Microfoundations of Macroeconomics
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Lecture - 13
Ricardian Equivalence II

Welcome back, and now we are going to start a somewhat new topic. We already discussed the Ricardian equivalence and we wanted to talk about that, how and what happens when we have the borrowing constraint on the individuals when we have certain individuals in the society who are not allowed to borrow in the open market with the given rate of interest. And if there are any changes in the government stand for example if the government is going to change the taxation policy.

If there is some tax relief given in the budget then, how the individuals are going to react? Ricardian equivalence is a very powerful concept, especially in the context when we are trying to understand that, when you have given a scenario that, there is a government and government is trying to enforce some kind of either the giving you an incentive or it is having some kind of conditions under which it is increasing the tax.

How do individuals are smoothly maintaining their consumption in both period 1 and period 2? smooth transfer of a tax burden or any government burden and if it is without compromising on consumption, then we say that the Ricardian equivalence hold and if it is not smooth, then we say that Ricardian equivalence does not hold. Now, in a condition where we already looked at two types of consumers a and b.

But now we are trying to look at a 3-period setup. we have done so far, the one-period model under that we examine the framework of consumption and leisure. Then we introduced the 2-period model and then in 2-period model, we discussed the behaviour of consumption and saving and how the representative agents are behaving in certain scenarios when they are having an increase in income current and future.

In this session, we will be having a different exposure to what we call the 3-period model. Finite life scenario, we are still not into the infinite, we have already discussed the infinite utility, and we have already derived the infinite lifetime budget constraint. But here we will be looking at only in 3 periods. There will be an overlap of one period, so overlap of one period in the sense that.

If we are having an old, if we divide the three period into a scenario where you have old and young and the young comes in period 2 when the old retires. How the smoothing is taking place? So far, what we have done is that we have decided exogenously that, if there is increase or decrease in taxes, then how the individuals are going to react with their current and future consumption.

Here, we will be trying to decide endogenously which means that, if we have these two generations; one is old another is young. Now, the young is coming in period 2, so the old is taking care or sharing some burden of the young by transferring some amount of wealth. A typical example could be that, our parents when we are born, our parents also take care of us what will be the livelihood scenario in future?

How he will be doing? Some kind of financial safeguard they always have. Some kind of benevolence we see that the olders will have some kind of special care or inheritance given to the younger generation and this younger generation uses that for further consumption. We will be trying to look at that.

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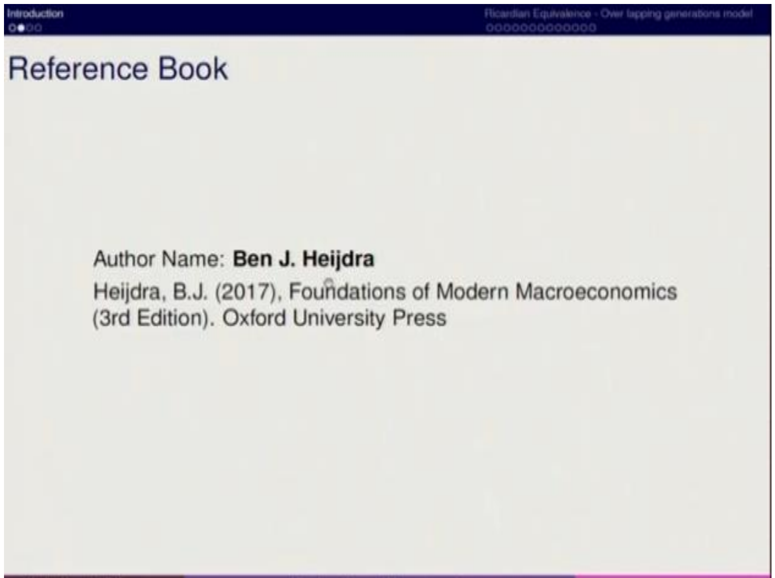

Here, we are referring the Ben J. Heijdra. Unlike we have so far referred to Williamson, in this particular session we will be referring to Ben J. Heijdra, the foundations of modern macroeconomics, it is the third edition and it appeared in 2017 with its third edition. The book is really good and it gives you a sufficient mathematical elaborations.

But yes, if those of you who are not having a proper macroeconomics background, then you can still try and read about these concepts in this particular book, you just need the understanding of calculus and even the difference equation, so that will be more than enough to understand. But the underlying idea remains that we are trying to understand a situation, where if you have the inter-generation wealth transfer happening from the older generation to the young generation.

Then, how the exogenous shocks like a tax increase or if the younger generation is facing some kind of uncertainty how he can cope up. But we will be looking at it only from the perspective of tax increase because here we have the Ricardian equivalent scenarios where we look at the smoothing of consumption of current and future.

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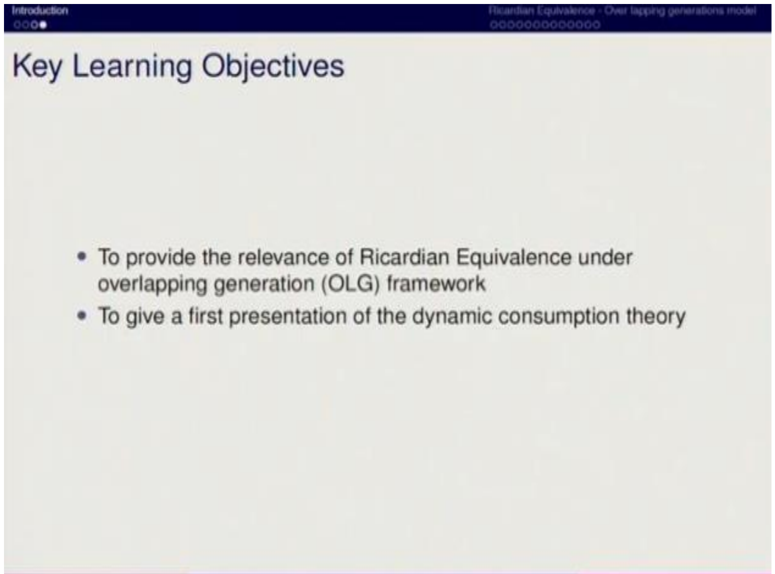

Here this is what we are trying to understand and we can also call it three-period consumption theory, as a dynamic consumption theory and we will be also deriving the consumption function.

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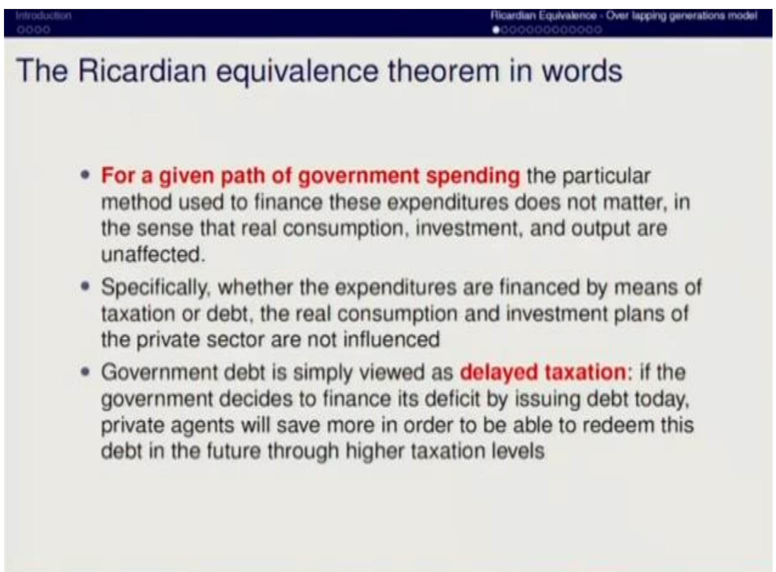

We will be also looking at the framework that we have discussed so far in terms of current and future consumption. For a given path of government spending, this is what we try to mention, if you have a government expenditure it means that, if the government is going for increasing the expenditure, it means that it will increase the taxes. If it is going to increase the taxes, then here it is assumed that the real consumption investment output and all the other factors are unaffected.

This is one argument that most macroeconomists often mention. And here there is also an argument that, whether the government uses the tax as an instrument or goes for debt for borrowing, in both scenarios if your government is going for borrowing, then it also means that, in the future, the tax scenario is not going to be good, so individuals will be saving, so in both scenarios, you have the same reactions, so that is what we say.

Because, if the government is going to have the government debt, suppose in the Covid scenario government is borrowing some amount of money from the external sources or from different sources, whatever burden that government is incurring now, it will allow the individuals to share in future, whenever they will find it convenient time. You might be saying that in some countries now the talk has started that you have the rolling back of the stimulus.

This means; now we are in a very comfortable situation and the economy is doing good. Then, in that situation, the government will be rolling out all the physical stimuli, and tax incentives and then, which means not enjoying the same benefit that you had. But those benefits are being now transferred into the higher taxes or the rolling back of incentives. You may not be having the same scenario that you had earlier.

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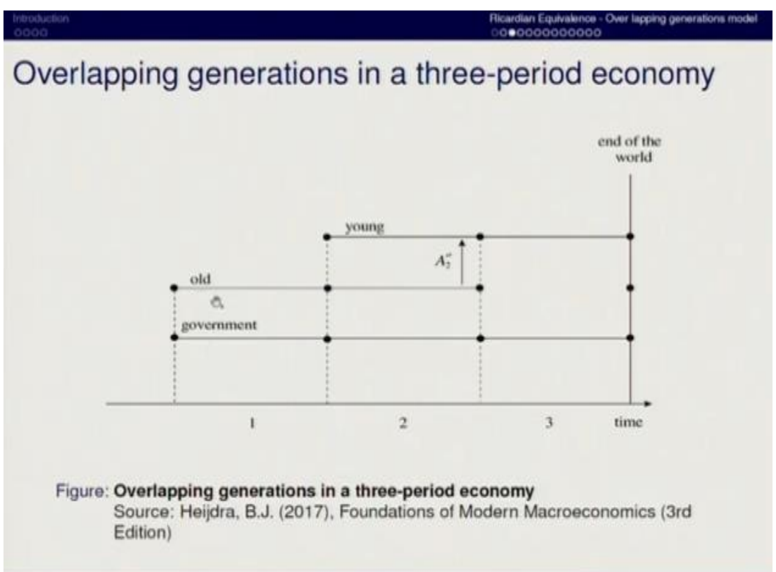

Here I am talking about the government which exists in all periods, so here we are saying that in three period time, so here one, two and three periods that you have. Here you have the old generation entering here and old generation lives for only two periods, the young generation also lives in two periods, so old generation comes here and dies here. The young generation comes here in this particular stage and it dies in this stage.

Beyond this, we are not saying. what we are assuming is that we want to see that, if there is an overlap of one generation here, which means that the old is also there and the young is also there. During this period what happens? Will there be any kind of interaction happening? We say that the old generation saving because now there is a possibility that the old generation can look beyond two periods, which the young generation is not allowed.

This old generation whatever he saves for the future period, can still keep aside some amount of wealth that he has and pass it on to the young in this period and the young will be using that to smooth the consumption in both periods. This is the beautiful idea that how in real-life scenarios, we often see such things when our parents also they are young.

They think about the younger generation, which means that they will be saving some amount here that this saving will have the future value, which means that some kind of extra income they will be getting. Then, this income they will be used for their consumption, but some amount of buffer that they will have, they will pass on to the younger generation. But, here one thing is important the young generation cares about the old generation.

The old generation saves only when he is caring about the young generation. If the old generation does not care about the young generation, A_0^o then this will disappear. This is the amount that asset that this particular old generation is transferring to the young generation as an extra income you can say or extra asset or extra wealth and this young generation will be using this amount to smooth the consumption.

If you go back to this side, then, you can think about that old generation will also have some amount of wealth starting and this wealth will be interest bearing and this wealth that this particular individual is having apart from income, he also earns some kind of inheritance, so it starts not with 0 but some amount, so this we will be defining now.

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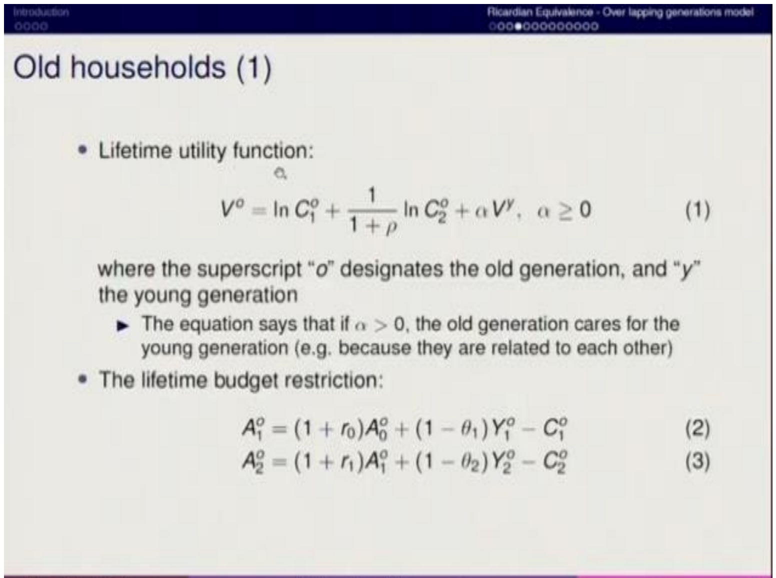

Here in this case, we are having the lifetime utility function.

$$V^o = \ln C_1^o + \frac{1}{1+\rho} \ln C_2^o + \alpha V^y, \quad \alpha \geq 0$$

In the lifetime utility function, this o subscript is nothing but represents the old generation. Here $\frac{1}{1+\rho}$, can be also written as beta, because this shows the sensitivity. For the time being if you do not want to write this as $\frac{1}{1+\rho}$, which mention the preference over the future over current or current over future, assume it to be β. Here α shows the magnitude of benevolence, that how much the old is carrying the young generation and since it is the transfer of wealth, so it has to be positive, it cannot be 0 and it cannot be negative, so the negativity constraint is ruled out where the subscript o designates the old generation and y the young generation.

Equation says that if $\alpha \geq 0$ the old generation cares for the young generation, this is what we mentioned about. That is because they are related so maybe father and son, father and daughter, so we are mentioning about these two things. Here we have less, so the utility function is quite straightforward, the only thing is that we are adding one more component here, αV^y.

Now, we are mentioning the lifetime budget constraint. Here the lifetime budget constraint looks like this,

$$A_1^o = (1 + r_0)A_0^o + (1 - \theta_1)Y_1^o - C_1^o$$

$$A_2^o = (1 + r_1)A_1^o + (1 - \theta_2)Y_2^o - C_2^o$$

Now, if we since we have to derive the lifetime budget constraint, so how do we derive this? we can just go substituting A_1^o here and solve it.

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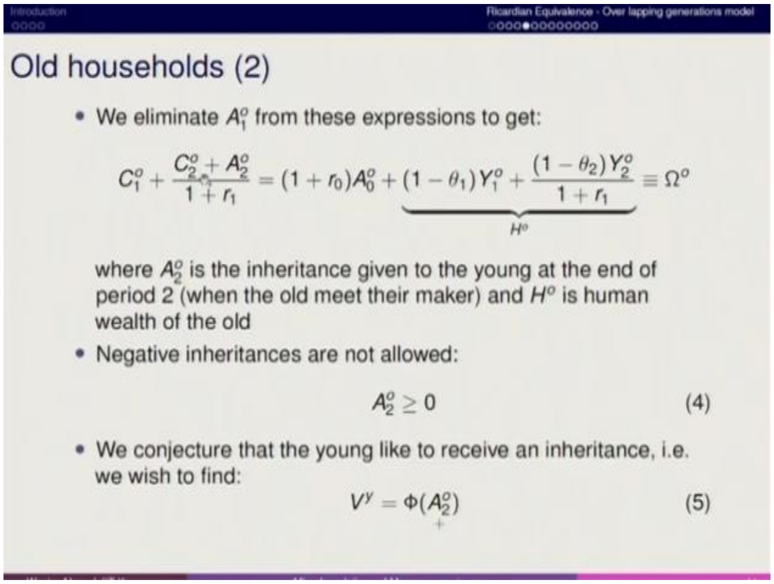

Here we have,

$$C_1^o + \frac{C_2^o + A_2^o}{1 + r_1} = (1 + r_0)A_0^o + (1 - \theta_1)Y_1^o + \frac{(1 - \theta_2)Y_2^o}{1 + r_1} \equiv \Omega^o$$

where A_2^o is the inheritance given to the young at the end of period 2 (when the old meet their maker) and H^o is human wealth of the old here this is what when he dies, he transfer this. But, if he is not caring about anything, then this may also be 0, because since he is caring about young generation, so he is saving in this period 1 and that is what is being transferred to period 2. This period when he is not caring at all, not benevolent or if do not see any bequest, then it will be 0, so bequest is important. Here negative inheritance are not allowed so,

$$A_2^o \geq 0$$

Now, here we are also mentioning,

$$V^y = \Phi(A_2^o)$$

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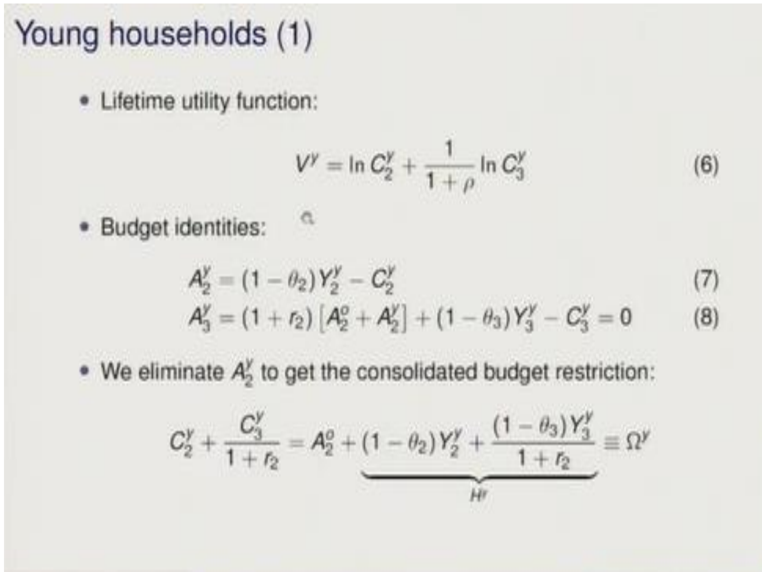

Let us go by the lifetime utility function, so this is what so we are done with old household, so we will be arriving at this very soon and this can be represented by this. Φ we are not clear about from where Φ is coming, so we will be explaining that. But we have now worked out with the old household, now we will be working with the young one how the young one is looking? Since, we are not allowing young to think beyond 3rd period, which means that young is entering in period 2.

You can see here, young is entering here and now he is not allowed to think beyond this, we are not allowing that. If he is going to period 3 then this is the terminal period for the young, then how we can see the reaction. Here we have

$$V^y = \ln C_2^y + \frac{1}{1+\rho} \ln C_3^y$$

The budget identities are what? Here we have,

$$A_2^y = (1-\theta_2) Y_2^y - C_2^y$$

$$A_3^y = (1+r_2) A_2^o + A_2^y + (1-\theta_3) Y_3^y - C_3^y = 0$$

In period 3 when we have A_3^y, which is the asset in period 3 is nothing but $(1+r_2) A_2^o$ he is also getting from the old generation in period 2 the transfer of wealth. It is added and then in period 3, whatever is the income difference that he has, so this amount and this amount has to be absorbed only in period 3.

The beauty of this is that this is not coming from the young generation; it is coming from the old generation. And this A_2^o will play very important role in smoothing any kind of exogenous shock coming from the government, either increase in debt due to that you have the increasing taxes or direct increase in taxes. In that situation this particular individual will be using A_2^oas a quotient against any shock, unexpected shock.

We eliminate A_2^y here and substitute it here ultimately what are we getting? Here, we are getting

$$C_2^y = \frac{1+\rho}{2+\rho}\Omega^y$$

$$C_3^y = \frac{1+r_2}{2+\rho}\Omega^y$$

This is what we he is getting and rest of these values are same as old, only thing is that here we have period 2 and period 3 in case of old it was only period 1 and period 2. This H^y is the human wealth of the young generation.

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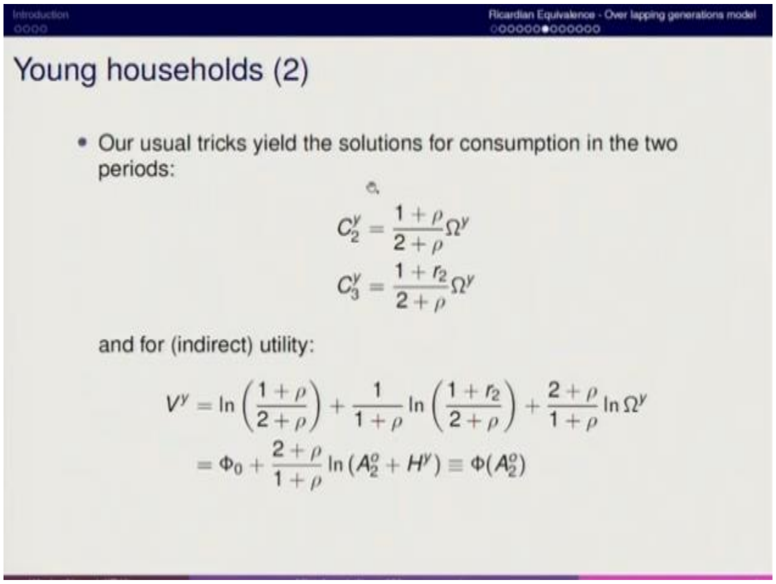

Now, here if you just go about solving for consumption in both period for the young household, so consumption for young household will be the same. That will be deriving at the other condition and then you will be solving it and after solving it you get this expression. Now, here we have the younger generation, so if you think about substituting it here, that how does it if substituting it here, the value of $C_2^y$and C_3^y.

If I substitute here then this is how it looks like

$$V^y = \ln\left(\frac{1+\rho}{2+\rho}\right) + \frac{1}{1+\rho}\ln\left(\frac{1+r_2}{2+\rho}\right) + \frac{2+\rho}{1+\rho}\ln\Omega^y$$

If you go by the equation of the benevolence, then benevolence will be looking like, this is how we are solving it. Now, if I go for further solution, then we can write this as this

$$V^y = \Phi_0 + \frac{2+\rho}{1+\rho}\ln(A_2^o + H^y) \equiv \Phi(A_2^o)$$

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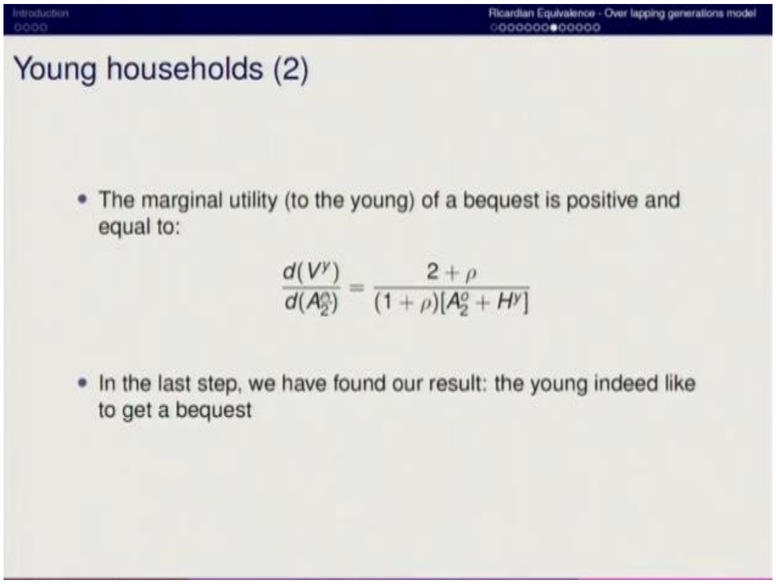

Now, if you want you can also solve for the marginal utility, so marginal utility of this will be with respect to A_2^o, whatever is the amount being transferred from old to young, if you want to see that how much the change of utility of the young generation with respect to the benevolence or the transfer of wealth is from old to young. Then, after the differentiation of this, it becomes

$$\frac{dV^y}{dA_2^o} = \frac{2+\rho}{(1+\rho)[A_2^o + H^y]}$$

This is what we have the rho, which means that the denominator term the A_2^o is still visible and this will create a problem if it increases. The marginal change that we are saying, we are saying that the A_2^o is still playing very important role.

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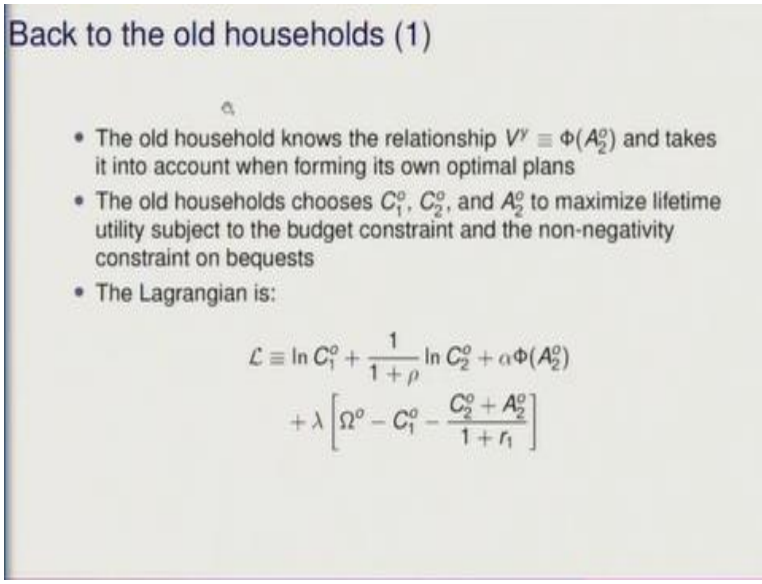

Now, we are seeing that, how are the situations under which the old is allowed to think about the young generation. The old house will know the relationship $V^y \equiv \phi(A_2^o)$ and so takes into account in forming the consumption pattern.

In period 1 this particular individual think that this $\phi(A_2^o)$is playing very important role. He will always have preference high for the future to save more so that in period 2, he can transfer some more amount to the younger generation. The old household chooses C_1^o, $C_1^o$and A_2^o to maximize the lifetime utility subject to the budget, which means that A_2^o is now inherently endogenous part of the old generation.

In the equation we can write this way.

$$\mathcal{L} \equiv \ln C_1^o + \frac{1}{1+\rho} lnC_2^o + \alpha\Phi(A_2^o) + \lambda(\Omega^o - C_1^o - \frac{C_2^o + A_2^o}{1+r_1})$$

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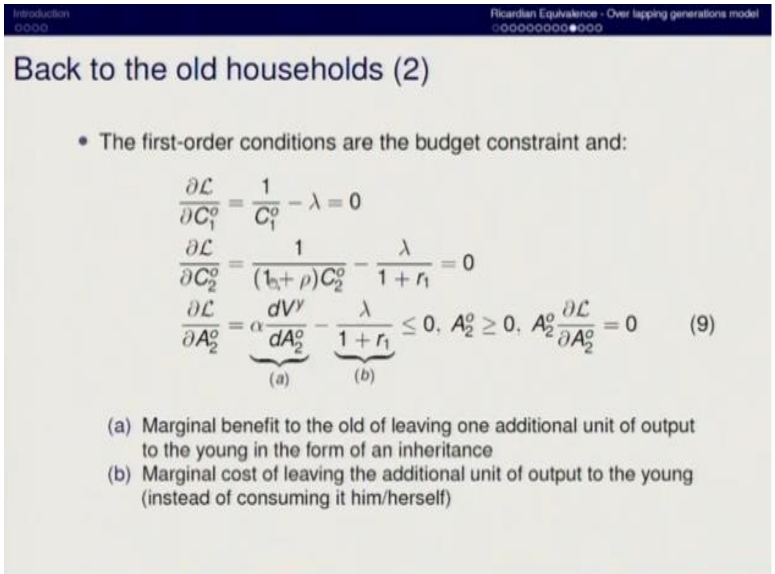

Now, if you go by the differentiation, so since we are looking at this household is going to choose C_1^o, C_1^o and A_2^o, so we will have to now differentiate with respect to all this C_1^o, C_1^o and A_2^o, so this is what we are trying to say.

$$\frac{\partial \mathcal{L}}{\partial C_1^o} = \frac{1}{C_1^o} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial C_2^o} = \frac{1}{(1+\rho)C_2^o} - \frac{\lambda}{1+r_1} = 0$$

$$\frac{\partial \mathcal{L}}{\partial A_2^o} = \alpha \frac{dV^y}{dA_2^o} - \frac{\lambda}{1+r_1} \leq 0,\ A_2^o \geq 0,\ A_2^o \frac{\partial \mathcal{L}}{\partial A_2^o} = 0$$

What is more important is this? $\alpha \frac{dV^y}{dA_2^o} - \frac{\lambda}{1+r_1}$, so if α is equal to 0 which means that no life for the younger generation, the old generation is not caring at all about the younger generation. $\alpha \frac{dV^y}{dA_2^o}$ particular expression will be 0 and you are left with $-\frac{\lambda}{1+r_1}$. If the individual is caring about, then only this marginal change will play very important role and this value we will derive here.

This is what we try to say that the marginal benefit that we have a and b to the old leaving one additional unit of output to the young in the form of additional inheritance. The marginal cost of leaving the additional unit of output to the young instead of consuming.

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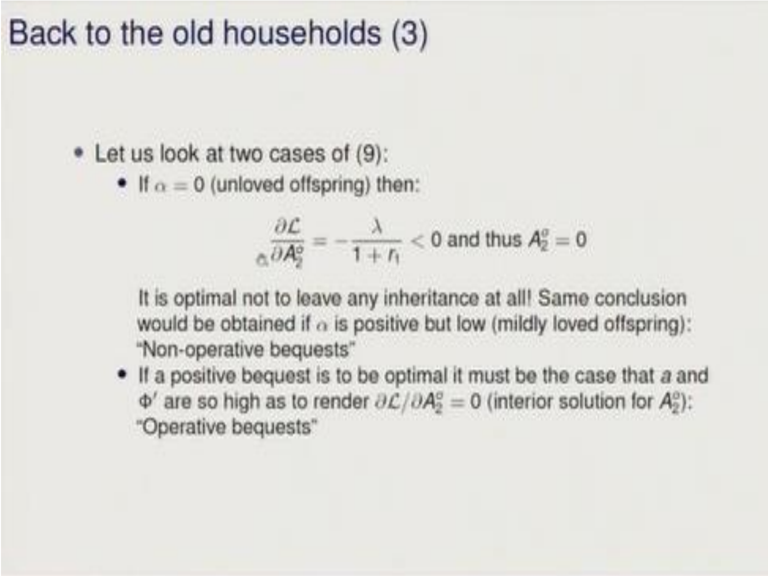

This is what is important here. As I mentioned that, if alpha = 0, then it becomes $-\frac{\lambda}{1+r_1}$ and this we call it as the operative bequest, where we say that the older generation is taking care about the younger generation. And, I hope with these 3 periods and understanding I would say 3 period dynamic consumption model, we are able to see that the Ricardian equivalence holds here because this A_2^o is acting like a grease or acting like a smoothing function for the younger generation.

Old generation whatever he has, because he is thinking at the same time C_1^o, C_1^o and A_2^o, so which means that the amount that has been transferred from the old to young. Young is using this A_2^o to smooth out the consumption of current and future and this makes the analysis interesting to understand.

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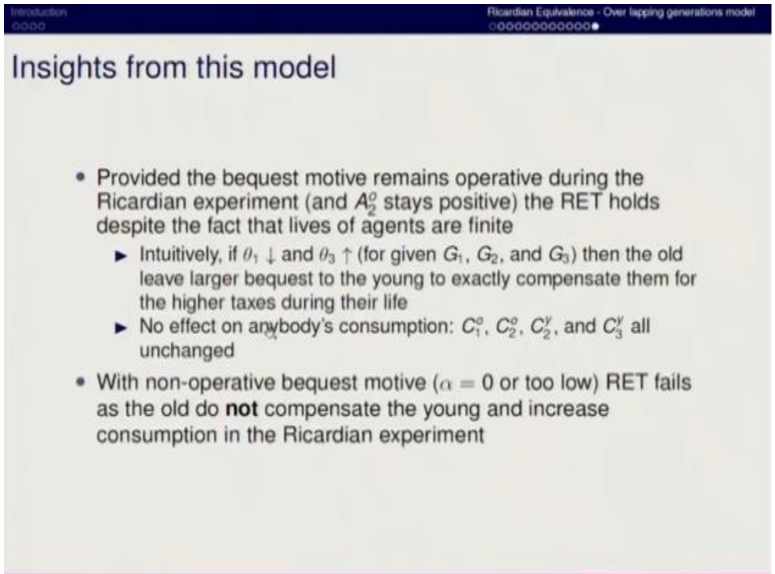

This is what we try and understand with non-operative bequest motive, alpha is equal to too low, RET fails. If A_2^o is positive, which means that if the transfer is positive, then RET holds, and this is what we try to understand. And then your θ_1 and θ_3 you can bringing the dimensions of the government, the tax change, so those things are always the part of, but one of the most important I would say contributions of this particular session is that.

You can understand the transfer of the wealth, inter-generation transfer does matter for the Ricardian equivalence and when we have inter-generation transfer then, the Ricardian equivalence holds. Otherwise, if the old generation is not caring about the young generation, if you do not have the inter generation transfer of wealth, then of course your Ricardian experiment will fail and we will not have the cooperative, I would say the benevolence, cooperative benevolence is important.

Sometime in the economy, so rich individuals do not care so much about the taxation, because they are already having lot of, I would say cautioning. Whereas, for the older generation it matters for the poor it matters a lot, because for them some kind of relief measures must be given. Those things we will be talking about now and will be looking at the credit market asymmetry in the next session.

And this topic will be discussing that, when you have different borrowing and lending rates, then how the consumer is going to think about either maximizing current consumption or future consumption. Current and future consumption frameworks will allow you to understand the great market asymmetry. I am stopping it here, I hope this session was useful to all of you, may be a new topic to be discussed.

But yes, you can think about now, how we in macroeconomics certain inter-generational transfers are taken care and with that we try to maximize the utility of both the overlapping and the old and young. Thank you so much.