Introduction to Econometrics

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Lecture 26

Multiple linear regression model and application of F statistics Part - 5

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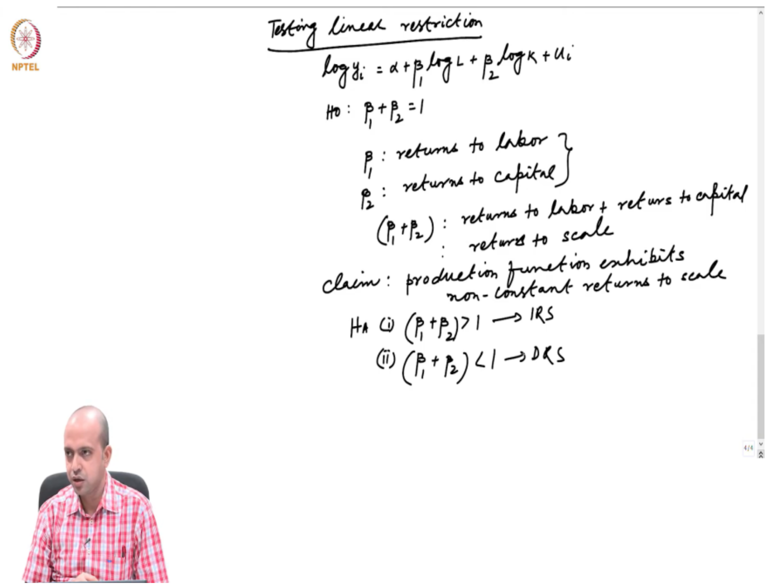

We will now move on to another type of hypothesis, hypothesis number 2 which is called testing linear restriction. For that, what we will do? We will take that same production function where log of yi equals to alpha plus let us say beta 1 log of labor plus beta 2 log of capital plus ui. This is the kind of production function that we are hypothesizing.

And we are testing linear restriction, basically let us say our null is this - beta 1 plus beta 2 equals to 1. Let us say this is our hypothesis, this is our null. Now, what is the meaning of this? Once again, we need to understand. When you are testing the validity, this is testing the validity of linear restriction actually. What is the meaning? So, testing validity of linear restriction means, let us say beta1 equals to beta2, beta1 plus beta2 equals to 1, that is what we are going to test.

What is the meaning of this? If this is a production function, then beta1 basically indicates returns to labor and beta2 indicates returns to capital. And if you take summation of these two, beta1 plus beta2, then that is what, that means returns to labor plus returns to capital. And this has a different name, when you add returns to both the factors, then as you know, from your understanding of micro-economic theory of production function, that is basically called returns to scale.

That means when both the factors are changing, then it is called returns to scale. When one factor is changing, that is called returns to factor. And as you know, in a production function, we may have 3 alternative cases, increasing returns to scale, decreasing returns to scale and constant returns to scale. So, our claim, what is our claim? Our claim is, let us say, production function exhibits non-constant returns to scale. We do not know whether it is IRS or DRS, but at least our claim is that it is not constant.

And then when you nullify that claim, so that means our alternative is basically, alternative are two types, beta 1 plus beta 2 could be either this or beta 1 plus beta 2 could be either this, this is IRS and this is DRS. So, we are hypothesizing actually the possibility of these two cases, that is our claim, and when we nullify our claim, our null becomes beta1 plus beta2 equals to 1, constant returns to scale. If that is the case, then how will you test?

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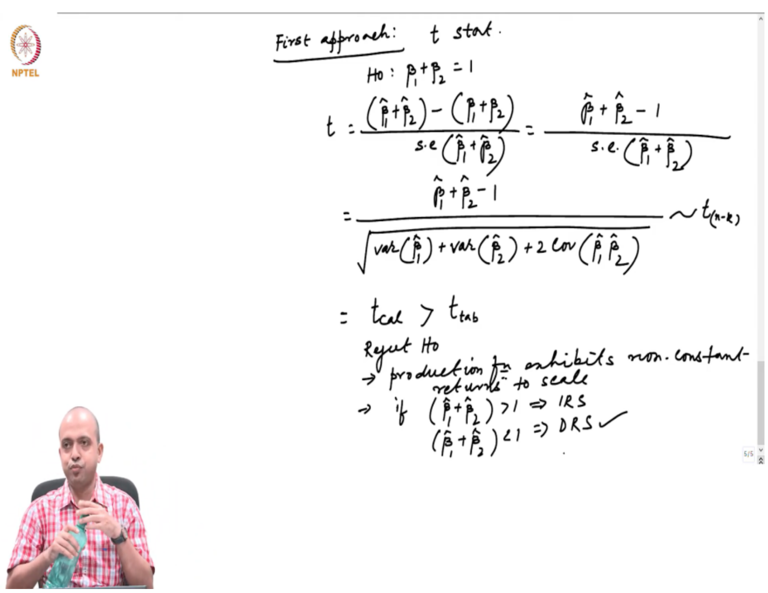

So, first we will follow, there are two approaches. The first approach is, this is first approach, I will write this as first approach where I am going to use t statistic. So, your null is this - H naught beta 1 plus beta 2 equals to 1. So, if that is case, then how will you formulate your test statistic?

So, it would become estimated value of your population parameter, that means beta 1 hat plus beta 2 hat minus hypothesized value of the population parameter. So, that means this first I will write beta1 plus beta2 divided by standard error of beta1 hat plus beta2 hat. That is what we learnt. Now, hypothesized value of this, what is the value? We have hypothesized beta1 plus beta 2 equals to 1, that is why your numerator would become beta1 hat plus beta2 hat minus 1 divided by standard error of beta1 hat plus beta2 hat.

See this test statistic expression is little different from the earlier cases. Earlier, we used to write beta1 hat plus beta2 hat minus 0. Or beta1 hat minus 0 divided by standard error of beta hat. So, that means in general when nothing is mentioned, we hypothesize the value of the true population parameter is 0, because our objective is to test the statistical significance of the variable, whether a particular variable, whether the particular coefficient is statistically different from 0 or not. That is our hypothesis.

So, obviously if that is the hypothesis, then what will happen? It would become minus 0. But here, this is different. It is beta1 hat plus beta2 hat minus 1 because we are hypothesizing the value of the true population parameter is 1. And what is the population parameter here? It is beta 1 plus beta2. And again, what is the standard error of beta1 hat plus beta2 hat? It is again square root of the variance. So, this would become beta1 hat plus beta2 hat minus 1 divided by variance of beta1 hat plus variance of beta2 hat plus 2 into covariance of beta1 beta2.

And once again, after doing all this, you will get a t value which is called t calculated. And if that is greater than t tabulated, so this will follow a t distribution with n minus k degrees of freedom and this would be t tabulated. If this is this, then reject our null. And what was your null? Beta1 plus beta2 equals to 1. When you reject your null, you can only say the production function exhibits non-returns to scale.

But you are not sure whether it is increasing returns to scale or decreasing returns to scale. So, when you reject your null, you can only say that production function exhibits non constant returns to scale. But which particular non constant you will get? That depends on if beta1 hat is greater than beta2 hat, sorry if beta1 plus beta2 hat, if beta1 hat plus beta2 hat is greater than 1 then we will say that this is IRS. If beta1 hat plus beta2 hat is less than 1, then we will say that DRS.

So, that depends on, so that means once you reject your null, then in the second stage, you have to see which particular case, that means you have to see the value of beta1 hat and beta2 hat, you sum it up and then see if beta1 hat plus beta2 hat is actually greater than 1, then we will see that the production function exhibits constant returns to scale.

If beta1 hat plus beta2 hat is less than 1, then we will say that production function exhibit diminishing returns to scale. So, that means immediately after rejecting your null you cannot say anything. Immediately after rejecting your null you can only say that production function exhibit non-constant returns to scale.

After that, you will look into the values beta1 hat and beta2 hat, you will sum it up, then you will see which among these two alternative cases actually is suitable in a particular context. That is how you need to apply your t statistic to test this type of hypothesis. This is the first approach. There is one more approach to test this hypothesis and that is the second approach.

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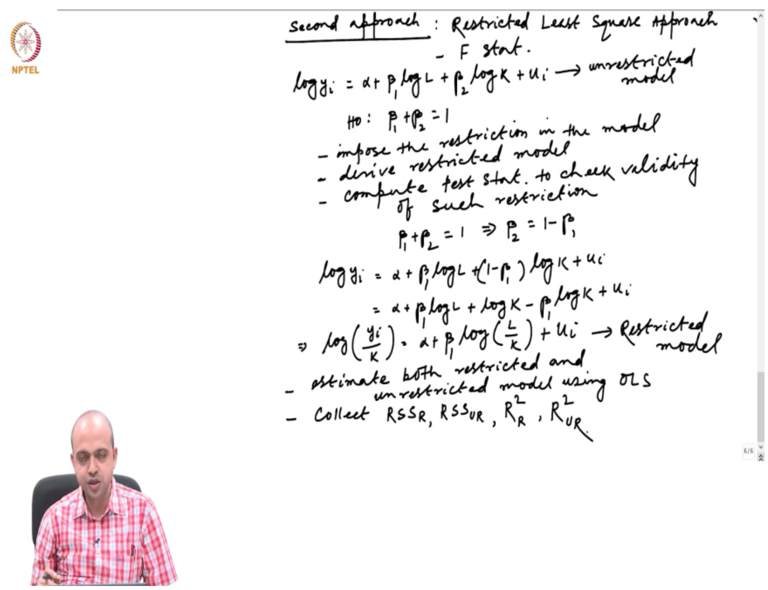

And the second approach is known as restricted least square approach using F. So, here restricted list square approach will use F statistic. So, I will write the production function once again. Log of yi equals to alpha plus beta1 log of labor plus beta2 log of capital plus ui. And what is the hypothesis that we are going to test? H naught is beta 1 plus beta 2 equals to 1.

Now, this least square approach, this alternative approach says if you want to test this, you impose this restriction in the production function itself and then derive a restricted version of the production function and then you derive your test statistic. And then you check the validity of such restriction. So, what they are saying? You impose the restriction in the production function, derive a restricted version of the production function, estimate your test statistic and then you see the validity of the restriction.

So, what I am saying? The first step is to impose the restriction in the model and second step is then, derive restricted model, then you compute your test statistic to check validity of such restriction. So, that means what I will do? How can you impose the restriction here? If beta 1 plus beta 2 equals to 1, that implies that, I can say that either beta 1 equals to 1 minus beta 2 or beta 2 equals to 1 minus beta 1.

So, what I am saying? I am saying let us say, beta 2 equals to 1 minus beta 1. Then you impose this restriction here. If we impose the restriction, then we can say that log of yi equals to alpha plus beta1 log l plus 1 minus beta 1 log k plus ui. So, this would become alpha plus beta1 log l plus log k minus beta1 log k plus ui. So, you can take this log k this side, so that would become log y minus log k which will in turn become log of yi by k equals to alpha plus beta 1 log l minus log k, which you can write plus log of l by k plus ui.

Let us say this is our restricted model. So, this is our unrestricted model. And this is our restricted model. What we have done? We have simply imposed the restriction. You can also put, here I have given beta 2 equals to 1 minus beta 1, you can say that beta 1 equals to 1 minus beta 2. You can derive another version of restricted model.

Then the next step is, once you estimate so the next step is to estimate both restricted and unrestricted model using OLS. And when you estimate the restricted version of the model using OLS, that is known as restricted least square approach in the literature. You can remember this name in the literature whenever you are estimating a model, restricted version of your original model using OLS, it is known as restricted least square approach.

So, estimate both restricted and unrestricted model using OLS and collect RSS residual sum of square from restricted model, residual sum of square from unrestricted model, also R square from the restricted model and R square from the unrestricted model. Once you collect this residual sum of square and R square from both the models, you will have your test statistic using this formula.

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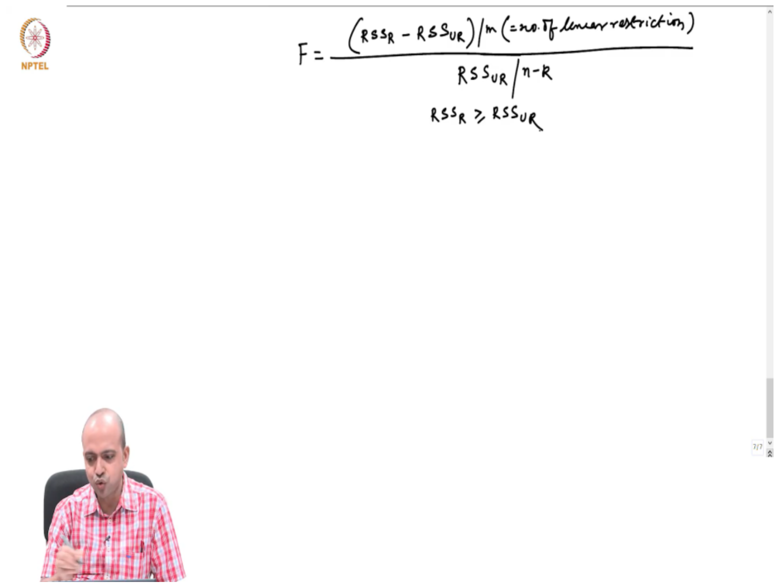

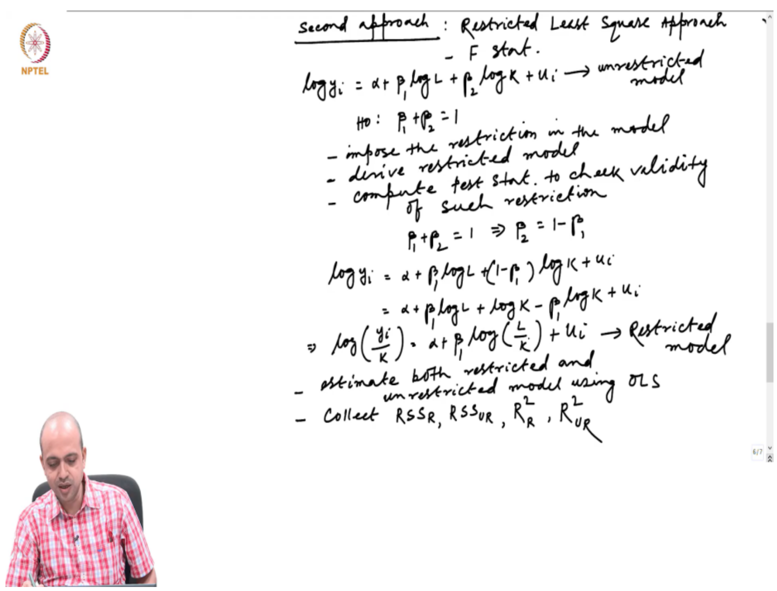

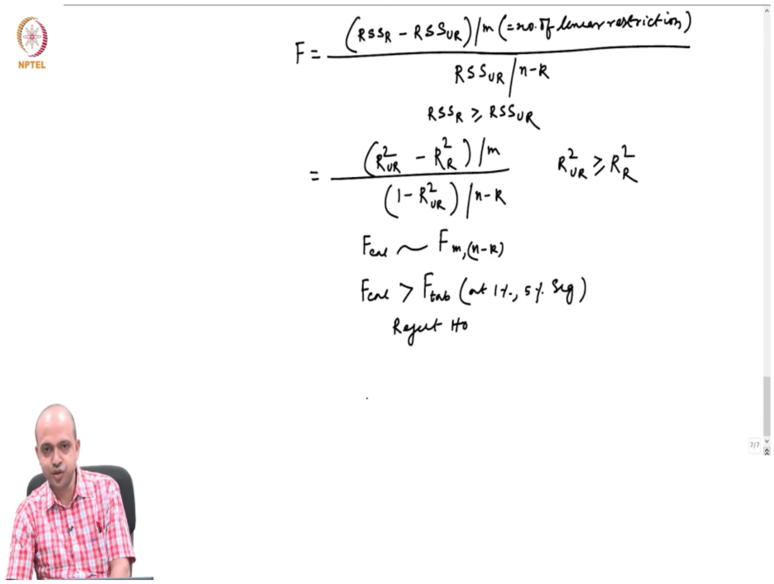

What would be your test statistic? Your test statistic F is defined in this way. RSS_R minus RSS_{UR} divided by m where m is number of linear restriction. What is the number of linear restriction here? That is 1, divided by RSS_{UR} divided by n minus k, where k is the total number of parameters in the unrestricted model. Now, you may get sometime confused that in the numerator should it be RSS_R minus RSS_{UR} or RSS_{UR} minus RSS_R.

You have to keep one thing in mind, that this is the relationship between RSS_R, between RSS_{UR}. The restricted RSS, that is RSS from the restricted model is always greater than or equal to RSS_{UR}. Why this is so? In the restricted model, you have only one explanatory variable, that is l by k. However, in the unrestricted model, you have two explanatory variables. So, obviously when number of explanatory variable goes down, the RSS would become higher, that is what, or at least higher or equal. At least it will not come down. So, that means RSS in the restricted model should be more than or equal to RSS_{UR}, since in the restricted model, you have less number of explanatory variables.

If you want to get a R square version of this formula, then that would become like this. So, RSS_R you can write as, that means this would become R square. Now, among R square UR and R square R, you can easily understand that R square in the unrestricted model should be greater than, this would become m and this would become 1 minus R square UR. Why am I writing this? Because R square UR is greater than or equal to R square R, that is also clear.

Because in the unrestricted model, since you have (())(23:42) variable, your R square is higher, because R square and RSS, there is some kind of inverse relationship. When R square increases, RSS will come down. When R square decreases, RSS will go up. That is the relationship between R square and RSS and that is why, if this is the RSS version of the formula, this would become the R square version of the formula.

And then again, this will give you F calculated which will follow a F distribution, what would be the degrees of freedom? m for the numerator and n minus k for the denominator. And if this calculated F is greater then, F tabulated at these degrees of freedom and 1 percent or 5 percent level of significance, then reject your null. That means, beta1 plus beta2 equals to 1, that hypothesis you can reject.

And if you reject this, then depending on the values of beta1 hat and beta2 hat, you will decide whether there is increasing returns to scale or decreasing results to scale. So, this is basically the idea of restricted and unrestricted models of computing your F statistic. So, with this we are just closing our discussion on hypothesis testing.

Next day what we will do? We will take some data set, preferably the production function, we will estimate the production function and then we will test these two important hypotheses, equality between two regression coefficient and beta 1 hat plus, and the validity of a linear restriction. So, these two we will test using a single data set in our next class.

Thank you.