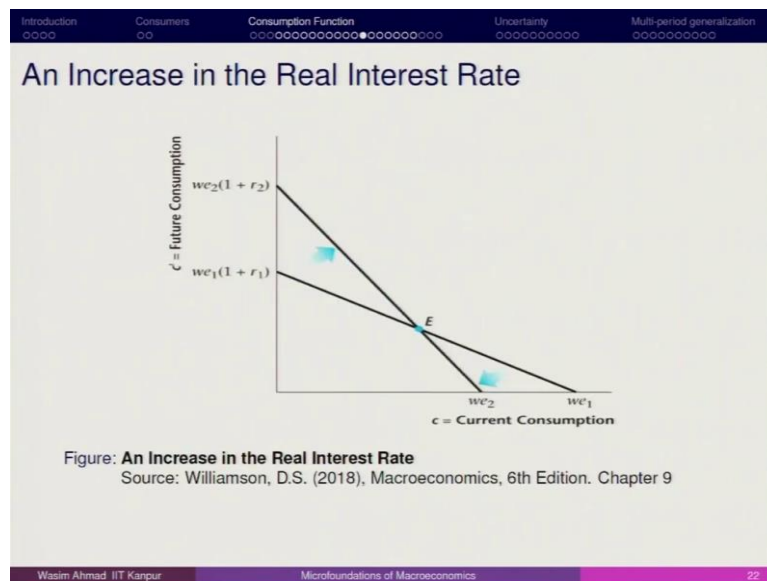**Micro Foundations of Macroeconomics**
Prof. Wasim Ahmad
Department of Economic Sciences
Indian Institute of Technology – Kanpur

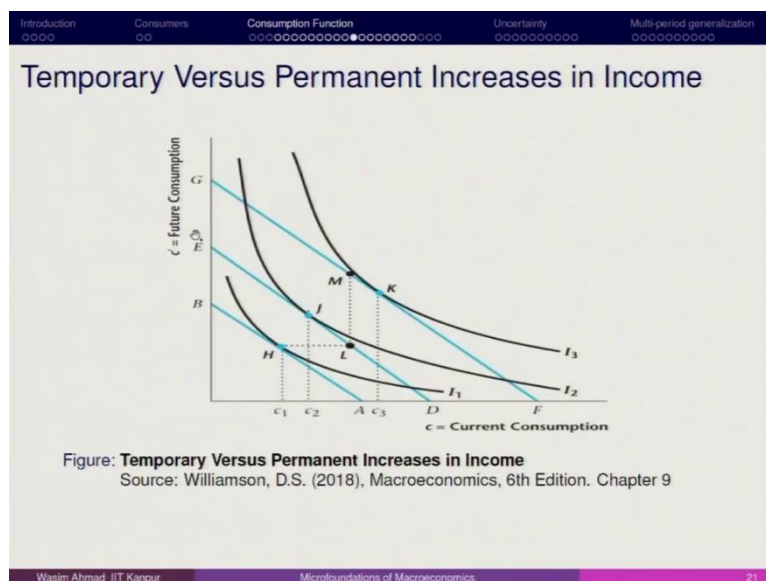**Lecture – 09**
Two Period Model IV

Let us start. Welcome back, we are going to discuss further extensions of the two period model. And we just covered certain dimensions like what happens when we superimposed the conditions. So comparative statics that we discussed with regard to the current period and future period and then we also had discussion on how we can derive the consumption function.

And now, we will be moving to a scenario where we are introducing the interest rate. We also discuss about temporary and permanent income. So, now we will be talking about the interest rate scenarios.

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Figure: **An Increase in the Real Interest Rate**
Source: Williamson, D.S. (2018), Macroeconomics, 6th Edition. Chapter 9

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Figure: **Temporary Versus Permanent Increases in Income**
Source: Williamson, D.S. (2018), Macroeconomics, 6th Edition. Chapter 9

So, here if you can see, the budget line is shifting, almost parallely. So, it is parallel shift, which means that slope of the budget line does not change. So, once I am seeing the slope of the budget line does not change. What is it? $-(1 + r_t)$. So, once I am saying that the slope of this budget line does not change, it means that there is not much impact on the current future period consumption. But now, since $-(1 + r_t)$ is the slope.

But if there will be any change in the interest rate, this will be reflected in your current and future period consumption. What does it mean? So, you can see that at point E, the budget line pivots. Pivot means that if you have the increase in real interest rate, your endowment is going to come down in the current period and you try to get more in the future. What it means here that if the rate of interest is going to be higher.

You try to save more in the current period itself. Your current consumption may be lower, and it tends to be lower, and your future consumption will be higher. Because whatever you save here, it is going to transform by $1+ r$.

So, once it pivots, it means that here, you are going to save this much amount compared to the previous position. And this amount is being transferred to your future which also means that the banks are offering you a higher rate of interest. What it leads to? It means that you should go for more saving, and then you can earn better interest in the future.

And then you can have better consumption not in the current period but in the future period. So, this is what, it means here. That once I go for extension of this, then I have this extension,

so here you are losing, here you are getting rewarded. You can see that this shift is smaller, and then here you have the larger shift. Here you are moving up that also adds $1 + r$ dimension.

Here the slope of this budget line is different than this slope of the budget line here it is more steeper. If you are thinking about increasing the real rate of interest, then your budget constraints slope is also going to change and it is going to be steeper. Here this is what steeper implies. So, where you are pivoting? On your endowment point, so it pivots around the endowment point.

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So, this is what it is. An increase in the market real interest rate for a lender and increasing the market real interest rate decreases the relative price of future consumption in terms of current consumer. What this lender will be thinking about? One thing is for sure this lender would go for lending when the rate of interest increases. But how much and whether the current consumption will increase, decrease or the future consumption is going to increase?

And this is partly because of the rate of substitution that he is going to find here, and the substitution effect will be dominant and the income effect also have the same. So, this is the income and substitution effect for the consumption. Now once I am talking about the rate of interest change, and then we have the pivoting of budget line, then we have the dimensions whether the income effect will override on the substitution effect.

Although the substitution effect will be overriding on the income effect, these two dimensions are important to understand. So, here it is.

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Figure: **An Increase in the Real Interest Rate**
Source: Williamson, D.S. (2018), Macroeconomics, 6th Edition. Chapter 9
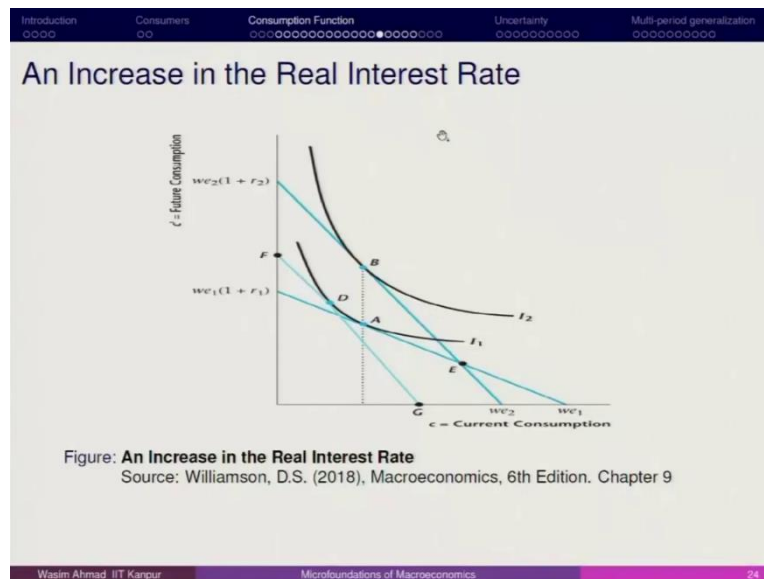
An increase in the real interest rate that we have, let us deal with this scenario. This is for the lending scenario. So, here it is that in both the periods, here he was at A now. Because of this the budget line, because real interest rate increase, there is a pivot. Pivot in the sense that now the budget line was original this it pivot to this line. Now, it is this line moving from we_1 and we_2.

Now, it is certainly clear that his future consumption is going to increase because now, he can see the income effect happening here. And now, how the income effect looks like, so, this amount is being saved already. This amount is being transferred to future because this was the original endowment after the rate of interest this is the scenario that you have. So, this is the original endowment and this is the future endowment.

So, here we have the rate of increase in saving. So, once we have the saving increasing this will have impact here. Now, from A to B it is clear that whatever he is saving, this is being transferred to the future consumption from A to B. But, we are not still sure that how much he is having the substitution effect dominant. So, we just want to make sure that if I just take out the extra income that he is going to get because of the rate of interest increase.

So, how does it look like if I tried to bring him on the original budget line. For, that I draw a budget line parallel to this. So, I have the budget parallel to this, which have same slope like this. We also have to assume that both are the normal goods. Now, here at point D, let us talk about point D.

At point D what is happening, here we are seeing that the current consumption is declining, but future consumption is increasing. So, which means that compared to original even after adjusting with income. This representative consumer is going to have more of the future consumption less of the current consumption. So, the substitution effect is the movement from A to D.

And D to B is the income effect. So, A to D that we have the point here. Here it is clearly showing, so, if the substitution effect is having a role here, then, both the substitution effect and income effect are in favour of an increase in future period consumption increase. The substitution effect shows that there is a decline in the current period consumption also.

So, if the substitution effect is going to dominate then the current period consumption will come down future period consumption will increase. If the income effect is going to dominate, then we have only the future consumption increase, we do not have the current consumption increase it remains the same.

So, in this particular case, for a lender. If this particular lender is saving more in the current period, because of the rate of interest increase, it is bound to have future period consumption increase. So, once we have future consumption increasing this is bound to have this. So, this is what it looks like here. Now, here we have the borrower. So, let us deal with the borrower.

The borrower scenario is a different case here. Here we are finding it much easier. So, here we have the scenario where this particular representative agent is at point E. So, this is the endowment point. But now, he has to move down because of the real interest increase. So, what is happening? This is the original endowment point.

So, he is having income of we_1 much, he is consuming a point. So, here this is what consumption looks like. Now, here because of the interest rate increase, the line pivots. Once we have the pivoting of the line, so, this is how it pivots. So, earlier he was at point A. Now, because of this pivoting what is happening is that this representative agent is having less of the income compared to the first period.

Now, if you think about the consumption, so, where he is consuming? Earlier he was consuming more of the current consumption, and less of the future. But after this what is happening? That this particular representative agent is at point B. Now, we draw a parallel line here to the original budget line such that it is tangent to the original indifference curve that his level of satisfaction remains the same.

Once I am saying about the original indifference curves, so, here we are drawing a line, it is tangent at point D. At point D, what is the point that we are making? Now, with this scenario, what does it look like? It looks like that if we have moved from A to D. So, this is what it looks like. That is because of the substitution effect where at this point, this particular guy had the current consumption higher than the future.

But at point D, his current period consumption has come down, and also the future period consumption has gone a bit amount. So, A to D is the substitution effect and D to B could be called the income effect. Now, at point B, since he is making a lot of effort, and with that effort, he is still not compromising, he is still not going to do well, on the future consumption part.

Current consumption is still better but compared to original point A, the consumption of current period is lower. So, overall, what it looks like as compared to the future this representative agent will have more of current consumption with future consumption not going that in favour. So, here, income and substitution effects both are important to understand, if I am talking about the original endowment point, so, this is the original endowment point.

This is the original budget line that this representative agent is having, this is the indifference curve. At point A, it is clear that this representative consumer, if he's having an increasing real interest rate, he is going to be having more current consumption. And future consumption is lower, when the interest rate rises at this pivot. Now, the representative consumer is at point B.

At point B, the future consumption has not gone up by that amount, but current consumption has come down a lot. You can say marginally the future consumption has gone up. If I am just trying to see what happens if I try to put the representative consumer on the same indifference curve, so, I draw a line parallel to this. So, both have the same slope $-(1 + r)$.

At point D, we can see that the future period consumption increasing. But it is not the current period consumption, it is not as low as the point B it is still higher. So, here with respect to the borrower, it is more or less clear that this borrower will have an impact on the current period consumption and about future period, we are not very sure. So, future period consumption it looks like, it will be on a downward trend.

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So, this is what we try to summarize. The effects of an increase in the real interest rate for a lender. So how does it look like for a lender the future period increase is certain this is what we saw here that the future period increase is certain. So, if the substitution effect is leading, then there will be an increase in saving.

But if the substitution effect is not leading, adjust the income effect then these two will remain uncertain.

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Effects of an Increase in the Real Interest Rate for a Borrower

Current Consumption	Decreases
Future Consumption	?
Current Savings	Increases

Now, here we have the current consumption decrease. As I mentioned here that the current consumption is decreasing in all cases, this is what we infer that it is the same. The future consumption whether it is if it is the substitution effect playing a role then, of course, will have the future period income increase, otherwise, it will not be the same.

The savings, once we have the savings, which means that it is going to increase for the borrower. But future consumption will remain uncertain. So, this is what we try to conclude here

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IEs and SEs of changes in income, consumption and IR

	$\uparrow Y_t$	$\uparrow Y_{t+1}$	$\uparrow r_t$
C_t	$+$	$+$	$-$
C_{t+1}	$+$	$+$	$+$

If you try to see respect to income substitution effect changes in income consumption and IR. So, this is how it looks like when you have Y_t increasing the current consumption if your current period income increasing. So, this has both positive effects, if you have future period income

increasing then this both have the positive effect and this I have taken from the Eric Sim's macroeconomics notes.

Especially on consumption and it is very helpful and this here it is Y_{t+1}. So, here it is once it is increasing, then here we have both positive. But with regard to the interest rate, it becomes interesting that, if you have the interest rate increase, then you have the current consumption going down future consumption increasing. And this partly because we have the if it is a lender, then he will go for saving and lend it to somebody to get more in future.

If it is borrower, he will go for borrowing and then he'll paying back in the future period. So that is the scenario we have.

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Let us have an example on the consumption function. How does it look like? Suppose we have considered two-period economy or two-period representative agent wants to maximize the value of utility defined over consumption in two-period C_t and C_{t+1}. Utility is represented by,

$$\max_{c_t, c_{t+1}} u(c_t) + \beta u(c_{t+1}), \text{ with } \beta < 1$$

So, $u(c) = lnc$ non–consumer receives Y_t and Y_{t+1} respectively.

The agent can freely borrow and lend in the financial market suppose the agent received y^P income, the income accrues to the individual every period where y^T is the transitory income Find the inter–temporal budget constraint for the consumer. So, first thing we have to write about the budget constraint.

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So, how does the budget constraint look like so, in this case, the budget constraint will be same as

$$\left(1 + \frac{1}{1+r}\right)y^P + y^T - c_t - \frac{c_{t+1}}{1+r}$$

So $1 + r$ will be in the present terms here.

Now, find the optimal consumption derive the marginal utility of consumption.

$$L(c_t, c_{t+1}, \lambda) = \log c_t + \beta \log c_{t+1} + \lambda\left[\left(1 + \frac{1}{1+r}\right)y^P + y^T - c_t - \frac{c_{t+1}}{1+r}\right]$$

$$\text{FOC: } \frac{\partial L}{\partial c_t} = 0 \text{ and } \frac{\partial L}{\partial c_{t+1}} = 0$$

Once I derive the budget constraint that I have written directly with lambda, then you can write the Euler condition, we can write Euler condition as,

$$\frac{c_{t+1}}{c_t} = \beta(1 + r)$$

By substituting Euler equation into lifetime budget constraint.

$$c_t = \frac{1 + \frac{1}{1+r}}{1+\beta}y^P + \frac{1}{1+\beta}y^T$$

Now, so, if you go by the partial derivation, so, then you can easily write the consumption. So, how it will look like so, if we go with respect to y^P, so, this will be $\frac{1+\frac{1}{1+r}}{1+\beta}$.

So, if we go with respect to y^T, it will be $\frac{1}{1+\beta}$, which means that the future consumption that you have it all depends upon how your beta is going to play. So, transitory component is a more to link with the behavioural aspect. Whereas permanent income it has to do more with the interest rate and beta. So, if beta is going to be lower, which means that you are giving more preference to the current consumption.

Then if the rate of interest remains same, you are going to get a higher value for y^P. Whereas, if $\frac{1}{1+\beta}$, see beta is going to be lower here, it means that you are going to consume more. So, the behavioural coefficient will not be having that much effect.

So, that is why since transitory income is subject to behavioural coefficient, and it is more to deal with the preference of the representative consumer that whether this representative consumer is going to give more importance to current or future.

So, I hope this makes the understanding of your macroeconomics better, and you can easily understand it.

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Now, we are dealing with the uncertainty scenario, how are you dealing with this scenario? Suppose the representative consumer faces uncertainty about his or her future income. Now, here you have to keep in mind about the averse and risk lover. When certain outcomes are not very certain. Your income is linked to some outcome based learning.

Which means that you try to attach with some kind of random components or maybe when you toss of when a flip a coin then if you get heads then you get 100 rupees, if you get tail you get zero. So, the average out phenomenon that we have that plays very important role here, that how when you face uncertainty then how this uncertainty is playing a critical role here.

Suppose the representative consumer faces uncertainty about his or her future income. Suppose, the future income has two values. So here we have Y^h_{t+1}, and here we have Y^l_{t+1}. Now, here, when I am saying about future income. Here, we are saying that since we are superimposing two conditions so high and low. Here $Y^h_{t+1} > Y^l_{t+1}$

Let the probability of the high income state occurring be p, with the probability of low income $1 - p$. So here it is that your expected income that you have. So here it is,

$$E(Y_{t+1}) = pY^h_{t+1} + (1 - p)Y^l_{t+1}$$

So here we have,

$$C^h_{t+1} = Y^h_{t+1} + (1 + r_t)(Y_t - C_t)$$

So here it is, this is what it is looking. Now when so this is the consumption when you have high income.

$$C^l_{t+1} = Y^l_{t+1} + (1 + r_t)(Y_t - C_t)$$

So here we are defining the future consumption. So, this particular part he is going to get is the high income this particular part low income. And this Y t – C t is coming as saving that he this representative consumer is having in the future period.

Expected value of future consumption is,

$$E(C_{t+1}) = pC^h_{t+1} + (1 - p)C^l_{t+1}$$

Suppose the future income takes two values which means that I am focusing more on the future value. So, even in Euler equation, there will be role of the future expectation. So, this is what we try to, say expected value consumption and second period is going to be $E(C_{t+1})$

So, this will be continuing in the next session. From here we will start in the next session, and we will try to understand it further. Thank you. Thank you so much.