Introduction to Econometrics
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Goodness of fit measure, ANOVA and Hypothesis Testing Part 3

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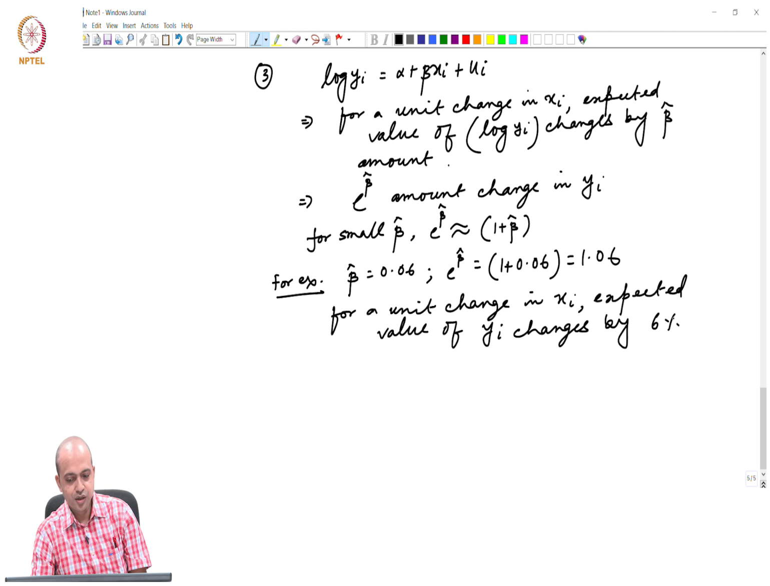

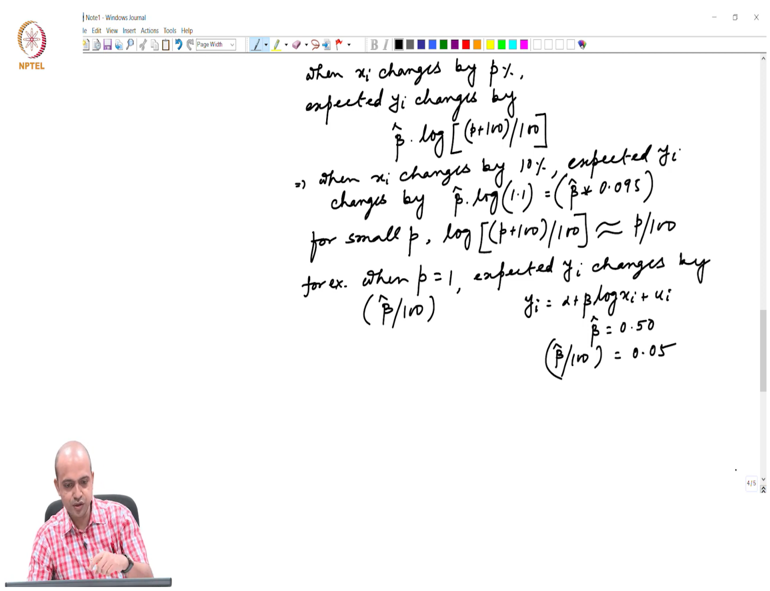

The third case is this. $\text{Log}(Y_i) = \alpha + \beta x_i + u_i$. Now, if I apply the interpretation of model 1 here, then what I can write that for a unit change in x_i, expected value of $\log(Y_i)$ changes by $\hat{\beta}$

amount. When x changes by 1 unit, expected value of $\log(Y_i)$ changes by $\hat{\beta}$ amount . That is the interpretation.

But as I said, understanding change in log of Y_i is little difficult because what we want is, we want to get the changes in Y_i. So, this is equivalent to $e^{\hat{\beta}}$ amount change in Y_i. When x changes by 1 unit, Y changes by $e^{\hat{\beta}}$ amount. Now, for small $\hat{\beta}$, $e^{\hat{\beta}}$ equals to actually $1+\hat{\beta}$.

For example, let us say that $\hat{\beta}$ is actually 0.06. So, this implies $e^{\hat{\beta}}$ equals to 1.1 plus 0.06 equals to 1.06. So, that means what we can say for a unit change in x_i, expected Y_i value changes by 6 percent. So, when $\hat{\beta}$ equals to 0.06 in this type of model when you estimate log linear model, when x_i changes by 1 unit, then your Y_i will change by 6 percent.

Previously, look at the definition difference. Here, when your x is log, what I am saying, for 1 percent change in x_i, Y_i changes by this much - $\hat{\beta}$ divided by 100 equals to 0.05. So, this is percent, this is unit. This is just the opposite. For a unit change in x_i, Y changes by 6 percent if your $\hat{\beta}$ equals to 0.06. This is the third model.

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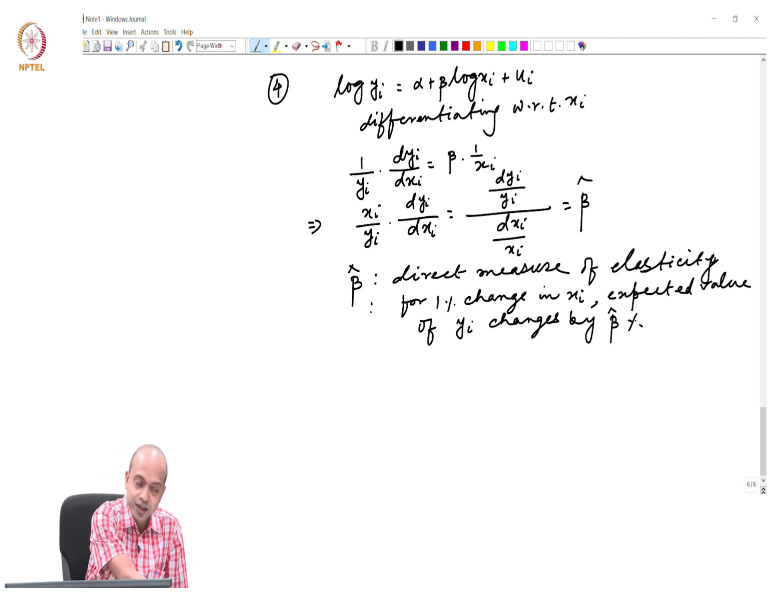

What is the fourth model? Fourth model is log-log. $\text{Log}(Y_i) = \alpha + \beta \log(x_i) + u_i$. And what is the interpretation of this model? To understand the interpretation of $\hat{\beta}$, this is the most familiar, one of the most used econometric model is log-log and I will let you know what is the interpretation of $\hat{\beta}$ here.

Now, what I am saying, differentiate this with respect to x_i, what you will get? If you differentiate this with respect to x_i, you will get 1/Yi * dy/dx equals to β * 1/x. And if you multiply both sides by x_i, then you will get x_i by Y_i into dyi/dxi which you can write as dyi/yi divided by dxi/xi. Now, if you look at, can you recall these relationships?

Suppose this is my demand function- yi is the demand for the i^{th} product and xi is the price of the i^{th} product. Then in this context, this is nothing but, this is equals to your actually $\hat{\beta}$. So, in this context as you can see, $\hat{\beta}$ is nothing but the price elasticity of demand. That means for 1 percent change in xi, y changes by $\hat{\beta}$ percentage.

That means the interpretation of $\hat{\beta}$ is here, a direct measure of elasticity when your model is log-log. This is a quite interesting and useful model. Whenever you are interested in estimating direct elasticity, then transform this model into log-log and from the log-log transformation, you will get $\hat{\beta}$ as a direct elasticity measure.

So, that means I can say that the interpretation is for 1 percent change in x_i, expected value of y_i changes by $\hat{\beta}$ percentage. This is the interpretation of $\hat{\beta}$. So, these are the alternative four models. First model is linear, both y and x is linear and the interpretation is simple, which we have already learnt. For a unit change in xi, on an average yi changes by beta hat amount. What we did?

We have tried to apply the same interpretation in all these models and then, at the end, we derive the actual interpretation of $\hat{\beta}$ in that log transform model. Now, what we will do? We will take one data set and we will estimate these four alternative models and we will try to understand their interpretation.

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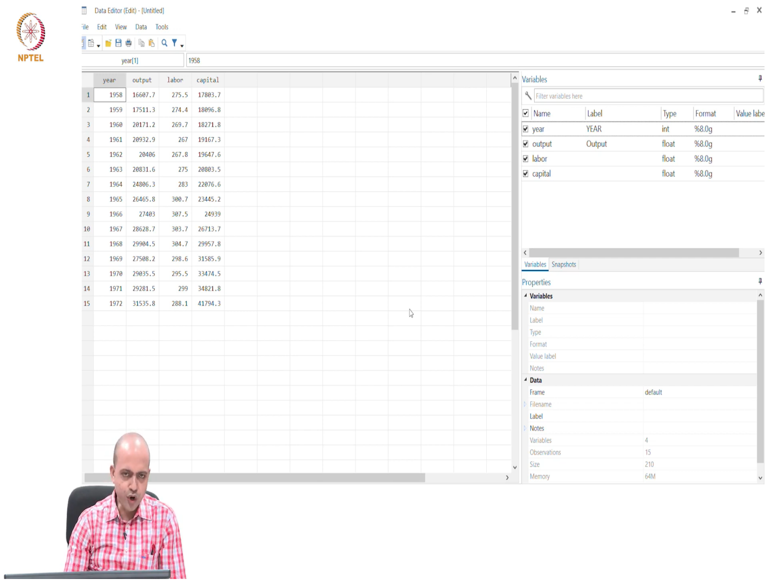

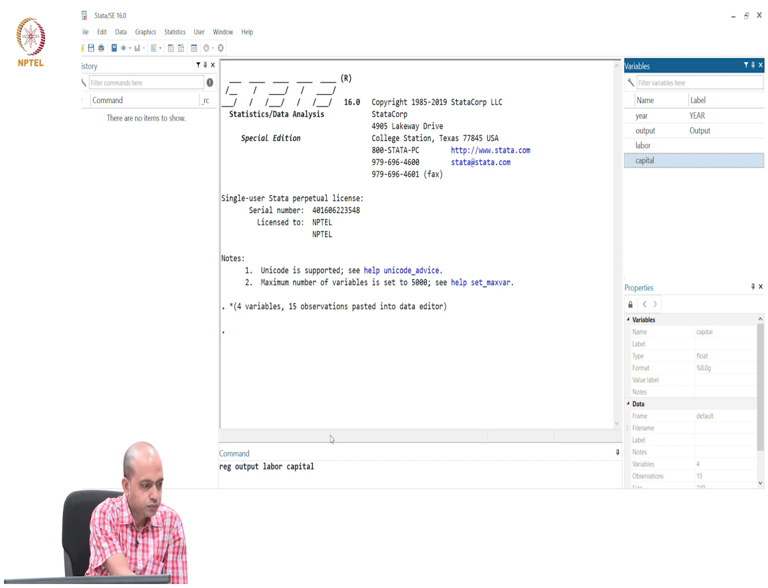

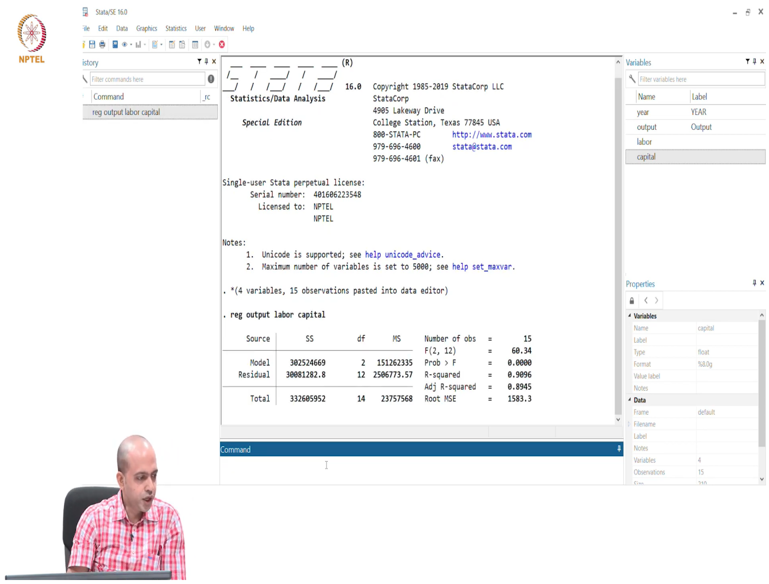

So, this is the data set. Actually, if you look at the data set, here we have data on output, labour and capital. Basically, we are estimating a production function, these data would be useful later on also, so this is basically a data on output, labor and capital. And we can estimate different types of production functions.

Let us first say that we are estimating the first model where both your yi and xi is linear. So what we will do? We will estimate these. output is my dependent variable. Then, my independent variables are labor and capital. This is the linear model we have estimated. So, how will you interpret the coefficient? This is very simple.

For a unit change in labor, on an average, output increases by 147 unit. Similarly, from the coefficient of capital, we can say that for a unit change in capital, output increases by 0.4035 unit. And both these variables are significant because if you look at the p values, if you multiply the p value, corresponding p value of labor, then you will get 0.002 multiplies by 100 is 0.2 which is still less than 1.

So that means both are significant at 1 percent level. Both labor and capital, they are significant at 1 percent level. That is what we have understood. So, this is the interpretation. Now, what we will do? We will estimate the log transform models. For that, we need to transform all these variables into log.

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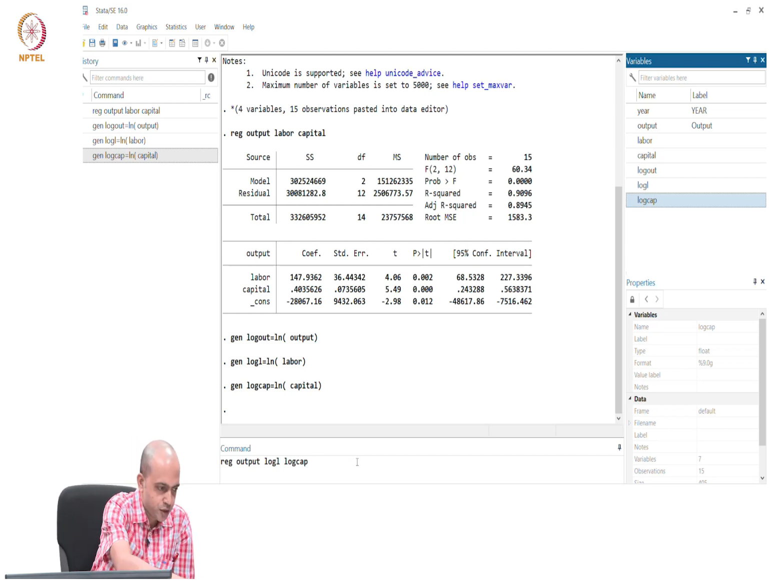

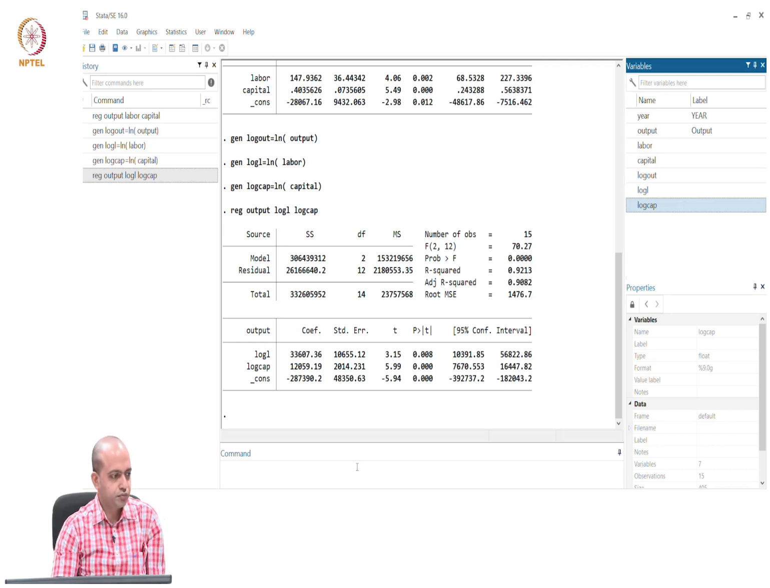

So, let us say, how to transform this variable? gen, let us say, log output equals to ln and then output bracket, this is the command. So, that means I have transformed my output into log and I have given a name called log out. Log out is the logarithmic transformation of the output variable. This is the command in Stata whenever you want to transform a variable in log, you have to use the “gen” command.

So, after gen, you put some name of the variable, I have given log out, you can give any other name as well. And then, after this what I am saying, this is the function. I am saying this is actually logarithmic transformation. So, ln within the bracket output. Similarly, gen, let us say, log labor or log l equals to ln within the bracket you put labor, so I have created another variable which is log l, that means logarithmic transformation of labor.

After that, you put this gen log capital or let us say I will put log cap equals to ln within the bracket, capital, enter. So, now what I will do? I will use the second model where x is log transformed but y is not. So, reg output and then I am putting log labor and log capital. So, this is called lin-log model; left hand side is linear, right hand side variables are log transformed and this is my model. Now, the question is, how will you interpret this coefficient?

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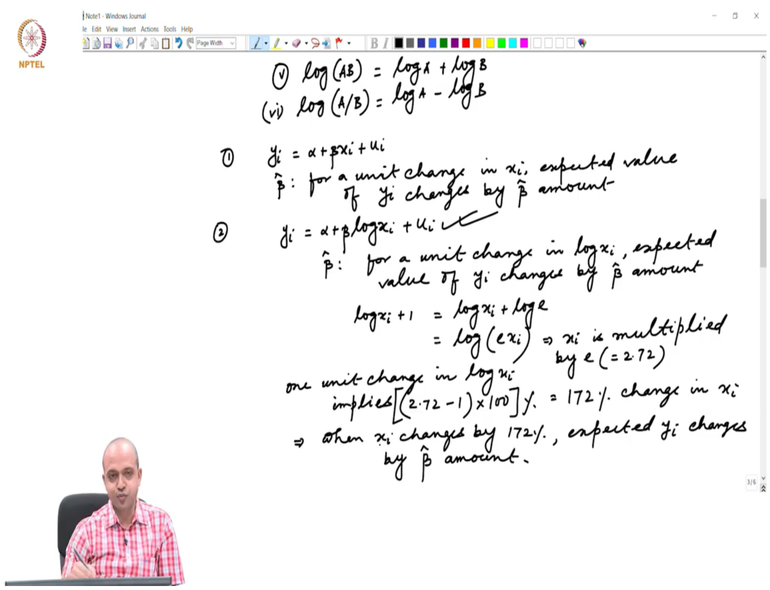

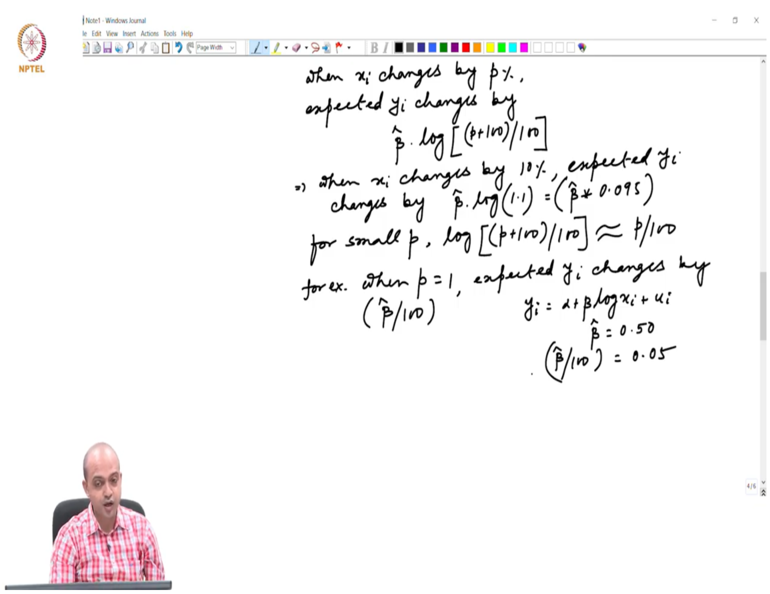

We will go back and see the interpretation. So, that means you are estimating this case, when y is linear but x is log transformed. So, that means here I can say that for a unit change for 172 percent change in xi, yi changes by that amount.

That means how we have transformed this? when p equals to 1, that means we can say that for 1 percent change in xi, your expected value of yi changes by beta hat divided by 100 unit. Now, if you go by that, that means beta hat by 100 is the unit change in expected value of y for 1 percent change in your xi.

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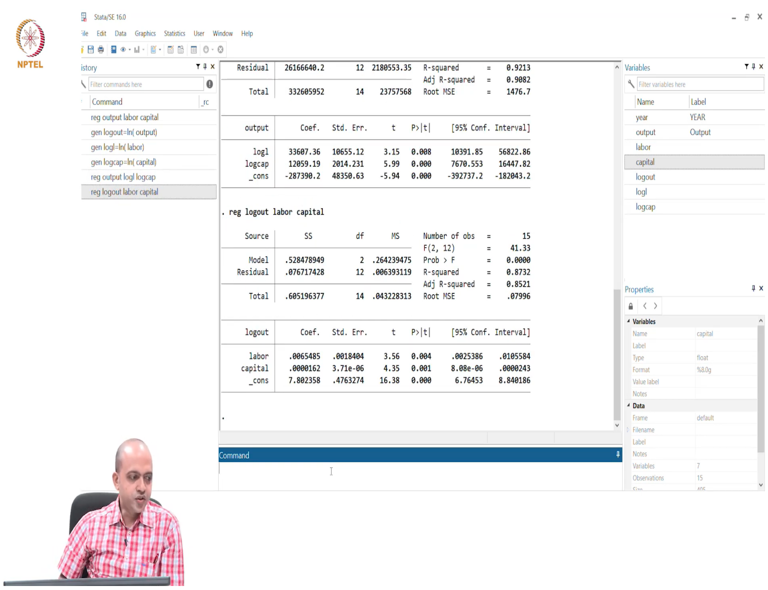

So, same logic will apply. So, that means for 1 percent change in your labor, your expected value of y will change by the coefficient beta hat is 33607. If you divide by 100, that means it would become 336.07 that is the interpretation.

For 1 percent change in labor, expected value of your output changes by 336.07 unit because we have derived the interpretation would be when p is small, that means we are assuming that p is here equals to 1, that means 1 percent change in your x, output changes by 336.07 unit.

Similarly, for 1 percent change in capital, output changes by beta hat by 100, it would become 120.59 unit. This is the interpretation. Is this clear? That is what we have derived for lin-log model, if you need to interpret the coefficient, what you do, you divide the coefficient by 100, that would be the unit change in your expected value of yi for 1 percent change in the xi.

When labor change is increased by 1 percent, output changes by 336.07 unit, when capital changes by 1 percent, output changes by 120.59 unit and this is what we got dividing beta hat by 100, very simple. Now, the third model. Third model is called log linear model. That means my dependent variable is log transformed but independent variables are not.

So, what we will do? We will regress this type of regression, reg log of output, sorry here I have given a different name. So, this is log output is named as this and then I will simply take labor and capital. This is the interpretation. This is the model. Now, what is the interpretation of log linear model?

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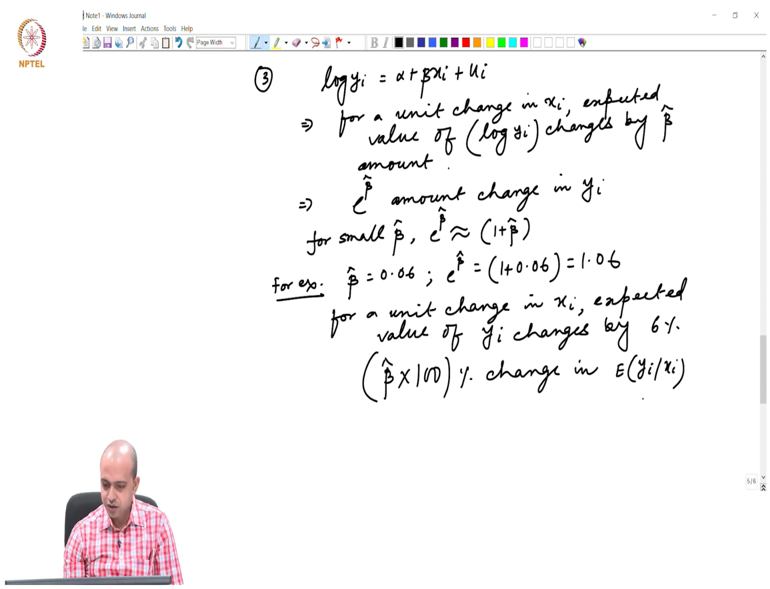

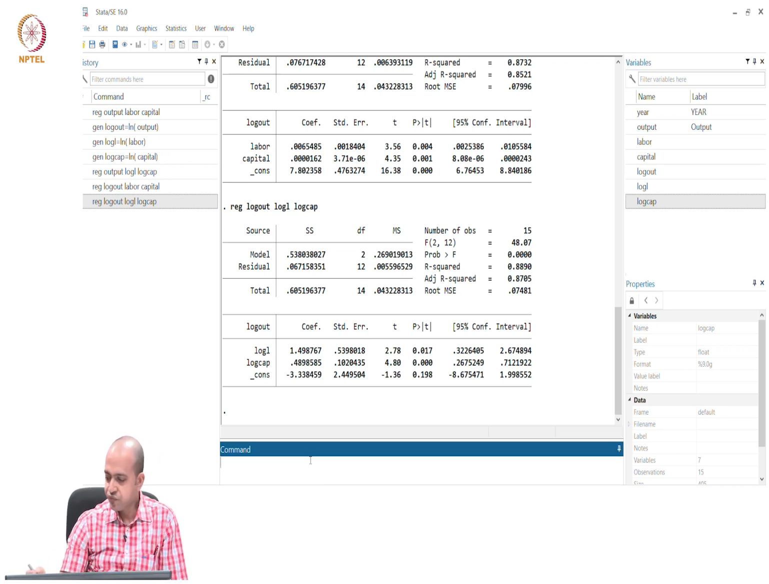

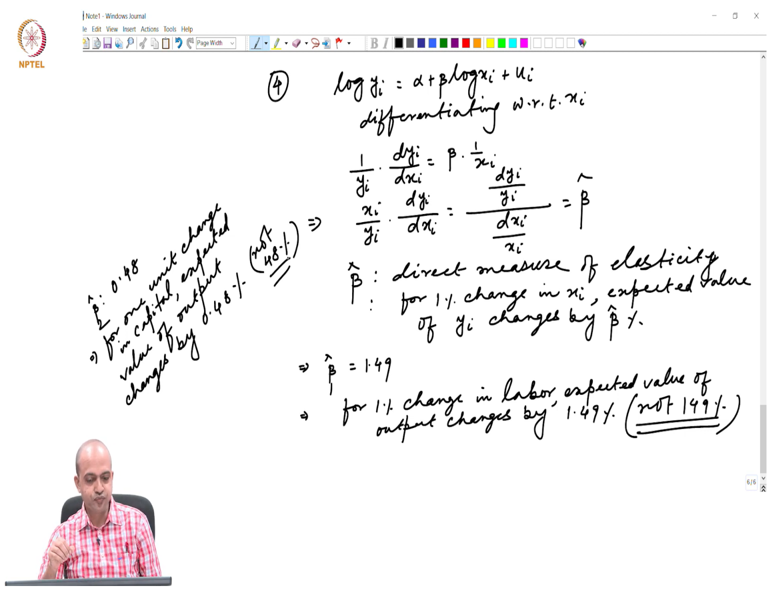

We have to go back and see what is the interpretation of log linear model. This is the model. So that means, if you want to interpret the coefficient, when your y is log but x is not, what you need to do? You need to say that y changes by $e^{\hat{\beta}}$ amount. That means $e^{\hat{\beta}}$ equals $1+\hat{\beta}$, so if $\hat{\beta}$ equals to 0.06, so that means it would be come 1.06 or 6 percent.

That means you multiply $\hat{\beta}$ by 100 and that would be the percentage change in expected value of y_i given x_i. So, you multiply β by 100. So, for a unit change in labor, output changes by, if you multiply this by 100 it will become 0.65 percent. For a unit change in labor, output changes by $\hat{\beta}$ multiplies by 100 percent, that means 0.65 percent.

Similarly, for a unit change in capital, output changes by this multiplied by 100, that means 0.0016 percent. So, when your dependent variable is log transform but independent variables are not, so that means the interpretation would be, you need to multiply your $\hat{\beta}$ by 100 to get the corresponding percentage change in y_i because you getting first $e^{\hat{\beta}}$ amount change in y which we do not want.

Then lastly, what we need to do, we will estimate the fourth model where both are log transformed. So, that means we will say that, we will estimate this type of model. Reg then log output and we are regressing on log of labor and log of capital, this is our model. And this model is called log-log model. Both side log transform. And if you put enter, this is the estimate.

How will you interpret the coefficient? You go back to the log-log model and see, this is the log-log model, log y = α + βlog x_i and you see differentiating both the side, we are getting $\hat{\beta}$ is basically equals to then dy/y * dxi/x. And as a result of which, $\hat{\beta}$ is a direct measure of elasticity in this case.

Direct measure of elasticity means β basically indicates for 1 percent change in x_i, expected value of y_i changes by $\hat{\beta}$ percentage. We do not have to do anything, it is a direct elasticity measure, so we will get direct percentage change in y_i for 1 percent in x_i. This is the log-log model interpretation.

And if you go back, now here, what we see that the coefficient of labor is 1.49, what does it mean? It means for 1 percent change in your labor, output changes by 1.49 percent. You have to clearly keep in mind. Generally, we commit a mistake; we again multiply this by 100 and say that for 1 percent change in labor, output changes by 149 percent which is absurd.

You have to clearly keep in mind that log-log model gives a direct elasticity measure. That means you do not have to do anything with this coefficient like the previous models. In previous models, we were either multiplying β by 100 or dividing beta by 100. But here, is a case where $\hat{\beta}$ gives you a direct elasticity measure. No need of doing anything to this models.

So, that means in this case, what will happen? You have to say that for 1 percent change in labor, output changes by 1.49 percent. I will write this and since this, and this is 0.48. So, from this, we can say that interpretation is, what is the coefficient of labor and capital? $\hat{\beta}_1$ equals to 1.49. This implies, for 1 percent change in labor, expected value of output changes by 1.49 percent.

Do not multiply this by 100 and say 149 percent, that would be wrong. And then, what is the value of $\hat{\beta}_2$? $\hat{\beta}_2$ is 0.48, that means this implies for 1 unit change in capital, expected value of output changes by 0.48 percent and not 48 percent. So, in the bracket I am writing, 'not 48 percent' - do not commit this mistake.

Here, I am writing 'not 149 percent', this you have to be very very careful. It is a direct elasticity measure, no need of doing anything with this $\hat{\beta}$ coefficient, it is a direct elasticity measure. So, this is a very very important topic as I said, this logarithmic transformation. Why, first of all, logarithmic transformation is required?

And secondly, if you transform either your dependent of independent or both the variables in log, then what is going to be the interpretation? So, this interpretation you have to clearly keep in mind. The interpretation is very simple actually, though we have derived many things, at the end of the day, it becomes very simple.

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Note1 - Windows Journal

(v) $\log(AB) = \log A + \log B$

(vi) $\log(A/B) = \log A - \log B$

① $y_i = \alpha + \beta x_i + u_i$

$\hat{\beta}$: for a unit change in x_i, expected value of y_i changes by $\hat{\beta}$ amount

② $y_i = \alpha + \beta \log x_i + u_i$

$\hat{\beta}$: for a unit change in $\log x_i$, expected value of y_i changes by $\hat{\beta}$ amount

$$\log x_i + 1 = \log x_i + \log e = \log(e x_i) \Rightarrow x_i \text{ is multiplied by } e\,(=2.72)$$

one unit change in $\log x_i$ implies $[(2.72-1)\times 100]\% = 172\%$ change in x_i

$\Rightarrow$ when x_i changes by 172%, expected y_i changes by $\hat{\beta}$ amount.

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Note1 - Windows Journal

when x_i changes by $p\%$, expected y_i changes by

$$\hat{\beta}\cdot\log\left[(p+100)/100\right]$$

$\Rightarrow$ when x_i changes by 10%, expected y_i changes by $\hat{\beta}\cdot\log(1.1) = (\hat{\beta} * 0.095)$

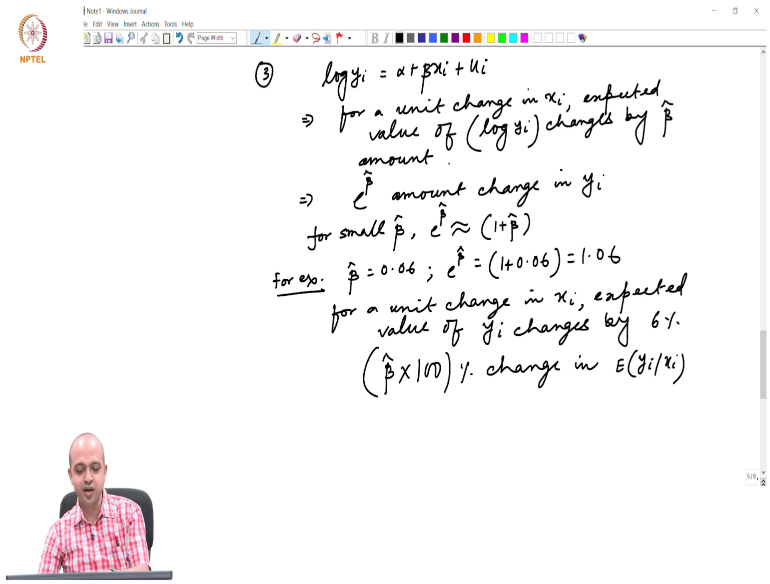
for small p, $\log\left[(p+100)/100\right] \approx p/100$

for ex. when $p=1$, expected y_i changes by $(\hat{\beta}/100)$

$y_i = \alpha + \beta \log x_i + u_i$

$\hat{\beta} = 0.50$

$(\hat{\beta}/100) = 0.05$



When your x is log transformed and y is not, then ultimately. You divide $\hat{\beta}$ by 100 and get that interpretation. So, that means for 1 percent change in your x, expected value of y changes by this much unit.

And when your y is log but x is not, then you multiply $\hat{\beta}$ by 100, you will get percentage change in your y. That is what you need to keep in mind. And when both are log transformed, you do not have to do anything because that is a direct elasticity measure. With this, we are closing our discussion today, thank you.