Research Methodology
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Lecture - 52
Order of Magnitude Calculations

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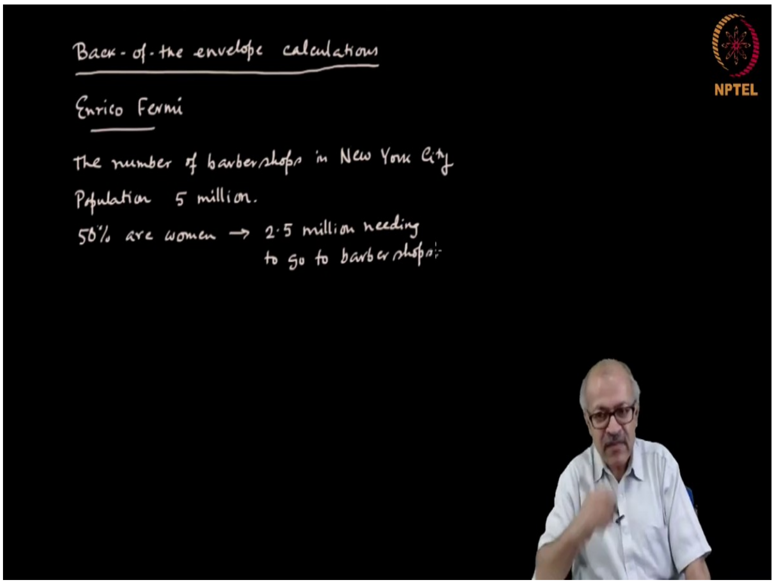

In a theorist's pursuit, it often helps to get some kind of a ballpark figure. Often we do that by some quick calculations. These are often called 'back of the envelope' calculations. The purpose of a back of the envelope calculation is not to obtain exact values. It is essentially to get some kind of a order of magnitude calculation and in many cases that helps.

The famous physicist Enrico Fermi was a master of this technique. Many times these method of calculation is called 'Fermi estimate', because Fermi showed how effectively such very quick calculations can lead to some idea regarding the order of magnitude of what you are trying to find. For example, you know that during the Second World War, in the United States a project called the Manhattan project was initiated which ultimately led to the development of the atom bomb. A large number of European scientists at that time worked in the Manhattan project because it was at that time believed that the Germans were also working on developing atom bombs.

So, as a counter to that, these scientists worked on that. When it was finally built, it was to be tested in a desert. Scientists were standing at a distance and the bomb was detonated. Fermi was also one of the spectators. He had a piece of paper in his hand. What he did was, he simply tore it up into shreds of small pieces and then dropped it from his hand.

As the blast happened, the blast took the pieces of paper to some distance before they fell. He quickly measured the distance they travelled in the horizontal direction in absence of any wind and how much did it fall from the vertical direction. Based only on that, on the back of an envelope, he calculated the approximate power of the bomb that was detonated at a distance (assuming that he knew the distance at which he was standing).

He got something like 10 kilotons as a ballpark figure. When, after a lot of trouble, a lot of calculation, the actual power of that bomb was calculated, it was found to be 21 kilotons, which means that he was right at least to the order of magnitude. So, these are typical Fermi estimates.

As a fun exercise, I can show you one such Fermi estimate: for example, how many barbers are there in the New York City?

The moment you are asked this question, you say I have no clue as to how many barbers can be there. Fermi would say that, ok, let us try to guess with some kind of ballpark figures. What is the population of New York City? The population of New York City is easy to find. Suppose something like 5 million people. Out of the 5 million people, half are women and therefore, they would not go to the barber shop; they would go to other places to get their hair cut. Since 50 percent are women, that brings it down to 2.5 million needing to go to barber shops.

Then, out of this 2.5 million, some will be babies: too small to need a haircut. Some will be bald and therefore, they would not need a haircut. Likewise there might also be some people who cannot afford hair cut: so poor that they cannot afford haircut.

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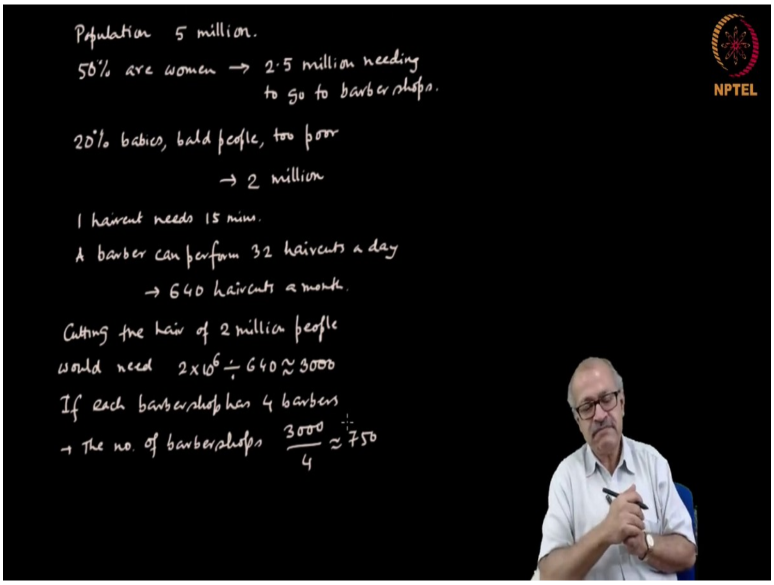

Suppose the number of such people may be, say, 20 percent. So, 20 percent do not need a haircut: babies, bald people and too poor. That brings the number down to about 2 million. So, 2 million people would be needing haircut approximately once a month. That is a ballpark figure because people would normally need a haircut once a month.

Now let us look at the barbers' perspective. Normally a barber would cut somebody's hair in about, say, 15 minutes. So, one haircut needs 15 minutes. How many haircuts can a barber perform in a day? Suppose the barber works for eight hours, each haircut needs 15 minutes, which means a barber can perform around 32 haircuts a day. Assuming weekends, he would work approximately 20 days a month. So, 32x20=640 haircuts a month.

So, one barber can perform 640 haircuts a month and there are 2 million people needing a haircut every month. So, cutting the hair of 2 million people would need how many barbers? It will be 2 million, i.e., 2 into 10 to the power 6, divided by 640. So, that would come approximately to 3000 barbers. So, New York City will need around 3000 barbers. Assume that each barber shop has some 4 barbers. So, if each has 4 barbers, then the number of barber shops becomes 3000 by 4, which would be about 750. This, then, is a sort of estimate of the number of barber shops that would be there in the New York City. Notice that when we started, we had no clue. But if you start arguing properly and if you use some kind of a ballpark figures, you can get meaningful estimates.

Notice the way the Fermi estimates go. This is a figure that we know, but these are sort of ballpark figures, which could be right, could be wrong, we do not know. How many peoples' hair can a barber cut per day? We do not know really, but we assume that it would be something like 32. If these ballpark figures are correct, then the estimate is more or less correct.

At least this should be correct to an order of magnitude. This is a typical example of a back of the envelope calculation and such 'order of magnitude' calculations are often used by astronomers. If you want to look at a particular part of the sky, how many galaxies would be there in that particular part of the sky? If you use this kind of a ballpark figure, you can estimate the number of galaxies that might be there in a particular part of the sky as visible to a particular telescope with a particular strength.

But probably the most famous Fermi estimate was the estimate of the number of alien civilizations that might make contact with us.

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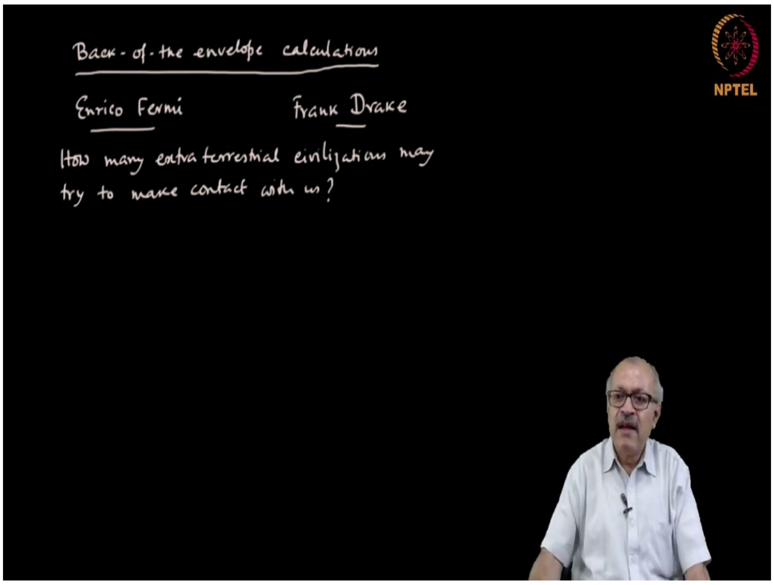

Very interesting calculation. In the 1960s, somebody called Frank Drake did a Fermi calculation of how many extraterrestrial civilizations may try to make contact with us. This is the question that was asked. Drake said, let us do a Fermi-type calculation and he wrote an equation like this.

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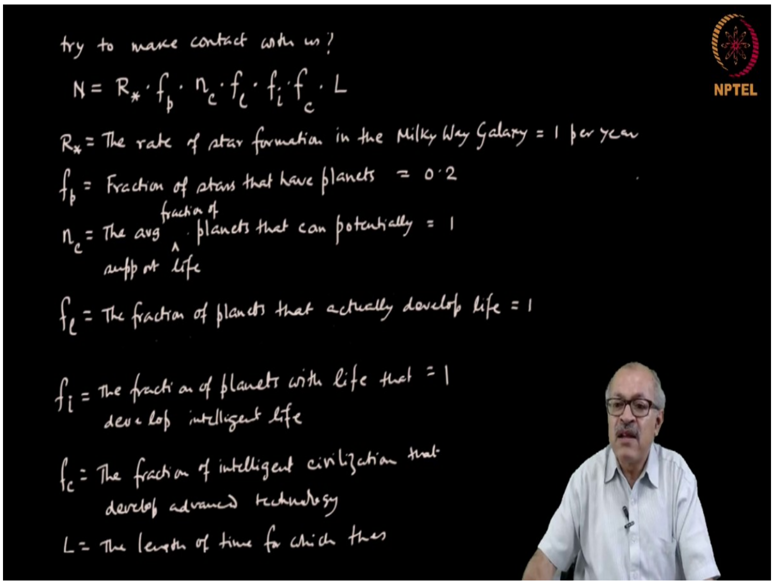

N is the number of extraterrestrial civilizations. He said that, let that be estimated as R star times f p times n c times f_l f_i then f_c then L. What are these? Well, N is the number that we are trying to estimate. R star is the average rate of star formation in our galaxy. We are assuming that extra terrestrial civilizations may try to contact us if they are within our galaxy. Otherwise they are too far off to be able to do that. So, the rate of star formation in the milky way galaxy.

You know, more or less, the rate at which stars are forming. But after that, you need to have some kind of a fraction of stars that have planets. Then we have the n_c, which is the average fraction of planets that can potentially support life. Then we have f_l, which is the fraction of planets actually develop life. f_i is the fraction of planets with life that develop intelligent life. Then f_c is the fraction of intelligent civilizations that develop advanced technology sufficient to contact us.

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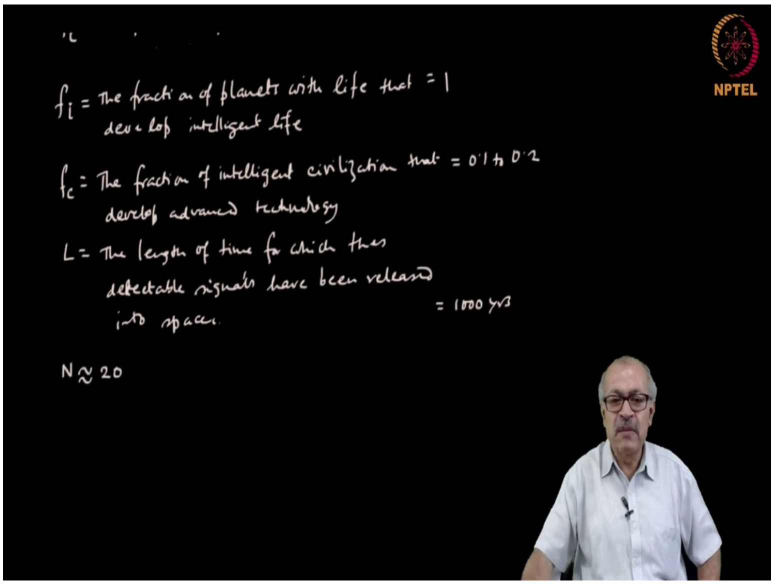

And finally, the length of time. The final term L is the length of time for which such civilizations have been trying to contact us, these detectable signals have been released into space.

Notice that each are reasonable things that we try to figure out. What Drake did was to take some ballpark figures. For example, he said let this be 1 per year, let f_p be, say 0.2. n_c was, say, 1. f_l was also 1. So, his point was that, if we are considering planets that can harbor life, then if they have sufficient time, then they would actually develop life. If life had sufficient time, then it would develop intelligent life. So, all these he assumed to be one. But f_c is not all that given: the fraction that develop high technology. He assumed it to be, say, 0.1 to 0.2.

L he assumed to be 1000 years. So, alien civilizations have been sending a signal for about 1000 years. So, if you multiply them out, he got the total time n as something like of the order of 20.

There have been enough controversy as to whether the assumptions are ok or not. There may be enough controversy. But the fact that he got a non-zero number aroused a lot of interest and that is what is at the basis of the search for extraterrestrial intelligence. All this because he got some non-zero number.

So, this is the basis of what is known as the ‘back of the envelope’ calculations or sometimes called ‘Fermi calculations’ and this often helps in formulating models and getting some idea regarding the ballpark figures.