Introduction to Econometrics
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Lecture 25
Multiple linear regression model and application of F Statistics Part - 4

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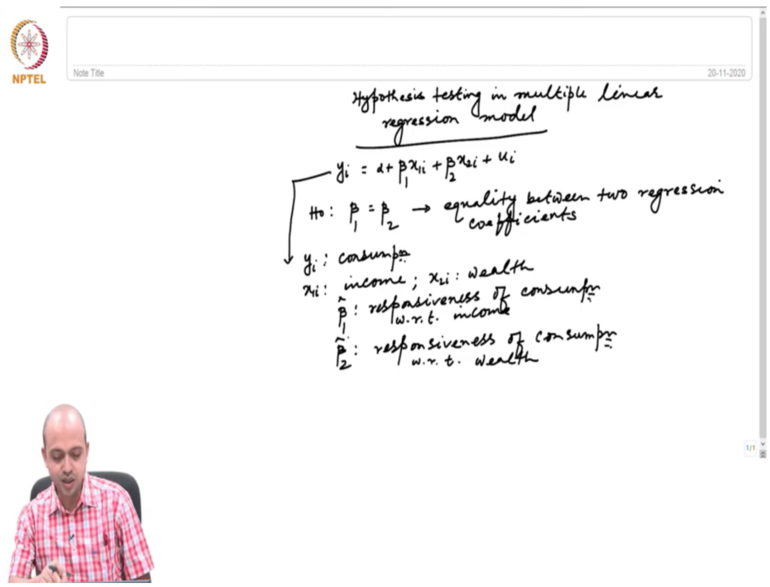

Welcome. So, we were discussing about different types of hypothesis testing in the context of multiple linear regression model. And today, we will discuss another two important types of hypothesis, that also we can test in the context of multiple linear regression model. So, this is hypothesis testing in multiple linear regression model.

Suppose, our model is like this; yi equals to alpha plus beta1 x1i plus beta2 x2i plus ui and the hypothesis that we are going to test is, whether beta1 equals to beta2. This is our hypothesis. So, that means we are testing equality between two regression coefficients. So, what is the name of the hypothesis? Equality. So this is known as equality between two regression coefficients. First of all, we need to understand, why do we require this type of hypothesis to be tested.

So, always we have to understand first the need. We should not do anything purely on a mechanical basis. So, we need to understand, in what particular context, this type of hypothesis is required to be tested. Suppose, in the present example, yi is consumption which is actually function of income as well as wealth. So, what I am saying, in the present context, yi is basically consumption and then x1i is your income and x2i is your wealth.

Now, we want to test whether the responsiveness of consumption. beta1 is basically responsiveness of consumption with respect to income. And beta 2 is basically responsiveness of consumption with respect to wealth. So, we are basically interested in purely empirical question. When income changes by 1 unit, consumption changes by beta 1 amount, rather I would say that this should be beta hat and after estimation this would be beta 2 hat, so when income changes by 1 unit, consumption changes on an average by beta 1 hat amount.

And when wealth changes by 1 unit, consumption on an average changes by beta 2 amount. And we want to know whether these two increments are the same. That means consumer's response towards additional income and towards additional wealth, whether they are similar or not, this is our research question.

If the research question is like that, when the consumer's income increases by 1 unit, what is the change in consumption expenditure? When the consumer's wealth increases by 1 unit, what is the change in consumer's consumption expenditure and whether they are same or not, that is our research question. If that is the case, then we can test this hypothesis whether beta 1 equals to beta 2.

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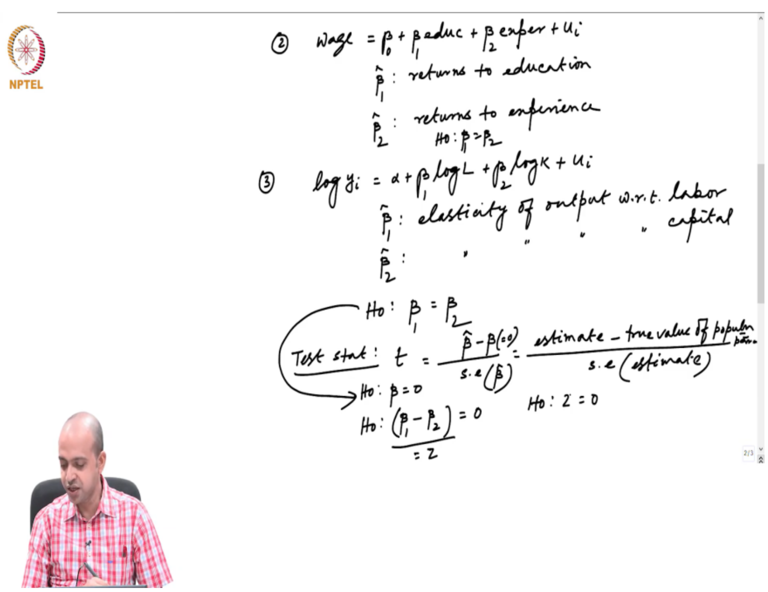

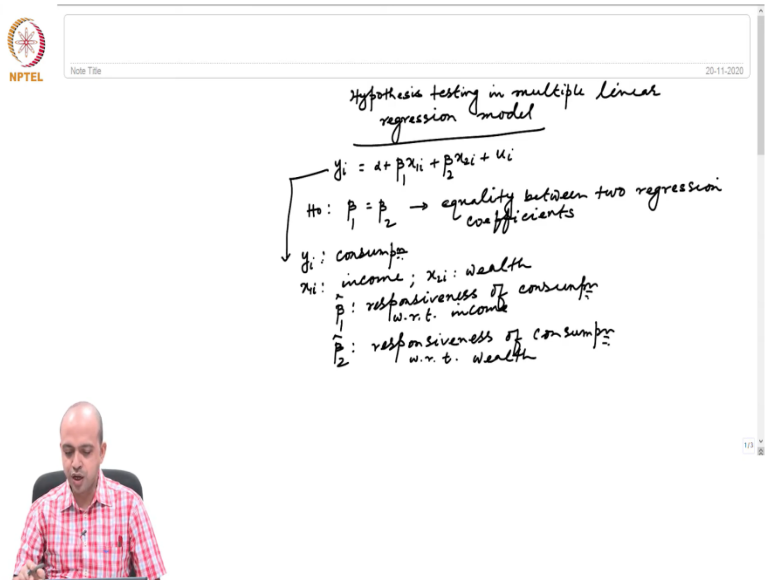

Similarly, we can take another example. Let us say that this is your wage function, this is example number 2, where wage is basically a function of beta 0 plus beta 1 education plus beta 2 experience plus ui. So, that means beta 1 here, after estimation, beta 1 is what, change in wage for a unit change in education, which we can call it returns to education.

And what is beta 2 hat? When you estimate, that will actually return to your experience. So, we want to know whether an individual's wage responds more towards education or towards experience, this is our research question. So, you may have this kind of hypothesis that let us say, education gives more return than experience in a particular job market, so in that case you might say that beta 1 hat is actually greater than beta 2 hat, so obviously, your hypothesis would be beta 1 hat equals to beta 2 hat, this is our hypothesis.

Or you can say that beta 1 equals to beta 2 in null hypothesis. So, that means when your objective is to test whether wage of an individual responds symmetrically towards education and experience or asymmetrically, then we can test this type of hypothesis. So, this type of hypothesis will tell you what is basically the important factor in determining the incrementing wage. Is it education or experience? This is example number 2.

Another example we can think of. Suppose we are estimating a production function and we have taken the logarithmic production function: log of y equals to sum alpha plus beta 1 log of labor plus beta 2 log of capital plus ui. This is our production function in (())(09:06) form. Here, we want to know, as a producer, let us say one producer, so what is beta 1 basically here?

That means the responsiveness of output with respect to labor. And since we have taken these equations in log form, we can say that this is basically direct elasticity measure. So, we can say that beta 1 is elasticity of output with respect to labor and beta 2 hat is elasticity of output with respect to capital. And you want to know whether these two elasticities are same or not. That means as a producer, producer is trying to allocate the given amount of money in two competing inputs labor and capital. So, of course, the producer will allocate more fund, more resources towards that particular factor which gives more return.

So, as a result of which, it is an important, a relevant question to the producers to know which particular factor gives more return. So, in this type of production function, beta 1 or beta 1 hat after estimation gives you the elasticity of output with respect to labor, that means we can say that, that is returns to labor and beta 2 or beta 2 hat is returns to capital and the producer wants to know whether the returns to labor is same as returns to capital or is it different.

If that is the case, then also you can test this type of hypothesis, your null would be beta 1 equals to beta 2. So, you are basically nullifying your claim. Our claim is one of these factors, either labor or capital, gives more return than the other. If you nullify this, then this will become beta 1 equals to beta 2. In this particular case, our claim is, let us say, returns to education is more than returns to experience. If you nullify that claim, once again the null would become beta 1 equals to beta 2.

And in the previous case, in example number 1, sorry, this is a consumption function and we are saying that this is basically responsiveness of consumption with respect to income and this is responsiveness of consumption with respect to wealth. And our claim is that, let us say, when income increases, consumer's pay in more of income on consumption rather than wealth. That is our claim.

So, what is our claim? Our claim is that a consumer's consumption expenditure is more responsiveness towards income than wealth. And if you nullify that claim, then it would become beta 1 equals to beta 2. So, these are couple of examples which, from which you can understand the reason for this type of hypothesis testing. What is the hypothesis we are testing? Equality between two type of hypothesis.

Now, how will you test that? To test this hypothesis, what is the test statistic that we are going to use? Let us say test statistic is, we are going to use t, t is test statistic. So, if this is the case, how will you formulate your test statistic? So, that means t in general, test this type of hypothesis, beta hat minus beta divided by standard error of beta hat.

So, that means what is the definition of t statistic if you recall? It is the estimated value, estimate minus true value of the population parameter divided by standard error of the estimate. So, when our hypothesis is this, when our hypothesis is beta equals to 0, you can formulate this type of test statistic. Now, when you are thinking what must be our test statistic when beta 1 equals to beta 2? Then what you can do? You can transform this type of hypothesis in this format.

And if you transform this hypothesis in this format, then what will happen? Your hypothesis would become beta 1 minus beta 2 equals to 0. And let this beta 1 minus beta 2 call as some other variable set. Is this clear? We all know that when our test null hypothesis is beta equals to 0, then our test statistic would be beta hat minus beta divided by standard error of beta hat and this would also become 0, because that is the hypothesized value of the true population parameter, so ultimately it will turn out to be beta hat divided by standard error of beta hat. Is this fine?

Now, when you are thinking what would be the expression of the test statistic when your null is beta 1 equals to beta 2, my suggestion is you convert your null hypothesis into this form, beta equals to 0. So, what will happen then? Your null would become beta 1 minus beta 2 equals to 0. That will make your life simple. And let us now call beta 1 minus beta 2 equals to z. So, what is our then in turn, our null hypothesis? z equals to 0.

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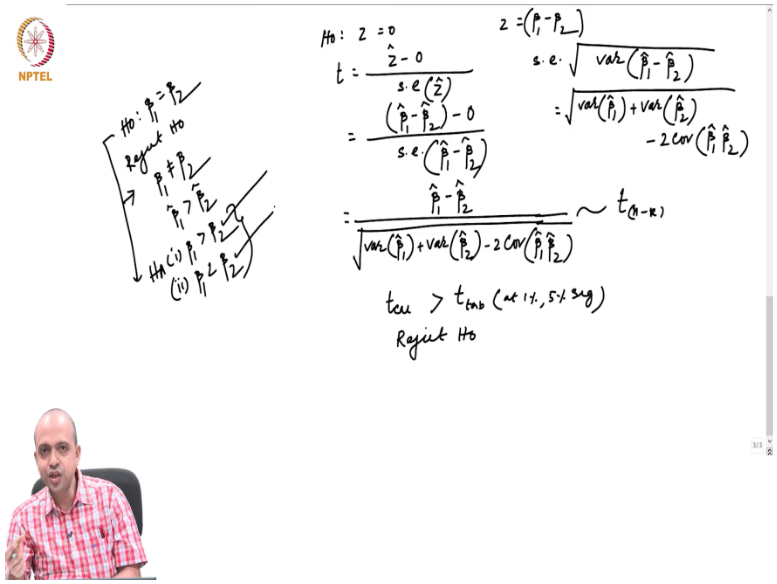

And if our null hypothesis is z equals to 0, so our null is z equals to 0, then what would be your test statistic? Your test statistic t would be again z hat minus 0 divided by standard error of z hat. Now, you replace what is your z? z equals to beta 1 minus beta 2. Now, you replace beta 1 minus beta 2 in place of z. Then what will happen? This would become beta1 hat minus beta2 hat minus 0 divided by standard error of beta1 hat minus beta2 hat. Then, what is standard error of this? Standard error of beta1 hat and beta2 hat is actually root of variance of beta1 hat minus beta 2 hat. That is the formula.

And what is variance of this? Equals to, this is equals to variance of beta1 hat plus variance of beta2 hat minus 2 into covariance of beta1 hat and beta2 hat. So, ultimately, your t would become beta1 hat minus beta2 hat divided by variance of beta1 hat plus variance of beta2 hat minus 2 into covariance of beta1 hat beta2 hat, this is the formula. And this will follow a t distribution with n minus k degrees of freedom or this is let us say, here I have, this is n minus k degrees of freedom.

So, what we have to do? We have to calculate this t and then we have to compare with the tabulated value, this is t calculated. So, once you estimate beta1 hat, beta2 hat and all these things, then you will get t calculated. And that you have to check, if this is greater than t tabulated at 1 percent or 5 percent significance, then reject H naught. That means, what was our null? Beta 1 equals to beta 2, so that means you will reject your null.

I am saying that these two regression coefficients, they are not similar, that means depending on your context, in the first example, that means you will say that consumers respond differently when there is a change in income and when there is a change in wealth. Then, in the second example when you are estimating returns to wage, then you have to say that in the labor market, wage responds more towards education, returns to education is different from returns to experience.

Now, when you reject your null, please keep in mind, our null is beta1 equals to beta2. Now, if you reject your H naught, that means we are only saying that beta1 not equals to beta2. That is we are saying. Now, next question comes then, what is contributing more towards somebody's consumption or somebody’s wage or somebody's output? That depends on your estimated value. If beta1 hat is greater than beta2 hat, then we will say that that means education gives more return than experience.

Or if we say that, if the example is production function, we will say that output responds more towards labor than capital. So, depending on the context, so you have to check what is the estimated value. If beta1 hat is beta2 hat, that means here, if you put beta1 minus beta2, obviously, you would be in the right hand side of the t distribution. And when you are in the right hand side of the t distribution, your decision will also change accordingly.

So, rejection of your null does not tell you anything about which among the two alternative hypothesis we are getting. So, alternative is, what is the alternative hypothesis? There are two alternative. Either beta1 actually greater than beta2, or beta1 is less than beta2, these are the alternative cases. That is why our null is composite hypothesis which among these two is actually true, that depends on the estimated value of beta1 hat and bet2 hat. That is very simple.

Rejection of our null is only telling they are not equal. It is not saying anything about which particular case. Which particular case you will have, that depends on the estimated value of the population parameter. If beta1 hat is greater than beta2 hat, then we will say that this is the case. If beta1 hat is less than beta2 hat, then we will say this is the case.

So, after rejecting your null, you will say they are not equal and then you will look at the value of beta 1 hat and beta 2 hat, and then you will confirm out of these two cases, which particular case is actually suitable in a particular context. This is an important hypothesis, testing equality of the two regression coefficient. That is what you can test.