Lecture - 06
Two Period Model I

Hi, everyone. So, let us start we are talking about the one period model consumption and leisure model. And, we have specified the model, and different agents in the model. We have also tried to look at the optimization condition. We were talking about the role of social planner which means that we are talking about the economic efficiency that whatever the competitive equilibrium that we have achieved whether it is socially efficient or not.

So, for that purpose, we had taken the help of certain tools and mechanisms that we normally use in welfare economics, so, Pareto optimality condition. And, in order to make sure that whatever we have achieved the competitive equilibrium, it remains efficient. We also have a new agent which is the social planner which means that it is basically the government that ensures that there is no exploitation of the consumer by the firm.

And since the consumer is also the wage receiver which means that whatever the market decides, he or she has to accept. So, in that context, we are trying to see and towards the end, I think we have covered most of the part. We are left with only a few. So, the reference remains same, the Stephan Williamson book and the Sanjay Chug that we have referred.

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Reference Book

Author Name: **Stephan D Williamson**
Williamson, D.S. (2018), Macroeconomics (6th Edition). Pearson International Edition, Boston, USA
Author Name: **Sanjay K Chug**
Chug, S.K. (2015), Modern Macroeconomics. MIT Press
Author Name: **Backhouse and Boianovsky**
Backhouse, R. E., Boianovsky, M. (2012). Transforming Modern Macroeconomics: Exploring Disequilibrium Microfoundations, 1956–2003. Cambridge University Press.

And, we are now we were talking about the optimality condition. And, in that, we mentioned in detail the competitive equilibrium. So, I think we had reached here.

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c) Competitive Equilibrium
Solution:

- We can write the above expression as

$$C = zF(K, N_d) - G$$

- Equation (14) is the expression for C. To find the expression of l, we can transform the consumer's optimization solution (Eqn. (14)) as

$$U_l'(C, l) - zF_N'(K, h - l)U_c'(C, l) = 0$$
$$U_l'(C, l) = zF_N'(K, h - l)U_c'(C, l)$$
$$\frac{U_l'(C, l)}{U_c'(C, l)} = zF_N'(K, h - l) = w$$

So, competitive equilibrium, we have defined. So, what is this condition? Let me give you a quick recap that what we were assuming competitive equilibrium in this context.

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Properties of Pareto Optimum

- In this model, the competitive equilibrium and the Pareto optimum are identical.
- We know this as, at the Pareto optimum:

$$MRS_{l,c} = MRT_{l,c} = MP_N$$

That marginal rate of leisure for consumption and marginal rate of transformation of leisure for consumption is equal to the marginal product of labor and which is also equivalent to the wage rate. So, this is the condition that we have to achieve in the case of Pareto optimality. So, we have already done that part to a certain extent.

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c) Competitive Equilibrium

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$$\frac{U_l'(C, l)}{U_c'(C, l)} = zF_N'(K, h - l) = w$$

So, here, first, we have already achieved this. I would say criteria about the competitive equilibriums which means that the marginal rate of substitution of leisure for consumption and the marginal product of labour is equal to the wage rate. And, this, the marginal product of labour can also be linked with the marginal rate of transformations.

So, once we have achieved this condition, then we say that we have now reached to the competitive equilibrium.

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d) Pareto Optimal

Solution:

- To introduce Pareto optimality condition, we first must assume the 'Invisible hand' called 'Social Planner (SP)' and how SP would choose consumption and leisure to maximise the welfare for the representative consumer, given the production function.
- The SP solves:

$$\text{Max}_{c,l}\ U(C, l) \text{ subject to } C = zF(K, h - l) - G$$

- To solve the SP problem, we set-up the Lagrange multiplier problem as

$$L = U(C, l) + \lambda[zF(K, h - l) - G - C]$$

Now, for Pareto optimality, we have to now assume certain aspects of the Pareto optimality. So, for that aspect, what is the third person that we are introducing into the model, it is the social planner. So, how social planner would choose consumption and leisure to maximize the

welfare of the consumer? So, for that, what we are supposed to do is that we are trying to optimize again.

And, for that reason, we have the maximum of $U(C, l)$. So, this is the utility that we have defined for the representative consumer. And here, we are also seeing that it is subject to $zF(K, h - l)$ which is the production function and this sum amount of the in the output that goes to the government. So, this is equivalent to the tax that you can say so to solve this problem, since it is focusing on consumption and leisure.

So, the modus operandi remains same. Here, not much change. So, here, we have the Lagrange multiplier problem,

$$L = U(C, l) + \lambda[zF(K, h - l) - G - C]$$

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> **d) Pareto Optimal**
>
> **Solution:**
> - To solve the SP problem, we set-up the Lagrange multiplier problem as
>
> $$L = U(C, l) + \lambda[zF(K, h - l) - G - C]$$
>
> **F.O.C**
>
> $$\frac{\partial L}{\partial C} = U'_c(C, l) - \lambda = 0 \tag{20}$$
>
> $$\frac{\partial L}{\partial l} = U'_l(C, l) - \lambda[zF'_l(K, h - l)] = 0 \tag{21}$$
>
> $$\frac{\partial L}{\partial \lambda} = zF(K, h - l) - G - C = 0 \tag{22}$$

And then, we are going for the first-order condition. So, first-order condition remains the same.

$$\frac{\partial L}{\partial C} = U'_c(C, l) - \lambda = 0$$

$$\frac{\partial L}{\partial l} = U'_l(C, l) - \lambda[zF'_l(K, h - l)] = 0$$

So, this is what we get the expression which is the marginal utility of the increase in utility due to change in the leisure.

$$\frac{\partial L}{\partial \lambda} = zF(K, h - l) - G - C = 0$$

Now, we solve in the same way that we have solved. And, we will be having the similar outcome here.

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d) Pareto Optimal
Solution:

$$\frac{\partial L}{\partial C} = U_c'(C, l) - \lambda = 0$$

$$\lambda = U_c'(C, l)$$

Substitute the value of λ in Eqn. (21)

$$\frac{\partial L}{\partial l} = U_l'(C, l) - U_c'(C, l)\,[zF_l'(K, h - l)] = 0$$

We have then

$$U_l'(C, l) = U_c'(C, l)[zF_l'(K, h - l)]$$

$$\frac{U_l'(C, l)}{U_c'(C, l)} = w = zF_l'(K, h - l) = zF_N'(K, Nd) = w$$

So, if you are solving equation one of the first-order condition with respect to consumption,

$$\lambda = U_c'(C, l)$$

And, if you just substitute this

$$\frac{\partial L}{\partial l} = U_l'(C, l) - U_c'(C, l)[zF_l'(K, \ h - l)] = 0$$

If you go again for solving this, then this is this expression can be written in this way which is the marginal utility of leisure for consumption and then you have the marginal utility of consumption.

And, if you just go for further solution of this. So, we are arriving at this condition.

When I say that this condition so, here, in this case, it looks similar to competitive equilibrium. So, here, we have the welfare idea. So, even with the condition of the Pareto optimality, we are achieving the same condition that we achieved in case of competitive equilibrium.

$$U_l'(C, l) = U_c'(C, l)[zF_l'(K, \ h - l)]$$

$$\frac{U_l'(C, l)}{U_c'(C, l)} = w = zF_l'(K, \ h - l) = zF_N'(K, Nd\ ) = w$$

That shows that whatever competitive equilibrium that you have achieved, it is socially efficient also. So, that means that when you leave the market if it is the free-market economy where the firms and labours are interacting in the same way, there is not much interference. Both are price taker and not the price decider. So, in that case, this is what we are going to see.

d) Pareto Optimal
Solution:

$$\frac{\partial L}{\partial l} = U_l'(C, l) - U_c'(C, l)\,[zF_l'(K, h - l)] = 0$$

We have then

$$U_l'(C, l) = U_c'(C, l)\,[zF_l'(K, h - l)]$$

$$\frac{U_l'(C, l)}{U_c'(C, l)} = w = zF_l'(K, h - l) = zF_N'(K, Nd) = w \qquad (23)$$

- Based on above expression, we conclude that Eqn. (23) solve for Pareto optimality quantities of C and I and it is identical to the CE condition.
- Therefore, we can conclude that the CE and the Pareto Optimum are the same thing in this model. Hence, the first and second welfare theorems hold.

So, the Pareto optimality condition that we have achieved here. What is the meaning of this? Which means that the last equation that we have equation 3 it solves the Pareto optimality quantities both consumption and leisure and it is identical to the competitive equilibrium that we have achieved. So, we can now conclude that as I mentioned that the competitive equilibrium that we have achieved so far it is fitting the criteria of social efficiency.

But, as I mentioned that in which all situations you have the failure of the competitive equilibrium to become socially efficient. You know the first thing that we had mentioned about the externality condition when we have negative and positive externality and the market failure condition then in that situation it becomes really difficult to attain this. Second thing, we mentioned about the proportional income tax.

If the taxes are going to be imposed as a percentage of on the wage of the labour, then the labour is going to have the limited income. And, this will lead to some compromise on consumption. So, this may not be efficient. The third thing is that if you are thinking about the market mechanism that we are mentioning here that you have the market and, in that market, both factors, the labour as a factor of production and the firm coming and interacting with each other and bargaining with each other.

But, here, if you have a monopoly situation, then this condition will not satisfy. So, the ultimate, the bottom line of this one period model is that with the help of the simple representation of

the consumer firm and the government we are trying to arrive at the macroeconomic condition. And, the macroeconomic condition is about what? So, here, you have 3 things cleared.

One is how you are going to decide about the role of the government. The role of the government is clear it is to collect the taxes and just make the expenditure and use that for expenditure. The second thing is that when we say that we have increase in government then what happens to the consumption. So, consumption decreases.

And then, you have consumption decreasing, then your leisure is also decreasing because of the substitution and income effect dimensions. There also we are clear about the macro that the increase in government expenditure it may not lead to a better outcome. Second, we discussed about the increase in productivity. So, what was the productivity increase?

When we have the productivity increase, it leads to increase in consumption, but for leisure, it is not very sure whether leisure will increase or decrease. It may decrease also because some people may satisfy with the increased wage rate. So, as compared to the government expenditure increasing, this leads to wage rate fall. And this further creates a scenario for extra working hours.

It will be in that case it is not very good. The third important thing is that we are bringing intervention by social planner. So, once you are bringing intervention by social planner into the condition that if you are leaving everything to the market that whether the outcome from that market it is socially efficient whether we can say that it fulfills the criteria of economic efficiency.

Then, that in this particular model, becomes easier to understand considering the externality, the proportional tax and market conditions keeping aside. So, I hope with this simple exposition of in a very simple context of this model which I have mostly adopted from the Williamson's, gives you a complete feel about the how the firms and the market and the consumers interact to each other and how the socially efficient outcome can be achieved.

Those of you who are doing it for the first time I would urge you to go and read some more books and explore these models. Because, if you want to understand the deeper context of the macroeconomics, then it is always good to explore these dimensions and try to link it at the

macroeconomic picture. So, I will stop here now for one period model. Now, I will be moving to the second period model.

So, now, we are going to see the two-period model. And, in the one period model, we had a very stringent condition that the representative consumer is not allowed to do anything in a dynamic fashion which means that whatever he or she is getting an income he or she has to consume it in that period not beyond that. So, now, we will be seeing the scenario where we have the inter-temporal framework which means that the consumer will be interacting not in just in the current period but also in the future period.

So, here, we will be just for the sake of simplicity taking two period, you can go for infinite period model. That I will also be showing with you with some examples. But, in two-period model, it becomes really interesting, and we will have this similar kind of dimensions over there also. And then, we will see that when we are restricting the individuals in only in one period, then what are the condition and situations.

When we are going to shift it to two-period model, then how the behavior of these agents change. So, there it becomes really important. In two-period model, basically, we try to see the behavior of the representative consumer from the perspective of consumption and saving. I have some amount of income. I have got 100 rupees. I know that I will be living only for two periods.

I have one option that I can consume the whole 100 rupees in the current period itself. Or, I save some amount for the next period. So, I have 100 rupees. I may be consuming 40 rupees in the current period thinking about the future period that in future period also I will have the opportunity to consume. But, beyond future period, you have nothing. So, second period is the terminal period. So, this particular guy has to spend 60 rupees there.

Now, what will be the incentive to save? So, in macroeconomics, we always think about the rate of interest as an opportunity cost of either consuming today more or saving it if it for the future. So, if you have the rate of interest higher, then it means that this particular representative consumer would like to save more for the future and less for the current period. And, in the same way, if you have some kind of expectation that you are going to get higher income in future, then you will also go for adjustment in the current period.

When you have scenarios like, what happens when you have a more income in the current period? So, you have lot of implications for what we see in macroeconomics what is called consumption theories. You have relative income hypothesis. You have permanent income hypothesis. So, in permanent income hypothesis, we assume that a particular consumer is having consumption behavior.

And, he tries to streamline it through the lifetime earnings. And, how he is going to have during his life expectancies. Suppose your job is for 60 years but the average life expectancy is around 75 years which means that you will be working for 60 years but you are going to die after 10 to 15 years. So, you will have to plan your income and consumption in such a manner that even after retirement you have some amount of income left for the remaining period.

So, that has a lot of implications in the macroeconomic dimension that whenever we have a tax increase whenever we see a some kind of extra incentive given, then consumers react. One term which is very common and we often use is called consumption smoothing. What is consumption smoothing? Consumption smoothing is called when you have certain scenarios.

For example, if suppose we are examining government and if the government is going to increase the tax in the current period, then how this particular consumer is going to react. Because, in the current period, if the tax is going to be increased, then this particular individual will have some kind of compromise on consumption.

So, how this particular consumer will smooth out for till the future period. So, that becomes the part of consumption smoothing process with the event of tax increase or decrease. So, if the tax is going to decrease for instance, he is going to get an extra income then how that is going to play a role in smoothing the consumption in these 2 periods. There are certain associated tools and techniques.

Like for example, we use the concept of social planner. We also try to measure that context in the inter-temporal framework by introducing the concept of Ricardian equivalence. So, there we try to see that with the adjustment in tax whenever you have increase or decrease, how consumers are going to behave with their consumption pattern. So, that becomes a really important tool to examine.

The second thing is about the basic concept. So, for example, we have the macroeconomics one. In macroeconomics, we study about the marginal propensity to consume and average propensity to consume. That comes from the Keynesian theory of consumption. So, marginal propensity to consume that how much you have change in consumption due to change in income.
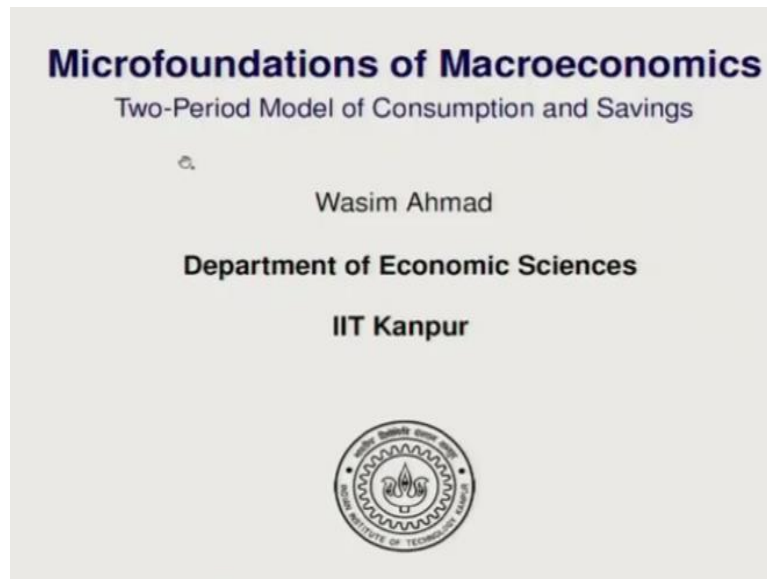
Now, if you try and see in the inter-temporal context with the help of the consumer with two period scenario that how much he is going to save and how much he will have. With that also you will have the clear-cut understanding that whatever in macroeconomics we study the change in consumption due to change in income we will have the simple or similar kind of derivation in the two period.

And then, we will be seeing that what actually determines the marginal propensity to consume whether just the income change or it is the behavioral change also. So, some kind of variable coefficient will be introducing. In the same way, we will be also seeing that what happens to consumption when there is a change in rate of interest, what happens to consumption when you have a future income change.

So, marginal propensity to consume you will not be just seeing in the current period, you will be seeing it with respect to future period of the income. So, then, it is in that context such analysis helps you to understand the macroeconomic theories in a more deeper way. And then, you can analyze those things and have some kind of the spillover context in a larger way where you can have to decide about the consumption pattern at the economy level.
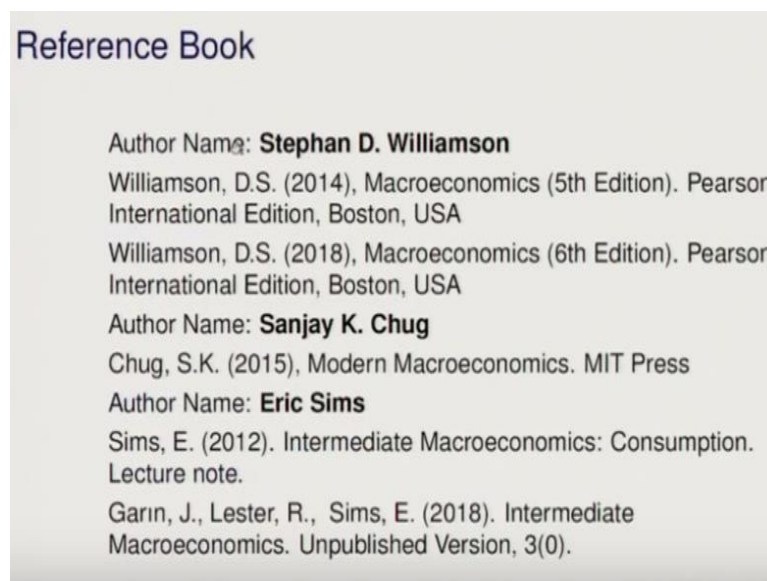
So, let us get back to that. So, here, we have the two-period model of consumption and savings.
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Microfoundations of Macroeconomics
Two-Period Model of Consumption and Savings

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So, in two-period model here, I have given you the sufficient background. So, I think it is more than sufficient. Here, we will have the same set of reference.

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Reference Book

Author Name: **Stephan D. Williamson**
Williamson, D.S. (2014), Macroeconomics (5th Edition). Pearson International Edition, Boston, USA
Williamson, D.S. (2018), Macroeconomics (6th Edition). Pearson International Edition, Boston, USA
Author Name: **Sanjay K. Chug**
Chug, S.K. (2015), Modern Macroeconomics. MIT Press
Author Name: **Eric Sims**
Sims, E. (2012). Intermediate Macroeconomics: Consumption. Lecture note.
Garín, J., Lester, R., Sims, E. (2018). Intermediate Macroeconomics. Unpublished Version, 3(0).

So, here, we have the Stephan D Williamson. I have also referred a very good textbook which has not yet been published. It is by Intermediate Macroeconomics by Eric Sims. This particular textbook is going to be very helpful especially those who are in senior undergraduate and the master level. So, this particular textbook is really good. Sanjay K Chug remains same.

This particular book is freely available. So, if you want you can download from the web page of Eric Sims and you can see and read those texts. The Williamson is having the recent edition 6th edition we have also referred that apart from the 5th edition.

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Key Learning Objectives

- Setting-up the two-period model and how two-period model helps understand the consumption behaviour in the inter-temporal framework.
- The process of consumption smoothing and how consumer adjusts his/her consumption within two period dimension.
- The optimization condition in the two-period model.

So, what will be the objective? As I mentioned, we will be working out with the agents in the two-period model and how two-period model helps understand the consumption behavior in the inter-temporal framework. The second thing is the process of consumption smoothing as I mentioned that when we have certain incidence of tax, how the representative agent is going to react.

Whether it will be a simple he will not be saving at all or he will be having some incentive to compromise on current consumption and saving for the future. So, those dimensions we will try and see. And then, we will have the optimization condition in the two-period case model. In the same way, the competitive equilibrium, we had the optimization condition.

So, for the consumer, the optimization condition remains same when we have the maximization of utility. So, we always try and see that how is the indifference curve and budget constraint that we try to define for the representative consumer. How both are tangent? How are these 2 conditions satisfy? So, the marginal rate of substitution is equal to the, in this case, it will be simply the rate of interest.

But normally, we have the price ratios for the macroeconomic textbook if you have gone through.

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Budget Constraints

- The consumer's current-period budget constraint:

$$c_t + s_t = y_t - t_t \tag{2.1}$$

- The consumer's future-period budget constraint:

$$c_{t+1} = y_{t+1} - t_{t+1} + (1 + r_t)s_t \tag{2.2}$$

Simplify

- Solve for s in (2)

$$s_t = \frac{c_{t+1} - y_{t+1} + t_{t+1}}{1 + r_t} \tag{2.3}$$

So, here, we have the budget constraint. In budget constraint, suppose, we have two-period model, in two-period model here we have the current period budget constraint. As I mentioned that we have the representative household. This representative consumer lives for only two periods. So, in period one, he gets some income. He is working and he gets some income.

So, that will be similar to what we have seen in case of one period model. Now, we are also superimposing one more period that is the future period. So, current and future periods, in both periods, he will have some amount of income that we are representing by y_t and y_{t+1}. And since, this particular guy is going to live for only two period so one opportunity that this particular person is having that he can go for saving in the current period.

But, because, in future period, it is a terminal period, so, in future period, anyway, this particular guy is going to die. So, it is better that he will be using everything. There is no role of inheritance or anything that he cannot pass on to someone who loves and whom he or she loves. That is not a possibility here. The terminal period means the end of the period. So, whatever this representative consumer is having he or she has to utilize.

So, here, we have this c_t, the consumption in period t. So, first period, let us deal with the first period budget constraint.

$$c_t + s_t = y_t - t_t$$

So, y_t is current period income and s_t is current period saving . We are also assuming that this representative consumer also faces some kind of taxation which is of lump sum tax. So, some amount of tax, some amount of income goes as tax.

So, this $y_t - t_t$, you can consider this as an disposable income. So, this is the income. This is the expenditure side. This disposal income is distributed in 2 parts. Some amount of money is being used for consumption and some will be saved.

Now, the consumer's future period budget constraint

$$c_{t+1} = y_{t+1} - t_{t+1} + (1 + r_t)s_t$$

So, what is this expression? It says that now we are looking for the future period consumption. Future period consumption is what? So, here, what it says. Future period consumption is equal to disposable future income that he is having plus the interest earning that he is having on his first period saving. What is $(1 + r_t)$?

Whatever money that he is saving in the current period when it is being moved to future period whatever will be the prevalent interest rate this representative consumer gets that income gets that extra income and that becomes the interest income $(1 + r_t)$. And, this we can simplify this further. If we solve for s t here, so, what we are getting.,

$$s_t = \frac{c_{t+1} - y_{t+1} + t_{t+1}}{1 + r_t}$$

It becomes easier. Now, you can substitute this here.

$$c_t + \frac{c_{t+1} - y_{t+1} + t_{t+1}}{1 + r_t} = y_t - t_t$$

So, here it becomes easier. So, here we are working out with what. Why are we doing it? We are doing it because we want to now derive the if we are assuming two period current and future period then we want to derive the lifetime budget constraint of this representative consumer that with the combination of first and two period in the present value context.

How much he will be having the lifetime consumption and lifetime income? Because, if we have to derive, then it becomes easier.

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Consumer's Lifetime Budget Constraint

- The consumer's current-period budget constraint:

$$c_t + s_t = y_t - t_t$$

- Substitute in the current-period budget constraint obtaining lifetime budget constraint:

$$c_t + \frac{c_{t+1} - y_{t+1} + t_{t+1}}{1 + r_t} = y_t - t_t \qquad (2.4)$$

- We can simplify this further and write it as:

$$c_t + \frac{c_{t+1}}{1 + r_t} = y_t - t_t + \frac{y_{t+1} - t_{t+1}}{1 + r_t} \qquad (2.5)$$

Now, we can simplify this further. What we are getting is this part,

$$c_t + \frac{c_{t+1}}{1+r_t} = y_t - t_t + \frac{y_{t+1} - t_{t+1}}{1+r_t}$$

Now, you will be seeing that the future values are in present value terms. Why? Because, this particular representative consumer has to decide in the current period itself, so, about future if you are going to decide about what will be the expected value. So, that will be always in the present value terms. So, here, it will be this is the overall expenditure of this. This is the lifetime I would say consumption expenditure.

And, this is the lifetime income of this representative consumer. So, we are able to drive with this simple transformation. We are able to drive the two-period model budget constraint of the representative consumer which is unlike the first period model. Only one period model it is much different. So, overall what we try to see is that the two-period analysis makes it easier to understand the budget constraint.

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Consumer's Lifetime Budget Constraint

Consumer's Lifetime Wealth:

$$we = y_t - t_t + \frac{y_{t+1} - t_{t+1}}{1 + r_t} \qquad (2.6)$$

Simplified Lifetime Budget Constraint

$$c_t + \frac{c_{t+1}}{1 + r_t} = we = y_t - t_t + \frac{y_{t+1} - t_{t+1}}{1 + r_t} \qquad (2.7)$$

So, this is how it looks like, the simplified lifetime wealth and lifetime budget constraint will be this. This is the lifetime consumption expenditure. And, this is the lifetime income of the representative consumer. And, with this, we will be further superimposing the condition of the consumer's utility and then we will be trying to examine the condition that how this representative agent is going to play an important role. I will be stopping it here, thank you so much.