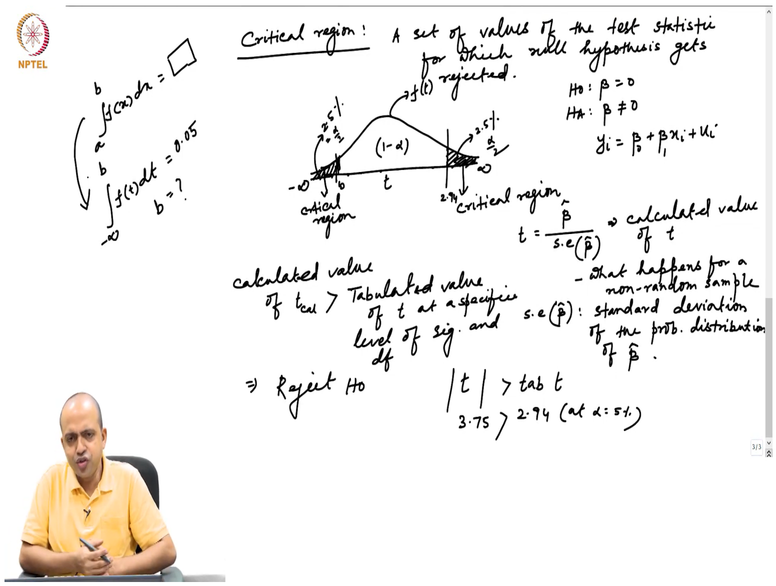
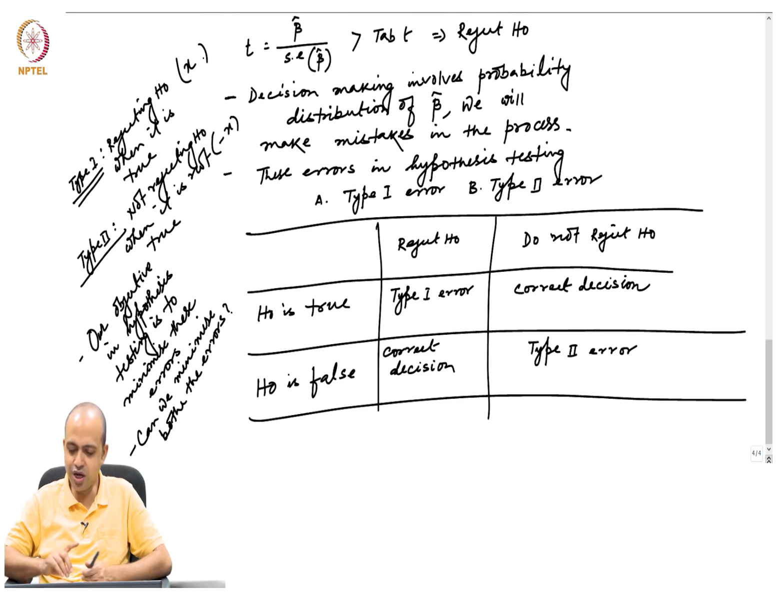
Introduction to Econometrics
Professor Sabuj Kumar Mandal
Department of Humanities and Social Sciences
Indian Institute of Technology Madras
Goodness of fit measure, ANOVA and Hypothesis Testing Part 5

(Refer Slide Time: 00:14)



Now, in this entire process what happens?

(Refer Slide Time: 00:27)



Since the decision-making rules says that $t = \frac{\hat{\beta}}{S.E.\hat{\beta}}$ that should be greater than the tabulated t. So, that means your decision-making rule involves some kind of probability distribution. So, this implies reject your null, this is your decision. Since the decision making involves probability distribution of beta hat, what will happen?

We will commit, we will make mistakes in the process because that is the nature of probability. If all 100 cases, your decisions are correct when you are following this rule, that tabulated t is less than the calculated t, if all 100 decisions are correct, then that cannot be a probability kind of situation. Since the decision-making rule involves some kind of probability distribution, we would definitely commit some mistakes or error in the decision-making process.

These mistakes or error in hypothesis testing are two types. So, type one error and secondly type two error. Now, I will explain this. Let us say that this is a situation H_0, let us say that reject H_0, this is do not reject H_0. Here, H_0 is true, H_0 is not true.

So, here I am just writing here H_0 is not true, H_0 is true. So, this about this case – H_0 is not true and you are rejecting this. Or I will write like this. Frist case is let us say, H_0 is true and this is H_0 is false. So, think about this case. Your null hypothesis H_0 is true and you are rejecting the null. That means you are committing mistake.

H_0, the null hypothesis is actually true, so you are rejecting a true hypothesis, so that means you are committing a mistake. So, rejecting the null when it is true is called type one error. What about this? H_0 is true and you are not rejecting also, that is a correct decision then. H_0 is false is not true and you are rejecting the null hypothesis, that is also a correct decision. Think about this case, H_0 is false and you are not rejecting also.

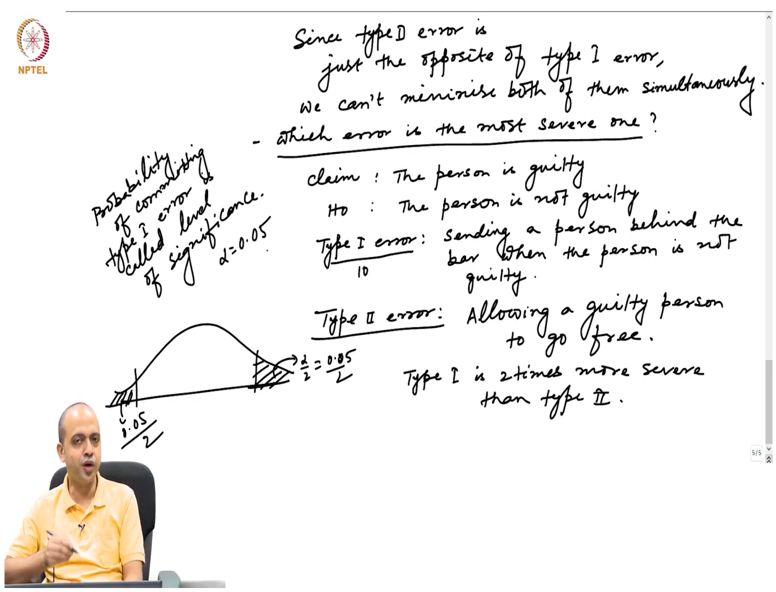
This is called type two error. So, that means how do you remember type one and type two error? Type one error is rejecting the null when it is true and type two error is do not reject your null when it is not true. This is how you have to remember. Rejecting the null, that means now I can define the type one error is rejecting H knot when it is true.

And type two error - not rejecting the null when it is not true. This is the simplest way to remember type one and type two error - rejecting the null when it is true is type one and not rejecting the null when it is not true is the type two error. Now, here in the hypothesis testing, as we said our decision-making rule involves some kind of error, we need to minimize these errors.

Now, the question is, can we minimize these errors, both the errors simultaneously? Can we minimize? So, our objective in hypothesis testing is to minimize these errors. Then the next question is, can we minimize both the errors simultaneously? Now, if you think about the nature of these two errors, you can see the type two error is just opposite of type one error.

Type one error says rejecting the null when it is true and not rejecting the null when it is not true is just the opposite of that. That means if I denote type one error is let us say by x, type two error is just denoted by let us say minus x. So, obviously, you cannot minimize x and minus x simultaneously because minimizing x indicates maximizing minus x.

(Refer Slide Time: 09:50)



So, that is why since type two error is just the opposite of type one, we cannot minimize both of them simultaneously. Then what is the idea? If you cannot minimize both the error simultaneously, what the statistician suggest is that you identify the most severe one. You identify out of these two errors, which is the most severe.

Now, I will give you a small story to understand which error is most severe one. Let us say that a person is caught by the police and the police suspect the person is related to some bank robbery but there is no enough evidence of that so you suspect him and now in the court, you have to produce the person, the guilty person in the court and then there would be trial in the court and the judge will decide whether he is actually guilty or not.

So, the person is caught and what is your claim? Your claim is that the person is guilty. Then what is your H_0? Your H_0 is that the person is not guilty. Then what would be your type one error in this context? Type one error. So you reject the null when it is true.

So, that means you are rejecting the null hypothesis, the person is not guilty, you are rejecting this. So, what will happen? The person is actually not guilty but you are rejecting it when it is true. So, you are sending this person behind the bar. So, that means type one error is sending an innocent, sending a person behind the bar when the person is not guilty. So, you are sending an innocent person behind the bar when you are committing type one error.

Then what is type two? Type two error says not rejecting the null when it is not true. So, that means the person is not guilty, that is not true. That means the person is actually guilty but you are not rejecting the null. The person is not guilty - is not true. That means the person is actually guilty but you are not rejecting it. So, you are accepting the fact that the person is not guilty but actually, the person is guilty.

Rejecting the null when it is not true, the person is not guilty is not true, so that means the person is innocent is not true. What is true then? The person is guilty. But you are not rejecting it. That means the person is not guilty, you are accepting that. So, what is type one error then? Allowing a guilty person to go free.

Now, if you think about these two cases, obviously when you are sending an innocent person behind the bar is most dangerous compared to the second one because the justice says that you can even let a person go free when he or she is guilty but you should never send an innocent person behind the bar. Why this is so? Because in the first case, you are not only sending an innocent person behind the bar, you are allowing the actual guilty also to go free.

This person is caught out of the suspect but the person who is actually guilty is moving around. So, that means we can say that type one error is actually twice severe than the type two error. Then statisticians say that since the type one error is most severe, you keep the person, you keep the type two type one error fixed at a p specified level.

That means in the process of hypothesis testing, we cannot allow type one error to be fixed out of the process, rather since this process is so, type one error is so severe, what we will do? In 100 cases, when you are taking the decision based on the decision-making rule specified earlier, we will say that we can afford of making type one error maximum 10, so 10 times.

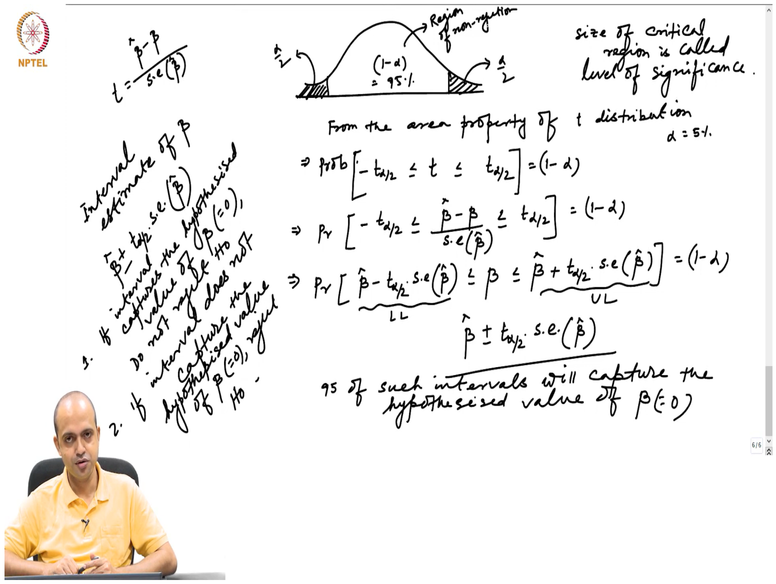
That is the number we can afford. Maximum 10 times, we can commit type one error. That means, and probability of committing type one error is called level of significance. Probability of committing type one error. When you are rejecting the null, what is the maximum, what is the minimum number of type one error that you are committing, that is called your level of significance.

That means in terms of the diagram when I am saying that this is 5 percent, that means this is alpha by 2 equals to 0.05 by 2 and this is 0.05 by 2. So, when alpha equals to 0.05, what we are saying? The probability of committing type one error is called level of significance. That means, the size of the critical region is also known as level of significance.

That means I am saying, when I am rejecting the null based on the rule we have just specified, that calculated t should be greater than tabulated t, we will commit minimum 5 times type one error, that is the meaning. And then, you minimize, keeping type one error at a prespecified level and then minimizing the type two error is the actual procedure. So, in the process, we will allow maximum 10 type one error to be committed. Lower is better.

So, that means from the diagram we can say that this alpha is 5 percent, that means alpha equals to 0.05 means I have specified probability of committing type one error 5 times that is a priory specified. Fixing alpha equals to 0.05, now we are trying to minimize. That probability of type two error. You can fix probability of committing type one error at 1 percent level also.

(Refer Slide Time: 22:15)



So, that means what is happening in terms of the diagram once again, let us say that this is our, this is $\alpha/2$, this is also $\alpha/2$. How the critical region and level of significance is related? We say that size of the critical region is called level of significance.

So, that means when your size of the critical region is 10 percent, you will commit ten type one error while rejecting the null. And we will say that the variable, let us say in our context, income, is significant at 10 percent level. When I am saying income is significant at 10 percent level, that means I am saying that while rejecting my null, I am committing 10 type one error.

When I am saying the variable is significant at 5 percent level, that means I am committing type one error 5 times. When I am saying the variable is significant at 1 percent level, that means I am committing only one type one error while rejecting my null hypothesis. So, these two areas $\alpha/2$ and this is $1-\alpha$

So, from this, from the area property of this distribution, what we can say? That t which will lie between $-t_{\alpha/2}$ and $t_{\alpha/2}$, the probability is actually $1-\alpha$. And what is t? $t = \frac{\hat{\beta} - \beta}{s.e.\hat{\beta}}$. Now, substituting t here, what I can write? That probability $-t_{\alpha/2}$ is less than or equal to $\frac{\hat{\beta} - \beta}{s.e.\hat{\beta}}$ is less than or equal to $t_{\alpha/2}$ which is equal to 1-α. Or from there what we can write? After some rearrangement, we can write that $\hat{\beta} - t_{\alpha/2} * s.e.\hat{\beta} \leq \hat{\beta} + t_{\alpha/2} * s.e.\hat{\beta} = 1 - \alpha$

Now, we have constructed one probability statement following the area property of the t distribution. Now, the question is what is the meaning of this probability distribution? So, that means when alpha equals to 5 percent, you can say that this area is actually 95 percent. Since these two regions are called critical region, region of rejection, we say that 1 minus alpha is called region of non-rejection.

So, what is the meaning of this? You have constructed that means some interval. When you estimate your beta hat, let us say that this is an interval and this is an interval. Let us say that this is the lower limit of the interval and this is the upper limit of the interval. How the interval is defined? $\hat{\beta} - t_{\alpha/2} * s.e.\hat{\beta}$ is called the lower limit of your interval, $\hat{\beta} + t_{\alpha/2} * s.e.\hat{\beta}$ is called upper limit of the interval. And this is 95.

So, what does this probability statement says? Apparently, it may so appear, you may like to interpret this probability statement as this interval will capture the true value of beta, the population parameter beta. Probability that this interval will capture beta is 95 percent. That may appear as an interpretation, ready-made interpretation. Since this is a probability statement, there are 2 intervals here. So, I will say that probability that this interval will capture the true population parameter beta is 95 percent.

But you have to sincerely keep in mind that interpretation is wrong. Why this is wrong? Because probability can always be attached only with a random variable. First of all, is this interval a random variable? Is this interval random? The answer is no. Why? Look at the interval, it is beta hat minus t alpha by 2 into standard error of beta hat.

Once you estimate the model, you know your beta hat, you know t alpha by 2, you know standard error of beta hat. That means this lower limit is known, upper limit is also known. That means the interval itself is known. So, when the interval is known, it will either capture beta or do not capture beta. Where is the probability here? We cannot attach a probability with an interval which is known to us. Then the question is, when the interval is known to us, we cannot attach probability, then what is the meaning of this probability statement?

The actual meaning of this probability statement is that if you construct this type of interval, 100 times following the formula, beta hat... so what is the formula we are following? beta hat plus minus t alpha by 2 into standard error of beta hat. This is the formula I am following. And following this formula, I am constructing 100 such interval from 100 different samples because 100 different samples will give you 100 such beta hat. And standard error of beta hat I will get from the probability distribution.

Then if you construct the interval, out of 100 intervals, 95 of such interval will capture the true value of beta. What I am saying? If you construct the interval following this formula, 95 of such intervals will capture the hypothesized value of beta in this case which is actually 0. 95 of such intervals will capture the true population parameter beta. Since the true population parameter value is unknown, we will say that 95 of such intervals will capture the hypothesized value of the true population parameter.

Now, in this process we have defined the interval estimation also. If you recall, we say that there are two type of estimates - interval estimate of beta hat, of sorry beta. So, instead of saying the beta hat as a point estimate, here I am now giving an alternative estimate of the beta which is called interval estimate. And how is the interval estimate defined? beta hat plus or minus t alpha by 2 into standard error of beta hat.

So, higher the value of the standard error, higher would be the size of the interval. The interval would be the size of the interval and lower would be preciseness. If your interval, size of interval is much wider, then obviously, that will lead to very lower level of precision. Lower the value of the standard error, you will give more preciseness in your decision making.

So, this is an alternative to point estimate which is more reliable because I can always represent a value of the true population parameter by an interval rather than just by only one point given by the point estimate of beta hat. This is how we can construct the interval estimates. So, how will you check the level of significance then? Once you estimate the model, you get your interval and if your interval captures the true value of the population parameter, obviously you do not reject your null.

So, do not reject your null when your interval captures the hypothesized value and reject your null when your interval does not capture the true value of the null. So, the rule in interval estimation says, if interval, what is the rule? If interval captures the hypothesized value of beta which is equals to 0, then do not reject H knot.

And if interval does not capture the hypothesized value of beta then reject H knot. This is the decision-making rule in the context of interval estimation. So, this is actually the entire process of hypothesis testing.

So, hypothesis is basically a case about the true population parameter and we have two types of hypothesis - null versus alternative. Null says when you are nullifying your claim. Alternative is just the opposite of that. And then, you get a test statistic based on your estimates, beta hat divided by standard error and your decision-making rule is says that when the estimated calculated value of that t is greater than the tabulated value of t, you reject your null.

And since the probability distribution is attached with this decision making, we commit some kind of mistakes or errors in this decision making. There are two types of error, Type one and Type two. Type one error is most severe as I discussed giving the example. We fix apriory the probability of type one error like this, that we are fixing actually the size of the critical region. When I am saying I am fixing the number of type one error, I am actually fixing the size of the critical region. And then we are trying to determine my area of non-rejection which is 1 minus alpha.

And then, once you specify and then you try to see what is your calculated value of t at that specific, whether your calculated value of t is greater than the tabulated value of t at that prespecified level of significance. That means size of the critical region is also known as level of significance. And it also known as probability of committing type one error.

So, probability of committing type one error, size of the critical region and level of significance, these three terms are interrelated - size of the critical region, probability of committing type one error and level of significance. So, when my variable is significant at 5 percent level, that means I am saying I am rejecting the null but I am committing 5 mistakes, that is okay.

I can afford of committing 5 mistakes. I can afford up to 10, but econometrician say that it is better to fix your type one error at 1 percent and 5 percent. If you commit more than that, while minimizing type two error, it may so happen that you are committing type two error more than 5 times or more than 10 times, then we will say that my variable is actually not significant. We cannot afford of committing that many type one errors. This is actually in the process, we are getting an alternative estimates of beta which is called interval estimate. How we are getting? From the area property of the t distribution and the interval is defined as beta hat plus or minus t alpha by 2, standard error of beta hat. This is how we are going to get the interval.

So, with this, we are closing our discussion on hypothesis testing. So, tomorrow we will bring back our original model, what we are discussing, what we are estimating and we will see how to apply this concept of hypothesis testing there to check whether a particular variable is significant following the level of significance approach or following the interval estimate approach. That we will discuss in our next class.

Thank you.s