns to scale. This is the null hypothesis. So, alternatively what we are assuming actually that  $RSS_R = RSS_{UR}$ . That is actually we are testing. This is an alternative formulation of our null hypothesis. Whether you are checking 4 or 5, since the formulation of F statistic is same, so what we are checking in the context of 4?

When I am saying that there is no significant difference between  $RSS_R$  and  $RSS_{UR}$  that means I am saying this restriction is a non-binding restriction. The restriction what I am putting  $\beta_1 + \beta_2 = 1$ , it is non-binding. This is not valid; this is not a valid restriction. If it is rejected, then we will say that the production function exhibits non-constrained inter scale.

And if you fail to reject, that means this restriction is actually valid and that means production function exhibits constant returns to scale. Similarly, in the context of 5 when you do not see if there is no significant difference between  $RSS_R$  and  $RSS_{UR}$  obviously the calculated value of F would be very low. And if the calculated value of F is low then you will

not be able to reject your null hypothesis. And what is my null hypothesis? That there is no structural break.

So, this is the basic philosophy of our F statistic. We have learned several ways of applying F statistic and we have also learned what is the STATA command for applying F test. Firstly, we learnt that when you are testing the validity of a restriction it is simply test command and then test, in case of labour and capital you have to just put the variable name. Let us say labour equals to capital or whatever name you give. So, this is one common we learn STATA command.

And we also learn test labour plus capital equals to 1. This is another command; labour plus capital equals to 1. Whatever exercise we are doing manually the same thing you can apply, same thing you can perform using the STATA command. It is very simple. So, with this we are closing our discussion on hypothesis testing in the context of multiple linear regression model. And in our next class, in our next lecture we will be discussing about the dummy variable models.

What happens when some of your explanatory variables are qualitative in nature? Because so far whatever model we have discussed, in all these models, all your explanatory variables can be quantitatively measured. But what we will do when some or all my variables in the right-hand side are qualitative in nature? We need to learn a specific technique to estimate that type of model and those variables are called dummy variables, that will discuss in our next class.

Thank you.