Microfoundations of Macroeconomics

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Lecture - 11
Two Period Model - VI

Welcome back, in this session we will be talking more about the generalized consumption function. The idea is that the two-period model of consumption that we have understood helps you understand the behaviour of agent in the intertemporal framework. But in many cases, for example if you are going to work out with the Modigilani life cycle theory or the Friedman's permanent income hypothesis then you try to assume that the individual is living for many periods. For example, he works for 30 years 25 years or maybe the 35 years scenarios then if I have to go for the calculation of his lifetime consumption that how much he is having the consumption then that matters a lot and then it gives you the overall framework.

If you read these days the journal papers appearing in the macroeconomic journals, they talk a lot about the lifetime consumption of the individual or the representative agent. The micro analysis helps you understand the two-period in a more contextual way we have moved from one period to two-period. Now we will be seeing the infinite period scenario how when we extend the individual's consumption for an infinite period.

Suppose period t then how the budget constraint of the consumer looks like? How the lifetime utility function of the consumer looks like? Here we will be trying to understand and will try to derive with simple exposition.

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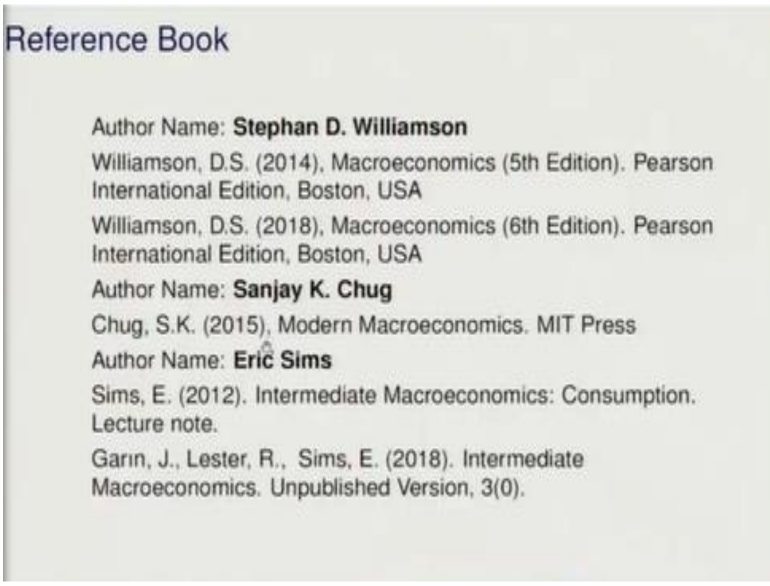

The reference remains the same. For this particular lecture, we are going to rely more on Eric Sims especially on his consumption chapter because that particular chapter is really good to understand the concepts. Let us start with the basic exposition of the model. I hope the uncertainty that we just discussed helped you understand the idea of precautionary savings that how precautionary savings plays a role in the economy.

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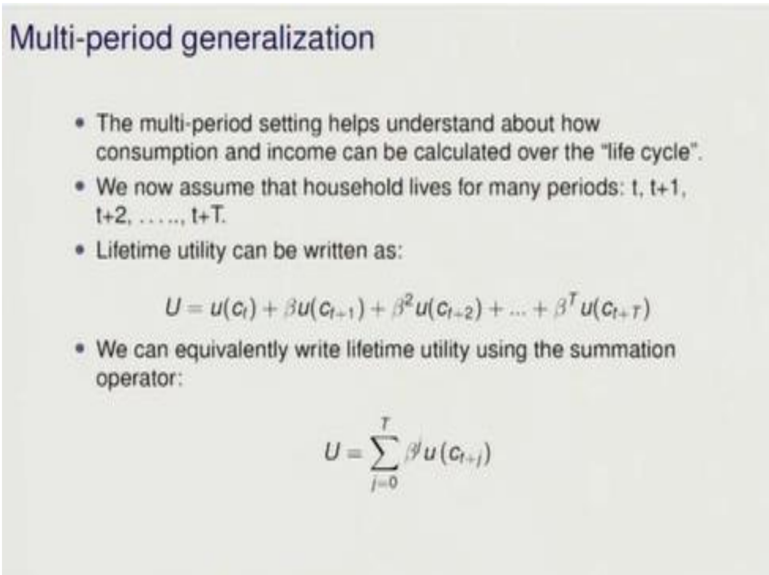

Let us start multiplied setting helps us understand how consumption income can be calculated over the life cycle. When I say in your macro textbook you will be reading about the life cycle theory of consumption. , how do you decide how do you calculate that the life cycle consumption theory? So we now assume that the household lives for many periods here you have $t, t + 1, t + 2$ and till $t + T$.

The lifetime utility can be written as

$$U = u(c_t) + \beta u(c_{t+1}) + \beta^2 u(c_{t+2}) + \cdots + \beta^T u(c_{t+T})$$

Now we can generalize this and try to write the expression in this way, we can equivalently write lifetime utility using the summation operator here

$$U = \sum_{j=0}^{T} \beta^j u(c_{t+j})$$

This is what the lifetime consumption of the representative agent is going to look like. Now from the utility side it is more like a clear case that the behavioural coefficient is going to have the polynomial order increasing. With the increase in the period the subsequent or with the addition of the subsequent period this is how it looks like.

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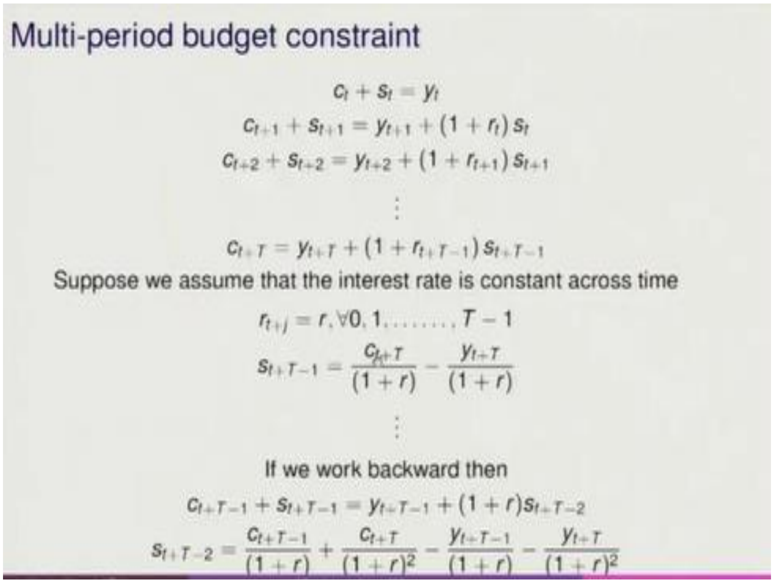

Now let us work out with the budget constraint. When we say about the budget constraint, let us work out with the first period. , what we had assumed when we were discussing about the two-period scenario.

$$c_t + s_t = y_t$$

This is the current period which means that if individual is going to have the income y t he is going to consume c_t amount and s_t amount is saving. In the future period what it becomes? here it becomes

$$c_{t+1} + s_{t+1} = y_{t+1} + (1 + r_t)s_t$$

This is again going to be bifurcated into 2 and whatever saving that he has made here is going to be interest rewarding. He will be getting me extra income on the saving attach which is equivalent to $(1 + r_t)$. If I go on adding such type of phenomena by or if I go on updating this particular equation with the period subsequent periods subsequent future periods.

If I go on updating this then this is how it looks like that in the future period, we had these two then subsequent periods.

$$c_t + s_t = y_t$$
$$c_{t+1} + s_{t+1} = y_{t+1} + (1 + r_t)s_t$$
$$c_{t+2} + s_{t+2} = y_{t+2} + (1 + r_{t+1})s_{t+1}$$
$$\vdots$$
$$c_{t+T} = y_{t+T} + (1 + r_{t+T-1})s_{t+T-1}$$

Here this particular individual will be saving again because in two period model we allowed the individual to save only in the current period not in the future period. Why? Because by the terminal period you have to exhaust everything you cannot save and dispose.

This is how but now when we are saying in a generalized framework then this individual can save in period 2 and period 3 and period 4 and till one period before the terminal period T, T – 1.

So, if he is going to die in 70 he can save till 69, in 70th year he is not going to save anything. Let us understand that. Here we have s_t , s_{t+1} , s_{t+T-1} and similarly in the final period when he is going to be in the terminal period what he will have? He will have

$$c_{t+T} = y_{t+T} + (1 + r_{t+T-1})s_{t+T-1}.$$

This will be the interest income that he is going to get before the terminal period just immediate before the terminal period and then he will have the saving. Now suppose we assume that interest rate is constant across time we are not going to change till the period t the interest rate is same not much change, this is how it looks like. Saving can be written in this way, this is how it looks like.

$$s_{t+T-1} = \frac{c_{t+T}}{(1+r)} - \frac{y_{t+T}}{(1+r)}$$

In the same way if I go for solving this here, If we work backward then

$$c_{t+T-1} + s_{t+T-1} = y_{t+T-1} + (1+r)s_{t+T-2}$$

$$s_{t+T-2} = \frac{c_{t+T-1}}{(1+r)} + \frac{c_{t+T}}{(1+r)^2} - \frac{y_{t+T-1}}{(1+r)} - \frac{y_{t+T}}{(1+r)^2}$$

What is the meaning? Meaning is that just one period before of the terminal period this will be the present value in terminal period this will be the square term because we already have 1 + r here. This is how it looks like, the two period back if you look for the savings that we have s_{t+T-2}.

One period before this terminal period, this is how it operates which means that your consumption and income both will have the similar characteristic. The only thing is that the terminal period c_{t+T} will have the polynomial if you are going to work out with this saving scenario s_{t+T-2}.

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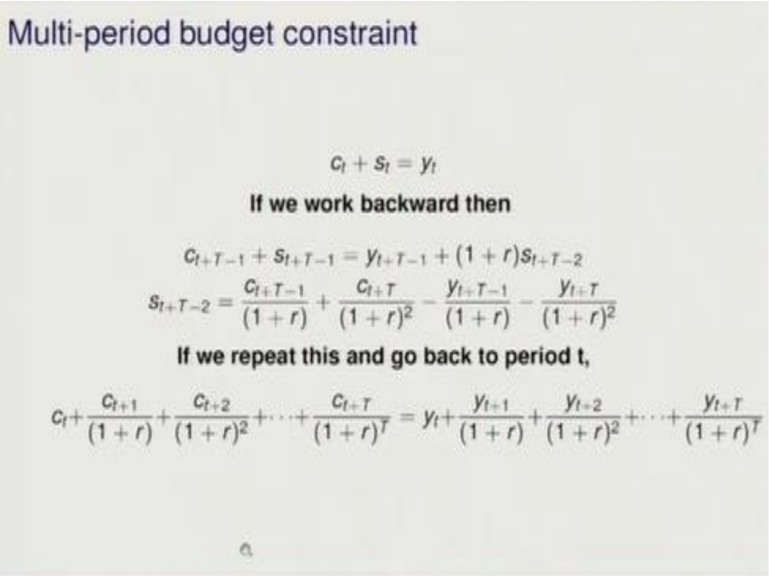

Now if you go back further then this is how it looks like,

$$c_t + \frac{c_{t+1}}{(1+r)} + \frac{c_{t+2}}{(1+r)^2} + \cdots + \frac{c_{t+T}}{(1+r)^T} = y_t + \frac{y_{t+1}}{(1+r)} + \frac{y_{t+2}}{(1+r)^2} + \cdots + \frac{y_{t+T}}{(1+r)^T}$$

If we just try to work it out further with this particular scenario backward and if we can repeat this and go to period t in this period go back again. If we go back to period t this is how it looks like that c_t it forms a pattern.

And this pattern helps you calculate the lifetime budget constraint of the representative consumer. The idea is that if you get back here the idea is that first we are saying that to what extent the individual can save what will be his saving and given the income the interest income attached what will be saving.

If I am looking at the savings that how much he saves given the terminal period scenarios , this is how is the savings of individual look like in period one. And if you go back further then this is how look it looks like if you go back till period t then we will have a such type of pattern appearing. Once we have such a pattern appearing then you can see that this is the lifetime consumption of the representative agent and this is the lifetime income of the representative agent.

We have not introduced any tax here; it is just the income and the consumption. , the subscript t + T it is representing the time period it is not representing any kind of tax.

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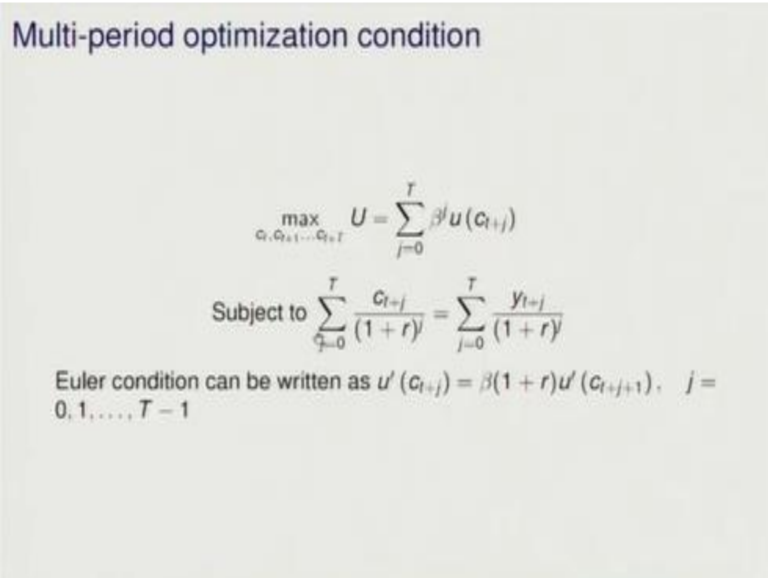

Subscript is the subscript originally, here if you try and see that how does it look like

$$\max_{c_t, c_{t+1} \ldots c_{t+T}} U = \sum_{j=0}^{T} \beta^j u(c_{t+j})$$

Subject to

$$\sum_{j=0}^{T} \frac{c_{t+j}}{(1+r)^j} = \sum_{j=0}^{T} \frac{y_{t+j}}{(1+r)^j}$$

The other condition can be written as the marginal utility of the future consumption. Here if I am mentioning about marginal utility of the consumption period t + j it is equivalent to the $\beta(1+r)$ into the marginal utility of future consumption it goes up to t + j + 1. You can say that the current period consumption of individual is nothing but beta multiplied by the savings that he is making in the current period, this is what is the same.

$$u'(c_{t+j}) = \beta(1+r)u'(c_{t+j+1})$$

At this level this is the Euler condition this is the generalized Euler condition of the representative agent. As compared to what we saw here in the two-period scenario, in the two-period scenario we had a Euler condition of we had the Euler condition going to this particular scenario.

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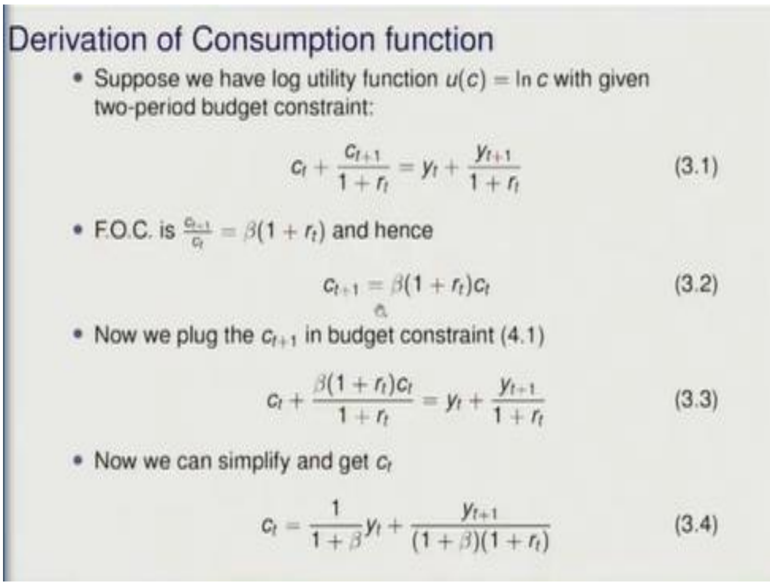

Here we have the Euler condition scenario you can see this. Now if I am going to see with uncertainty the Euler condition changed and Euler condition became expectation of s_{t+T-1}?. , this is how we had added.

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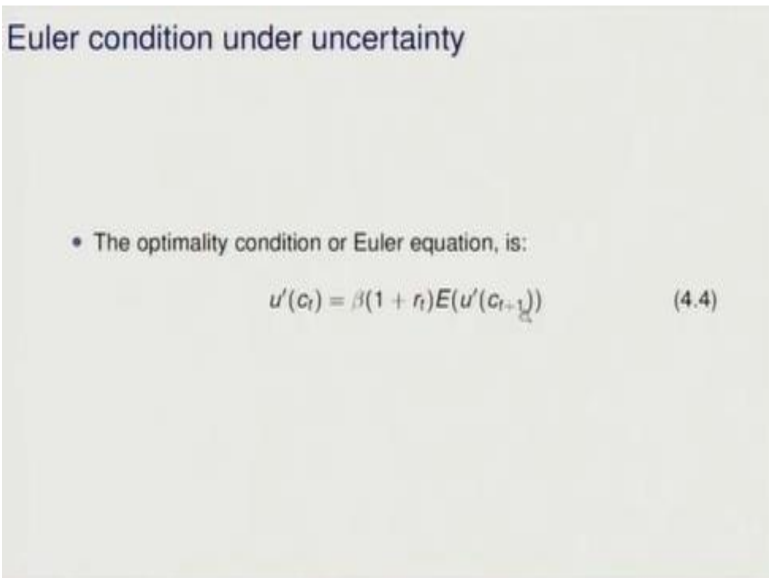

$$u'(c_t) = \beta(1+r)u'(c_{t+1})$$

This was the Euler condition of the representative consumer when he is facing uncertainty in the future period.

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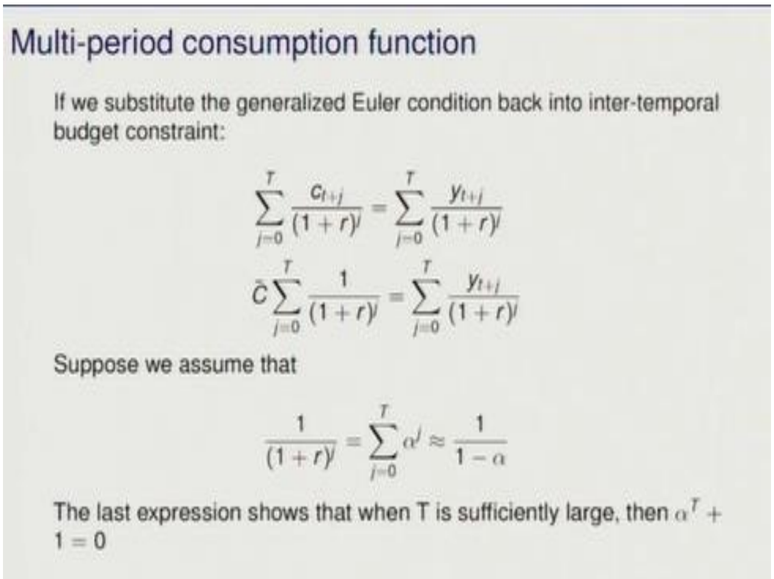

This is how we try to look at. But as we have mentioned in the previous analysis that, this is the dynamic optimization condition the Euler condition. It just says that whatever consumer will consume in the current period it is equivalent he if he saves that and consumes in the future.

In order to derive the consumption function in the lifetime you can go and substitute the budget constraint into this. You can formulate the log function of the consumption whatever expression you get you substitute back into the budget constraint what you get is this that if you substitute the generalized Euler condition back into inter temporal budget constraint which means that you can solve this and put it here.

$$\sum_{j=0}^{T} \frac{c_{t+j}}{(1+r)^j} = \sum_{j=0}^{T} \frac{y_{t+j}}{(1+r)^j}$$

$$\bar{C}\sum_{j=0}^{T} \frac{1}{(1+r)^j} = \sum_{j=0}^{T} \frac{y_{t+j}}{(1+r)^j}$$

Euler condition can be written as

$$u'(c_{t+j}) = \beta(1+r)u'(c_{t+j+1}), \quad j = 0,1, \dots, T-1$$

If you substitute back here you get the consumption function in the same way

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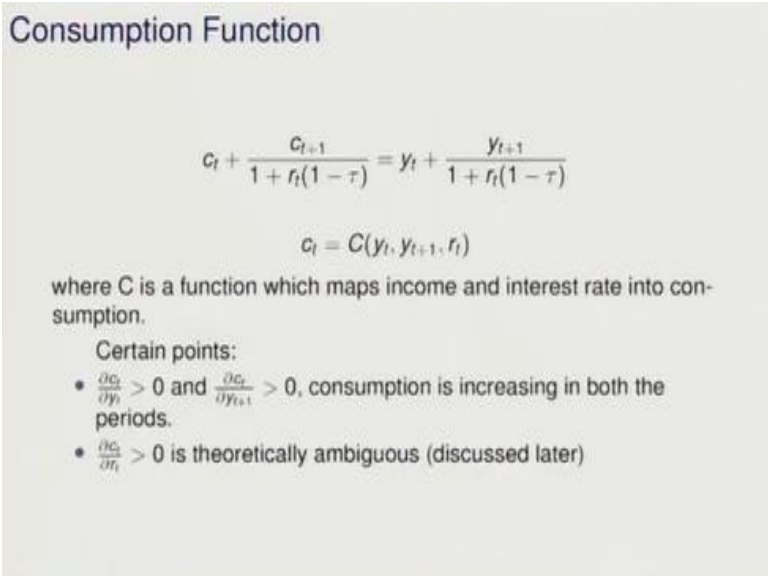

Maybe if I am talking about this that we can derive is equal to $c_t$is the function of y_t, y_{t+1}. I am talking about this derivation you can go for deriving the similar kind of derivation here. Here this is how it looks like. Here it makes sense here if you just try to substitute this is the consumption this is the income of the this is budget constraint if you try to work out.

$$\frac{1}{(1+r)^j} = \sum_{j=0}^{T} \alpha^j \approx \frac{1}{1-\alpha} = \frac{1}{1-\frac{1}{(1+r)}} = \frac{1+r}{r}$$

$$\sum_{j=0}^{T} \frac{1}{(1+r)^j} = \frac{1+r}{r}$$

The intertemporal budget constraint will be

$$\bar{C}\frac{1+r}{r} = \sum_{j=0}^{T} \frac{y_{t+j}}{(1+r)^j}$$

After further simplification, it can be written as

$$\bar{C} = \frac{r}{1+r}\sum_{j=0}^{T} \frac{y_{t+j}}{(1+r)^j}$$

The $\frac{r}{1+r}$ plays very important role as compared to 1 upon 1 + β that we got in two - period case here the rate of interest will matter. $\frac{r}{1+r}$ will matter and if you go for partial differentiation with respect to future income the expression will change. The underlying idea is that with this life cycle imposition or life cycle derivation of the utility.

And the budget constraint of the representative consumer we can easily see that how much this representative consumer is going to have the consumption, the change in consumption with respect to current income or I would say lifetime income. Here there will not be any kind of future as such but yes if you want you can add t + j + 1 and you can calculate but your lifetime consumption depends upon the rate of interest, how much you have the rate of interest.

And the earnings will decide about that how much you are going to save and if you are thinking about the giving preference to the future β is continuously going to be higher then you are going to give more importance to the future period that works. Generalized equation of the optimization this is how it looks like. , I hope it is clear to all of you that the consumption function that you normally see in macro textbook.

$$\bar{C} = \frac{r}{1+r}\sum_{j=0}^{T}\frac{y_{t+j}}{(1+r)^j}$$

Whether it is Keynesians whether it is the life cycle of Modigliani whether it is the permanent income hypothesis by Milton Friedman, all these theories can be explained with the help of simple mathematical exposition of the macroeconomic concepts and this macroeconomic background will help you understand these theories in a much better way. And you can also extend this further with many more additions.

And you can see that how those additions will help you understand not only the consumption but also the savings, for example we introduced the uncertainty and we saw that how this uncertainty when we introduced into the model it helped us understand the precautionary savings. To summarize now we will be moving to the government and we will be seeing that how in the two-period model setup the government is going to react when the individuals pay the tax to the government how government reacts to that, how government tries to finance the expenditure. Those concepts will be important and, in this consumption, if you go by the one by one we started with two period the we derive the consumption function.

And then we looked at the comparative statics introducing the income, interest rate scenarios then we introduce the uncertainty and uncertainty added further dimension of precautionary savings. And then we generalized the consumption and budget constraint of the representative consumers in a more robust way. And then we can see that with this generalization you can superimpose the condition of life cycle theory.

And see how or the permanent income hypothesis that how individuals will behave when they are introduced to this. Whether it will be Behavioural coefficient playing very important role or it will be the reward of the representative consumer. It also allowed us to impose the condition that if we are allowing the individuals to save in the subsequent periods then how the budget constraint of the representative consumer is going to be changed.

I hope such background will help you understand the recent developments in macroeconomics in a much better way. These are the new classical ideas, new classical ideas have such type of understanding that they added a dimension of the macro foundations. And we will be having different schools of economic thought and under that will be coming back to this again that how and from where this started and how it is exploring how it has been explored in the recent literature.

I would request all of you to have a look at the papers appearing in macroeconomics literature mostly from journal of macroeconomics and Journal of economic dynamics and control. These journals have a sufficient background about such formulations. And then you can easily understand at least you will have idea about that how this particular mathematical formulation helps and in which all areas such foundations micro foundations are applied to understand.

People try to understand the behaviour of the consumer, behaviour of the representative agents, behaviour of the individuals in a group when we superimpose the condition of rich and poor. All those dimensions are covered. I will be stopping it here and then I will be starting a new topic which is about the government. Let us have a background about the government.

And then we will be seeing that how governments react to such type of formulations. Now we are going to talk about the Ricardian Equivalence. Now in the two-period model we can also introduce the government that is the flexibility that it provides and since the mathematical treatment becomes more simpler in two period scenarios compared to infinite period or up to period T.

We will be, understanding that how individuals react to government decisions whether those government decisions impact the consumption behaviour of the representative agent. If those actions of the government are going to impact or if the government is going to give tax relief or tax burden on the individual then how these individuals are going to play important role how these individuals are going to work out with their consumption.

Whether the consumption smoothing pattern will remain same or it will change. Now I think those dimensions are important to look at and since you already have the background now. It will be easier to understand and the idea behind Ricardian equivalence if you see in a nutshell it talks about how individuals react to the change in the government decisions about taxation.

The concept starts with the basic premise that if government is going to give you tax relief in the current period it does not mean that you will not be paying in future. The government also goes through the same framework that we have seen for the consumer that it also optimizes expenditure maybe in the current period the government is going to reduce the taxes. But in future period the government will increase the tax.

Now individuals are also rational they have the similar kind of experience, learning’s and then they also have certain expectation. If the government is going to give a tax relief in the current period, then individuals are al smart, they know that we are going to get the we are getting the tax relief in the current period but we are going to pay this in future. They will save this amount to smoothen out the burden of tax in future period.

And then they easily smooth out or smoothen their consumption in two period scenarios. Ricardo in the late around 17 or would say 18th century he talked about such type of behaviour of the consumer when the government is going to take decision about the taxation. Here it is this concept is linked with the public finance theories public economics but it is more of a macro because it deals with consumption and all other variables.

But you can link it with the public finance the Ricardian equivalence tells that the tax cut is not a free lunch you will have to pay back in future. We will be understanding those dimensions here and we will be also trying to see that how this work.

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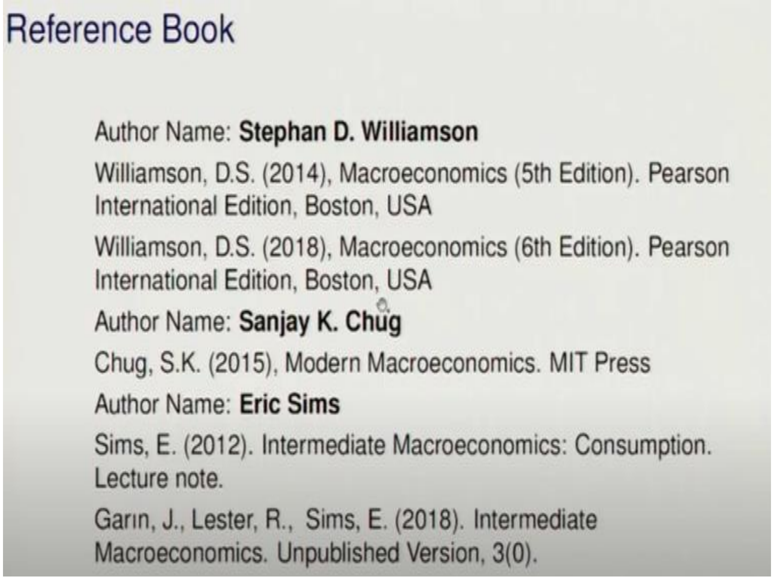

The reference remains same for this. I would request all of you to go through this Stephan D. Williamn book and this book is gives you the idea you can also go to the Sanjay K. Chug and this will gives a sufficient background.

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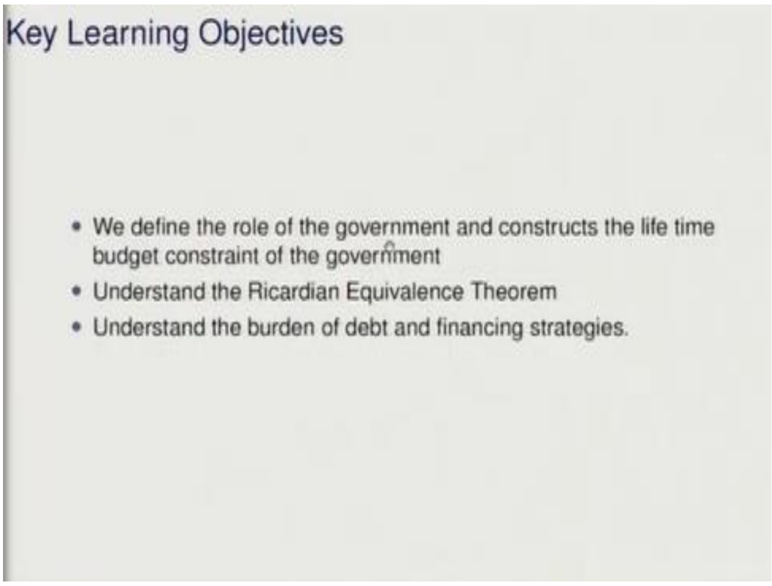

What will be the learning objective that how the government behaves in the economy what is the role, how government construct the lifetime budget constraint, how we can understand the Ricardian equivalence theorem how we can understand the burden of the debt and the financing strategies. These are this, but I will continue this in the next session and then we will start from there. Thank you much.