

Micro Foundations of Macroeconomics
Prof. Wasim Ahmad
Department of Economic Sciences
Indian Institute of Technology – Kanpur

Lecture – 10
Two Period Model V

Hi everyone. Welcome back. We are now going to discuss about the further extensions of consumption theory which means that we were talking about the 2 period a model of consumption and under that we derive certain aspects and we try to understand the behaviour of the representative agent under a 2 period framework. It made us understand that how with the help of micro foundations we can have any kind of generalization on the macro theory of consumption. We will be looking at certain dimensions that we cover. For example, in today's session, I will be mostly covering the uncertainty and the lifecycle theory of the Friedman. Friedman gave the permanent income hypothesis, which means that he mentioned how the individuals when reacting about their income.

Whether they are bothered about the constant increase in income or whether it is transitory, the sudden rise in the income. Those aspects we will be dealing with today. We will be trying to add the dimension of uncertainty in the macroeconomic concept, especially in the two-period context that how when we see any kind of uncertainty, which is going to arise in future, then how the consumer is going to react.

There is a term in macro, we often use in our analysis called precautionary saving. What is the meaning of precautionary savings? When I say that savings, which means that the sum the individual is facing any kind of uncertainty in future, in order to make sure that the future consumption remains his smooth, there should not be any problem. in order to maintain the current level of consumption in future given the uncertainty, this representative consumer goes for saving in the current period. As you have further increase in uncertainty, this leads to more precautionary savings, precautionary savings are not good for the economy as a macroeconomic variable because individuals compromise on their level of consumption. Once the individuals are going to come to a compromise on the level of consumption then of course there will be a compromise on the productivity, there will be a compromise on the output. Precautionary savings are good, because they will save for the rainy days, but it is also not

recommended to have the increase because of the uncertainty. But in a very broad framework, we try to understand the behaviour of the representative agent given the uncertainty.

So far what we have covered is comparative statics that we had about what happens when the current income is going to rise, what happens if the representative consumer is going to see a rise in income in the future period, and what happens when the representative consumer is going to see any kind of interest rates increase in future or in the current period.

The borrowing and lending scenarios will matter if the interest rate is going to be higher. Then there will be the role of the future income effect. And then we examine that aspect in today's session, it will be interesting, and we will be seeing the dimensions of the consumption and with this, we will conclude. And then we will move to the government and we will see that under 2-period model how government behaves in a more intertemporal framework that whether they have to go for transferring of tax to the current generation or it will be future how they manage , in economics we always say that the tax cut is not a free lunch because if you are seeing the lower taxes in the current period, then you have to be ready that you have to pay more tax in future.

In that scenario, how the representative consumer is going to smooth out the consumption in the intertemporal framework, we will be trying to understand that.

(Refer Slide Time: 04:25)

Reference Book

Author Name: **Stephan D. Williamson**
Williamson, D.S. (2014), Macroeconomics (5th Edition). Pearson International Edition, Boston, USA
Williamson, D.S. (2018), Macroeconomics (6th Edition). Pearson International Edition, Boston, USA

Author Name: **Sanjay K. Chug**
Chug, S.K. (2015), Modern Macroeconomics. MIT Press

Author Name: **Eric Sims**
Sims, E. (2012). Intermediate Macroeconomics: Consumption. Lecture note.
Garrin, J., Lester, R., Sims, E. (2018). Intermediate Macroeconomics. Unpublished Version, 3(0).

Now here references remain the same. Here we have the Steven D. Williamson. And then Eric Sims , Sanjay K. Chug is a particular reference in this. We will be out trying to understand

how the representative in agent is going to react to the uncertainty. Let us start with that setup. And we will try and understand in a more comprehensive manner.

In the last class, we were talking about uncertainty. Let us get back to the problem statement.

(Refer Slide Time: 04:59)

Uncertainty

- Suppose the representative consumer faces uncertainty about his/her future income.
- Suppose the future income takes two values: $Y_{t+1}^h > Y_{t+1}^l$
- Let the probability of the "high income state" occurring be 'p', with the probability of the low income '1-p'.

$$E(Y_{t+1}) = pY_{t+1}^h + (1 - p)Y_{t+1}^l \quad (4.1)$$

$$C_{t+1}^h = Y_{t+1}^h + (1 + r_t)(Y_t - C_t) \quad (4.2)$$

$$C_{t+1}^l = Y_{t+1}^l + (1 + r_t)(Y_t - C_t) \quad (4.3)$$

Here what we are saying is that the representative consumer is facing uncertainty about the future income, not the current. , why we are facing future uncertainty? Why are we introducing future uncertainty here? Because we want to see how this representative consumer is going to adjust to the current level of consumption, and how the current income is going to be impacted when this representative consumer is facing uncertainty in future.

Suppose, the future income takes 2 values. Here we have $Y_{t+1}^h > Y_{t+1}^l$ this shows that the representative consumer because of this uncertainty he has this level of the scenario in future. His future income is going to face this either it could be higher or it could be lower and as far as Y is concerned we refer to it as income.

Now, once I am having the uncertainty then there will be a likelihood attached. We attach the probability concept here. Here we mentioned that the let the probability of high-income state occurring be p and probability of low income 1 – p. Here we are trying to put a particular state variable state scenario in the future income because at current income we are not bothered much because current income does not face any uncertainty. It is just the future income which is facing uncertainty. In future income, we will be introducing two scenarios now that what

will happen. , which means that if I am introducing this that high-income state occurring be p and probability of low income $1 - p$ then here it will be,

$$E(Y_{t+1}) = pY_{t+1}^h + (1 - p)Y_{t+1}^l$$

I attach probability p with Y_{t+1}^h and $1 - p$ is the probability of having a low income in future Now, once I define this, then to the corresponding income whatever I am assuming the given the level of probability, I will have to also specify the consumption attached with this and the consumption attached with this.

Consumption attached with this is that if he is going to have a high income in future with a given level of uncertainty, then

$$C_{t+1}^h = Y_{t+1}^h + (1 + r_t)(Y_t - C_t)$$

And in the same way, we can also write it as here

$$C_{t+1}^l = Y_{t+1}^l + (1 + r_t)(Y_t - C_t)$$

$(1 + r_t)(Y_t - C_t)$ will be the extra income of the representative consumer that he will have given the high and low-income scenarios that he is going to face in the future period.

And the corresponding consumption in higher and lower states are these. Here, we have defined the consumption in the future period when the income is high.

Here now, the expected value of consumption that , here we have the expected value of Y_{t+1} that how much this representative consumer is going to have the future income, which is going to be the stochastic term attached with certain probabilities.

(Refer Slide Time: 09:14)

Uncertainty

- Expected value of consumption in the second period is:

$$E(C_{t+1}) = pC_{t+1}^h + (1 - p)C_{t+1}^l$$
- The key insight to understanding how uncertainty impacts consumption is that expected marginal utility is not, in general, the same thing as marginal utility evaluated at the expected value of future consumption.

Here we are mentioning that the expected value of consumption in the second period is going to be

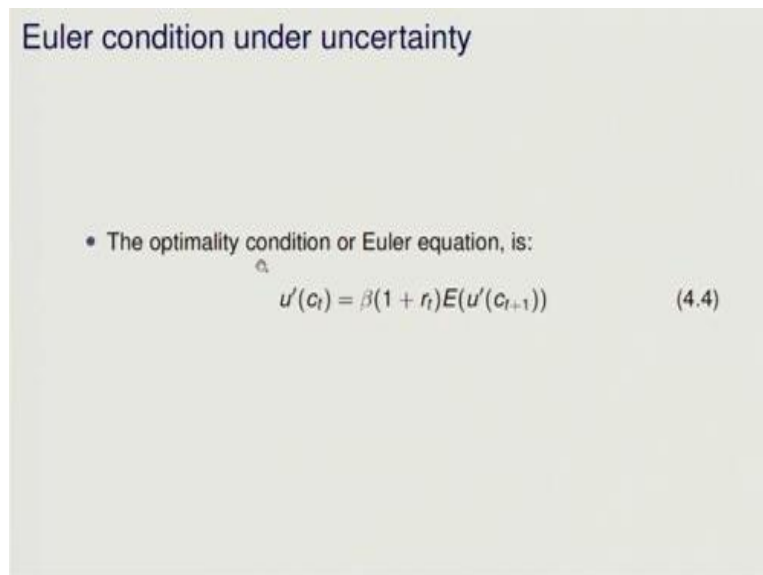
$$E(C_{t+1}) = pC_{t+1}^h + (1 - p)C_{t+1}^l$$

Now, I will be going for the expected level of consumption, I will have to attach probability with C_{t+1}^h and C_{t+1}^l .

The most important part of this is that we are going to calculate the expected future consumption given the level of probability that this representative consumer is going to face. Now, here you have to note that before in the Euler condition, we mentioned the marginal utility of future consumption. Here we will try to understand that how uncertainty impacts consumption and this consumption which is going to impact how it is going to be. Whether the expected marginal utility that I am going to get, will this be equal to the given level of probability that I am attaching. , which means that I will have to now deal with this h and l. The consumption calculated and h and l. Marginal utility at these two points whether it will be equal to the overall expected marginal utility.

Those things we are addressing, if I am saying that the high income state is having this consumption, low income state is having this consumption, then there will be any kind of expected marginal utility of future consumption.

(Refer Slide Time: 11:19)



Euler condition under uncertainty

- The optimality condition or Euler equation, is:

$$u'(c_t) = \beta(1 + r_t)E(u'(c_{t+1})) \quad (4.4)$$

Whether this future consumption is equivalent to these two, how we can average out. Average out scenarios are important to understand here. Now, if you look from the Euler condition, your

Euler condition comes out to be that the marginal propensity to consume in the current period or multiple studies of the marginal utility of consumption in the current period is nothing.

But $\beta(1 + r_t)$, the expectation is attached with this because the expectation is dealing with the future consumption. Once I have the expectation about future consumption, then this is the expectation of marginal utility of future consumption. , in order to arrive at the Euler condition, we will have to calculate this expectation, the expected value of future marginal utility of consumption.

Marginal utility of current consumption is equivalent to $\beta(1 + r_t)$, the expected marginal utility of future consumption which is not the marginal utility of consumption, which is not the same as marginal utility of consumption calculated at these points, these two stated points, which is p and 1 – p attached with these two point probability.

Which means that the consumer is indifferent about the marginal utility of current consumption given that he is able to save this much amount $(1 + r_t)$ is what he is going to get. The interpretation remains same, the only difference that we see is that here you have the expectation operator attached with the marginal utility of future consumption, because we are introducing uncertainty here.

This is the marginal change that we see with the other equation, given the uncertainty that we have set up this. The expected marginal utility of future consumption is going to be the added component here. Expectation is playing an important role.

(Refer Slide Time: 13:17)

Euler condition under uncertainty

Suppose we have following utility function with expected future utility:

$$U = u(c_t) + \beta E(u(c_{t+1})) \text{ with } \ln(C) = \ln(c_t) + \beta(\ln(E(c_{t+1})))$$

Inter-temporal budget constraint can be derived in the same manner.

$$c_t + \frac{E(c_{t+1})}{1+r_t} = y_t + \frac{E(y_{t+1})}{1+r_t} \quad (4.5)$$

To get the Euler condition, we can opt for either constrained or unconstrained optimization:

$$\max_{c_t} U = u(c_t) + (u((1+r_t)(y_t - c_t) + E(y_{t+1})))$$

$$\frac{dU}{dc_t} = u'(c_t) - \beta E(u'((1+r_t)(y_t - c_t) + y_{t+1}))(1+r_t)$$

F.O.C would be:

$$u'(c_t) = \beta(1+r_t)E(u'(c_{t+1})) \quad (4.6)$$

Now, let us work out , suppose we have the following utility function with expected future utility.

$$U = u(c_t) + \beta E(u(c_{t+1}))$$

$$\text{with } \ln(C) = \log(c_t) + \beta(\log(E(c_{t+1})))$$

We know that since the uncertainty is attached with the future period, it is going to be this.

Your lifetime budget constraint of the inter-temporal lifetime budget constraint of the representative consumer looks like this.

$$c_t + \frac{E(c_{t+1})}{1+r_t} = y_t + \frac{E(y_{t+1})}{1+r_t}$$

We can opt for either constraint or unconstrained optimization. Here we have,

$$\max_{c_t} U = u(c_t) + \beta E(u((1+r_t)(y_t - c_t) + E(y_{t+1})))$$

You just have to solve for the $E(c_{t+1})$. Here if you just solve then this is how you try and get it. You can introduce it here go for differentiation, U with respect to current consumption. , this is what we get the first order condition,

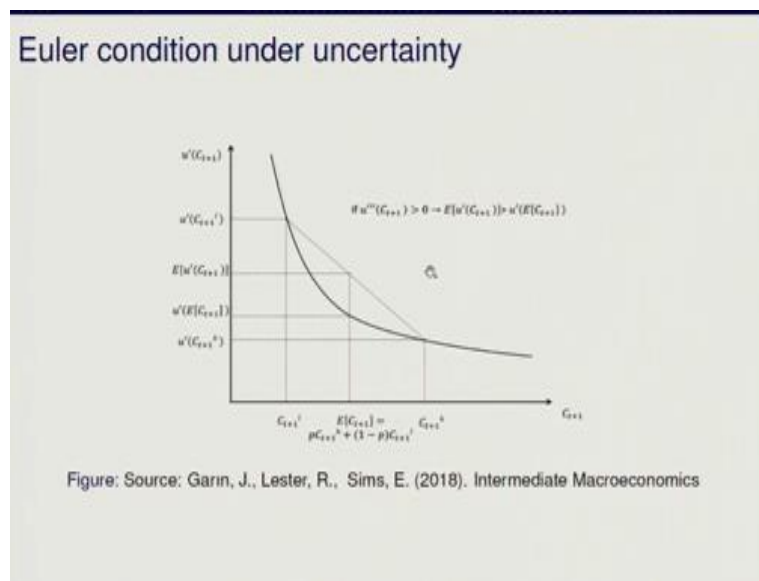
$$\frac{dU}{dc_t} = u'(c_t) - \beta E(u'((1+r_t)(y_t - c_t) + y_{t+1}))(1+r_t) = 0$$

I just write c_{t+1} it instead of the whole expression.

$$u'(C_t) = \beta(1+r_t)E(u'(C_{t+1}))$$

I am writing it here, which is nothing but the marginal utility of current consumption equal to $\beta(1+r_t)$ with the expectation of the marginal utility of future consumption. This is the Euler condition that we get when we are introducing the uncertainty in the future period. Let us now deal with it, how it looks like.

(Refer Slide Time: 15:13)



Here, what we the important point to note is that, this particular curve that we have the indifference curve, it is bowed in which means that the further second order conditions will play an important role. As long as it is greater than zero, the convexity criteria fulfils.

This is what we have the risk lover and risk averse scenario that we always try to look at. Now, here, we are trying to see the expected marginal utility of future consumption. This is the midpoint. Let us start, since we are not having any uncertainty in the current period, we are not considering that here we are only dealing with the future period.

What is the future period? In case of future period, this is how it looks like. Here you have the c_{t+1} which is the future period consumption, here you have the marginal utility of future period consumption, because this is where we are struggling because of these two scenarios that we are introducing. As long as you have the third order conditions, the third derivative of $c_{t+1} > 0$.

Then we can have the expected marginal utility of future consumption greater than the marginal utility of expected future consumption. This is the difference that we have. If you think about that, we are introducing two scenarios. Here we have the highest state and here we have the lowest state. The moment I move from here to here.

Now, the expected marginal utility of future consumption this is what we were trying to get here the expected marginal utility of future consumption for the Euler condition. Here we are saying that if we can get the midpoint of the line connecting this indifference curve.

Which is this, decides about the marginal utility of expected future consumption. Here we have the model. As long as this particular line is bowed in, it is inside having curvature shape, then the gap will be much larger and then you will have this condition satisfied. As long as you have wide gap from this to this point.

If it is more deeper inside it is having a bowed in kind of scenario, but the condition is that the minimal condition has to be satisfied. Curvature of the indifference curve plays a very important role here. This is what we try to achieve, our objective for the Euler condition is to find this, we have already defined the low and high. If again take into account the midpoint of this line, then we can easily get this and with this.

We can also have the expected future consumption. This is the ultimate objective of ours that how much this representative consumer given the uncertainty that he or she is facing is having the expected future consumption and corresponding expected marginal utility of future consumption. We will be introducing these two and we will be trying to understand that.

But this condition is more to deal with the borrower scenario, which means that if for convexity if you have the further the curvature of this will decide about how much is the gap that he is going to see from the expected marginal utility of future consumption and the marginal utility of expected future consumption these two are different things and , this is what we try to mention.

Here it is about the curvature. And here it is about the expectation which is the averaging of or I would say midpoint of these two.

(Refer Slide Time: 19:29)

Euler condition under uncertainty

- Curvature of marginal utility does matter
- The expected marginal utility must be greater than marginal utility evaluated at expected future consumption.
- Mathematically, $\hat{E}(u'(c_{t+1})) > u'(E(c_{t+1}))$

Curvature of marginal utility does matter. Here this is what we say that expected marginal utility must be greater than the marginal utility evaluated at the expected future consumption. This is what we mean to say and this condition should be satisfied.

(Refer Slide Time: 19:47)

Euler condition under uncertainty

- The slope of the line connecting two extremes $u'(c_{t+1}^h)$ and $u'(c_{t+1}^l)$ is:

$$\text{slope} = \frac{u'(c_{t+1}^h) - u'(c_{t+1}^l)}{c_{t+1}^h - c_{t+1}^l} \quad (4.7)$$

- To find the value of straight line at the expected value of consumption $E(c_{t+1})$ call it χ

$$\text{slope} = \frac{\chi - u'(c_{t+1}^l)}{E(c_{t+1}) - c_{t+1}^l} \quad (4.8)$$

Now, the slope of the line if I try to calculate these two, that slope of the line connecting these two extreme points, which is nothing but the marginal utility of the consumption with high income and marginal utility of consumption of low income both in the future periods. Here you have this particular slope will be calculated at this right this minus this. This is how it looks like.

Now, to find the value of the straight line, because we do not know at this point what is the value? We are just introducing this part. $E(c_{t+1})$ if I am defining this will be equivalent to the

$$\text{slope} = \frac{x - u'(c_{t+1}^l)}{E(c_{t+1}) - c_{t+1}^l}$$

(Refer Slide Time: 20:48)

Euler condition under uncertainty

- We set both slopes equal and obtain the value for χ

$$\frac{u'(c_{t+1}^h) - u'(c_{t+1}^l)}{c_{t+1}^h - c_{t+1}^l} = \frac{\chi - u'(c_{t+1}^l)}{E(c_{t+1}) - c_{t+1}^l}$$

$$\chi = pu'(c_{t+1}^h) + (1-p)u'(c_{t+1}^l) = Eu'(c_{t+1}) \quad (4.9)$$

- This shows that the two (high and low) values evaluated at the mean consumption is equal to expected marginal utility.

If I solve for x here, then here we have the probability attached with the marginal utility of the future consumption given high income plus $1 - p$. The marginal utility of future consumption given low income is equivalent to what we get is this expected marginal utility of future consumption and this is what we wanted to get.

$$x = pu'(c_{t+1}^h) + (1-p)u'(c_{t+1}^l) = E u'(c_{t+1})$$

Two high and low states scenarios evaluated and mean consumption is equal to expected marginal utility.

(Refer Slide Time: 21:37)

Euler condition under uncertainty

- Euler condition:

$$u'(c_t) = \beta(1+r_t)E(u'(c_{t+1}))$$

$$\frac{u'(c_t)}{\beta(1+r_t)} = E(u'(c_{t+1}))$$

And with this, we will be satisfying the criteria. Now, what is this? The criteria is that if we want to know about what is the expected marginal utility or future consumption of this representative consumer, then this is how it looks like.

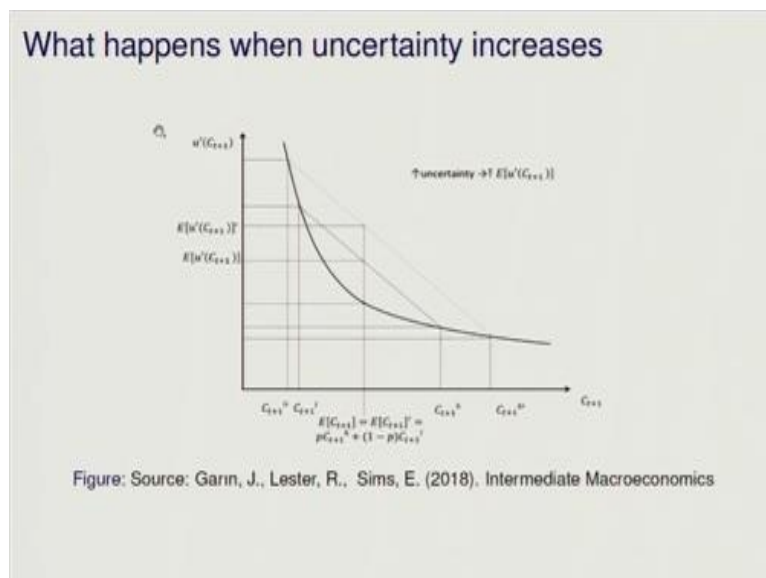
$$u'(C_t) = \beta(1 + r_t)E(u'(C_{t+1}))$$

$$\frac{u'(C_t)}{\beta(1 + r_t)} = E(u'(C_{t+1}))$$

You have the marginal utility of consumption going to be lower if r_t is lower than marginal utility of consumption is going to be higher, which means that the high interest scenario is leading to individual saving more and here it is just that this is the equality condition. , this equality condition says that the marginal utility of consuming or marginal utility of current consumption is equivalent to what the individual is going to save in the current period.

This is the same. Whether the consumer is going to consume more in the current period or his saving more in the current period both should be equal. This is what we try to achieve with this.

(Refer Slide Time: 22:41)



Now, here we are putting a comparative statics scenario where what we are doing is that what happens when uncertainty increases. Now, if are saying that once we have the rest of this scenario, same, here, we have the parallel line moving up when the uncertainty is increasing with the expected marginal utility of future income future consumption is increasing. This is what, if it was here and now, it is moving towards here.

Which means that, this leads to increase in marginal utility of future consumption, which means that the individual will be compromising more on the current consumption because current consumption does not face any kind of uncertainty. Once I say that the future expected marginal utility of future consumption is going to increase it means that this particular individual is going to save more in the current period.

If you have high uncertainty in future, that is why you have different indicators in the macroeconomic to measure the sentiments though how far or how efficiently, they measure the sentiment that is a different case. But you have different sentiment indicators and even central bank and different agencies try to collect the sentiment just to know that how individuals are forming the expectation about future.

If the industrials are forming the expectation about future not very good, very uncertain, which means that this is going to immediately impact the consumption pattern and once the consumption goes down, the aggregate demand is going to be impacted. And once aggregate demand is going to be impacted, then it will have a big impact on the macroeconomic outcomes. And , the economic outlook does matter.

And in that scenario, the precautionary saving idea comes into mind. If you are high uncertainty in future then individuals are going to save more. When we have COVID COVID-19 pandemic, then we saw that individuals those especially in the contractual jobs, they lost their job and as a result, we saw that the people started putting any kind of restriction on their consumption especially the lower income strata they have gone for.

I would say tremendous compromise on the current level of consumption. Considering the high uncertainty in future whereas, the reach and affluent class or those who are having the regular income those who are in regular job for them, it does not matter because they are sure that their permanent income is constant or it will increase. At least they are going to have any cash flow on a regular basis.

Their consumption uncertainty we are ruling out, you can link this kind of concept with those people who do not have the regular job and who get income whose income is representative upon the macroeconomic outlook of the economy the business cycle, the business cycle is more in favour, if it is moving upward, which means that it is on the recovery.

Then if they are not getting good job opportunity, which means that their future income is uncertain. The moment I say future income uncertain, which is directly linked with the employment, then this representative consumer is going to save more in the current period itself. That he can smooth out the future consumption easily. That is why government and macro all the agency they always try to work out.

They tried to have the balance macroeconomic outlook, the moment the individual is started expecting uncertainty in future there will be immediate impact on the current consumption people will start saving money and once they save, then it will create me kind of extraordinary pressure on the consumption aggregate demand. The economy will further may further plunder and then it may create very adverse scenario again in future.

This will again add the uncertainty scenario in future. Such type of analysis is important to understand the consumption pattern in the overall economy. Precautionary savings that concept we have understood here, maybe in your textbook, you are reading the precautionary saving then it will say but if you want to understand in a more clear manner with the set of simple mathematical formulations, then it helps you understand the concept better.

This is all about the uncertainty that we are going to face in the next period. In the next session, we are going to talk more about the generalized version of the consumption that we have assumed, what happens individuals are going to leave for more than two periods those dimensions will be covering. Thank you. Thank you much.