

Neuroscience of Human Movement
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Lecture - 04
Nernst Equation

So, good morning. Welcome to this class on your Sense of Human Movement. So, in today's class we will be talking about Nernst equation.

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In this class...

- Balance of forces across a semipermeable membrane
- Nernst equation



We described a situation in which forces across a semi-permeable membrane could balance each other and what could happen in such a situation are an equilibrium is created and we suggested that there is a potential at which this equilibrium is achieved, and we call this as the equilibrium potential for that ion or the electrochemical equilibrium is achieved. Equilibrium potential is the potential at which this electrochemical equilibrium is achieved, right. So, how to compute this? This is achieved using Nernst equation. So, in today's class we will discuss this.

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Generation of an Na^+ diffusion potential 1. Concentration grad.
2. Potential grad.

Na⁺ selective membrane

Solution 1 Solution 2

Solution 1 Solution 2

When positive charge in solution 2 does not permit any more sodium to enter, "Sodium electrochemical equilibrium" is achieved.

So, here is a situation when there are two solutions separated by semi permeable membrane. So, here is a semi permeable membrane, there is solution 1 and solution 2. Solution 1 has a higher concentration of sodium. So, the font size here represents concentration or a higher font size means a higher concentration of sodium and chloride in solution 1 whereas, in solution 2 you have a much lower concentration of sodium and chlorine, right. Suppose this semi-permeable membrane was permeable only to sodium.

So, this sodium selective membrane if that is the case, then what happens is that only sodium diffuses across this membrane in that direction. Chloride cannot diffuse because this membrane is permeable only to sodium and not to chloride, but since only sodium is diffusing, this causes a buildup of positive charge on the right side. If only chloride does also able to diffuse, you would not have this situation, but in this situation when sodium alone is diffusing, you have a buildup of positive charge in solution too.

If sodium was not charged, if sodium was not an electrolyte, if sodium was not ion, if it was a non-charged solute right, then in that case the movement of this substance would be driven only by concentration gradient and that will not build any charge in the right side, right. In solution 2 that will not happen because that we are discussing does not have any charge. So, it will not build any charge separation between the two sides, however since sodium is an electrolyte is an ion, what happens is that since the transport

of sodium across the semipermeable membrane builds this charge separation, so on one side you have positivity and on the other side you have negativity.

So, what this creates is a potential gradient that potential is developed. So, now in this case the transport of ion is governed by two forces. One is the concentration gradient related force and the other is the force due to the potential gradient. Note that the potential gradient itself may be developed due to the movement of ions which happens due to the concentration gradient, but as the sodium continues to move further addition of sodium is resisted by the positive charge, that is present here and this resistance to further movement of sodium increases as the amount of positivity on the right side starts building up.

So, initially there is less resistance, as you go along, the resistance is very high and at some point what happens is that the sodium ion can no longer go from this side to that side. It is not possible for that moment to happen, for that transport to happen because the positivity on the right side is sufficiently high to prevent any more entry of cations, any more entry of positive ions or sodium in this case into the right side or into the solution 2.

The potential at which this happens is the sodium equilibrium potential and that equilibrium is called as sodium electrochemical equilibrium, right. So, that is a sodium equilibrium after chemical equilibrium and the potential at which it happens is called as a sodium equilibrium potential.

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Movement of ions

$F_c = RT \cdot \ln(C)$ ✓
 $F_e = ZFV$ ✓

Where

- R = Gas Constant ✓
- T = Absolute temperature (in K)
- Z = Charge of the ion (+1, -1, +2, -2) ✓
- F = Faraday's Constant
- C = Concentration of the ion
- V = Voltage

$F_{c1} + F_{e1} = F_{c2} + F_{e2}$
 $F_{c(1)} + F_{e(1)} = F_{c(2)} + F_{e(2)}$

C_1, V_1 (Low C) C_2, V_2 (High C) $C_2 > C_1$

Now, let us consider the situation one more time. So, suppose there is a greater number of particles or solute particles on one side and let us assume that these solute particles are charged these are ions, right. So, here as you see the right side has higher concentration and the left side has a lower concentration of this particular solute. Let us assume that we also know the concentrations these concentrations C_1 and C_2 . In other words, says C_2 is greater than C_1 is something that we know, but we also know the exact values of C_2 and C_1 let us see.

Also, I know for example the voltages between in this chamber and in that chamber as V_1 and V_2 if that is transport across this semipermeable membrane, let us assume this is a semipermeable membrane. If there is transport across this membrane, it turns out that the force due to concentration gradient will move these ions in that direction, right. Let us assume that the potential gradient is in the opposite direction Suppose now the question is, is it possible to develop a relationship such that we are able to compute the point at which the balance occurs are the electrochemical equilibrium is achieved. Can I somehow compute this? It turns out that it is possible for me to develop expressions individually for each of these forces F_c and F_e and that is these are already known. This is the force due to the concentration gradient that is $RT \ln C$ out and the force due to the potential gradient is ZFV .

These are already known where R is the gas constant, T is the absolute temperature. So, in Kelvin right are the temperature at which the computation is performed or the temperature at which these chambers are, so that temperature T in Kelvin, for example, in humans this would be 37 degree Celsius, 310 Kelvin. So, appropriately we will have to substitute that value. It can be different in different organisms or in different situations. So, you will have to consider that and Z is the valency or the charge of the ion. This can be you know plus 1 minus 1. For example, for calcium this is plus 2 etcetera. F is the faradays constant and C is the concentration of that ion and V is the voltage, right and it turns out that I could come up with the equation that balances these two, that is this equation that is basically $F c_1$ plus $F e_1$ is equal to $F c_2$ plus $F e_2$.

So, these two forces, sum of these two forces exactly balance each other at some point. The question is what is that point? That is what we are interested in finding out. What is the potential difference or what is the concentration gradient at which this will happen, right? In particular we are interested in computing this potential at which this happens.

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Balance of forces in the presence of an electric field

In a state of equilibrium, movements of all particles are counterbalanced: There are no net changes in the concentrations.

$$RT \ln(C_{out}) + ZFV_{out} = RT \ln(C_{in}) + ZFV_{in}$$

$$ZF(V_{in} - V_{out}) = RT(\ln(C_{in}) - \ln(C_{out}))$$

$$ZFV_{Eq} = RT \ln \frac{C_{in}}{C_{out}}$$

$$V_{Eq} = \frac{-RT}{ZF} \ln \frac{C_{in}}{C_{out}}$$



So, in this state of equilibrium what happens is that although there may exist a concentration gradient, there will be no more movement, are no more net moment. It is possible that in the previous case, one ion goes from right to left. Suppose one ion goes from right to left. It is also possible that the probability of another ion moving from left to right is also approximately the same, such that the net force or the net movement is

approximately 0 so that is the situation. So, in the state of equilibrium, the net moment is approximately 0 or the movement of all particles under counter balance. So, there are no net changes in concentrations anymore, although there may exist a gradient. This cannot be further changed because of the potential gradient rate.

So, that is the situation. So, if we are considering the cell membrane and the outside of the cell membrane and the inside of the cell membrane, I could rewrite these equations and this equation as this, right. Suppose the concentration outside were C_{out} and the concentration inside was C_{in} , the voltage outside was V_{out} and the voltage inside was V_{in} . I could rewrite this and a simple rearrangement of that equation gives me this. So, basically I am taking the concentration terms on one side and the voltage terms on the other side and then, I am naming this difference as $V_{equivalent}$, right. This is minus because $V_{equivalent}$ is V_{out} minus V_{in} . So, basically set of $V_{equivalent}$ will give me minus $RT \ln C_{in} / C_{out}$. So, I am interested in knowing this value be equivalent. So, effectively ZF comes to the denominator as it does here.

So, that is the equation of interest for us. So, this equation is called as Nernst equation and it is useful in computing the equilibrium potential of individual ions across the biological membrane. It is critical to note that this is only when considering one ion at a time. Suppose there are multiple ions, right the situation will vary. So, for this equation, for this situation we are only considering one ion at a time, right. So, that is the equilibrium potential and I could for example convert the lawn here as log.

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Nernst Equation

$$E_X = \frac{-2.3RT}{zF} \log_{10} \frac{C_i}{C_e}$$

$\frac{-2.3RT}{F} \approx \frac{-60mV}{z} @ 310K$

Where

- E_x = Equilibrium potential (mV) for a given ion, X
- R = Gas Constant ($8.31 \text{ J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$) @ 310K
- T = Absolute temperature (K)
- F = Faraday's Constant ($9.65 \times 10^4 \text{ C/mol}$)
- Z = Charge on the ion (+1 for Na^+ ; +2 for Ca^{2+} and -1 for Cl^-)
- C_i = Intracellular concentration of X (mmol/L)
- C_e = Extracellular concentration of X (mmol/L)



So, using this multiplication factor and I know RT for example, I know F. The only thing I do not know is z which changes depending on which particular ion I am considering. So, I could theoretically compute all these numbers. I also do not know C_i and C_e . Of course, I also do not know these things. Only those that I have marked in red are unknowns, those that I have ticked are all known's. For example, these are all things that I already know. So, I could compute what that value would be.

So, that is this is minus 60 divided by Z at 310 Kelvin. Once again note this is assuming T is equal to 3 Kelvin, right. I could analyze this for various ions and for various concentrations, let us do some example problems, right.

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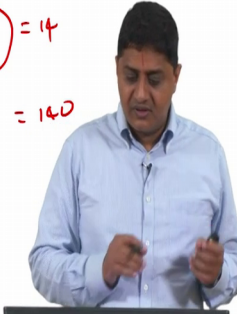
Equilibrium potential of Na⁺

- Intracellular concentration of Na⁺, C_i = 14 mEq/L
- Extracellular concentration of Na⁺, C_e = 140 mEq/L
- Charge in Na⁺, Z = +1

at 310K:

$$E_{Na^+} = \frac{-60mV}{z} \log_{10} \frac{C_i}{C_e}$$

(Handwritten annotations: C_i = 14, C_e = 140)



The intracellular concentration of sodium is 14 milliequivalent per liter and the extracellular concentration is 140 milliequivalent per liter for example, right and we already said that the Nernst potential is minus 60 millivolts by Z log base 10 C_i by C_e. So, I can substitute C_i as 14 C_e as 140. Once again this is computed at 310 Kelvin. Let us remember that suppose the problem asks you to compute this for a different temperature, then you have to go back to the general form of this equation and redo this problem, right but at 310 Kelvin you could substitute for these values.


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Equilibrium potential of Na⁺

- Intracellular concentration of Na⁺, C_i = 14
- Extracellular concentration of Na⁺, C_e = 140
- Charge in Na⁺, Z = +1

$$E_{Na^+} = \frac{-60mV}{+1} \log_{10} \frac{14}{140}$$

(Handwritten annotations: red checkmarks next to 14 and 140)



As we have done here 14 by 144, 10 by 140 is 0.1 or 1 by 10 or 10 power minus 1 like that.

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Equilibrium potential of Na⁺

- Intracellular concentration of Na⁺, C_i = 14
- Extracellular concentration of Na⁺, C_e = 140
- Charge in Na⁺, Z = +1

$$E_{Na^+} = -60 \log_{10} 10^{-1} mV$$

60 w.



That basically means this is 60 times log 10 based on it which is 1.

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Equilibrium potential of Na⁺

- Intracellular concentration of Na⁺, C_i = 14
- Extracellular concentration of Na⁺, C_e = 140
- Charge in Na⁺, Z = +1

$$\underline{E_{Na^+}} = \underline{+60mV}$$



So, that is actually plus 60 millivolts. So, what this means is if the membrane potential were at plus 60 millivolts, then there will be no more movement of sodium across the biological membrane. If ever the biological membrane reaches a potential of plus 60 millivolts, sodium will not bother to move from this side to that side or any movement of

sodium from side 1 to side 2 will be counter balanced by movement of another sodium ion from side 2 to side 1 for example, or there will be no net movement of sodium from one side to another side or from intracellular matrix to extracellular matrix. It is not necessary for the sodium ion to make a moment.

So, ideally this is the potential at which the sodium ion would like to have. Suppose the sodium ion has a mind of its own. This is the potential at which the sodium ion would like to have the membrane at. So, if at all it is possible for the sodium ion to have a wish, then it will say let us have the membrane potential at plus 60 millivolts, why? It is because this ion is less. It does not want to make any moment, right. So, let us consider the situation for potassium.

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Equilibrium potential of K⁺

- Intracellular concentration of K⁺, C_i = 120 *mEq/L*
- Extracellular concentration of K⁺, C_e = 4 *mEq/L*
- Charge in K⁺, Z = +1 *at 310K*

$$E_{K^+} = \frac{-60mV}{Z} \log_{10} \frac{C_i}{C_e}$$

120
4



Let us remember we discussed the typical concentrations of potassium sodium chloride in an earlier class. Let us remember that the concentration inside was approximately 120 million per litre and outside is for milli equivalents per litre. Now, once again I could substitute this. So, again the problem is to be solved at 310 Kelvin. Now, once again let us substitute the value of Z which is plus 1 here and substitute for C_i and C_e which is 120 and 4.

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Equilibrium potential of K⁺

- Intracellular concentration of K⁺, C_i = 120
- Extracellular concentration of K⁺, C_e = 4
- Charge in K⁺, Z = +1

$$E_{K^+} = \frac{-60mV}{+1} \log_{10} \frac{120}{4}$$



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Equilibrium potential of K⁺

- Intracellular concentration of K⁺, C_i = 120
- Extracellular concentration of K⁺, C_e = 4
- Charge in K⁺, Z = +1

$$E_{K^+} = -60 \log_{10}(30) mV$$



So, when I do that, so this is actually 30. So, I could compute log 30, right.

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Equilibrium potential of K⁺

- Intracellular concentration of K⁺, C_i = 120
- Extracellular concentration of K⁺, C_e = 4
- Charge in K⁺, Z = +1

$$E_{K^+} \approx \underline{-88mV}$$



So, then the equilibrium potential comes out to be minus 88 millivolts. So, once again this is the potential at which there is no net movement of the potassium ion across the biological membrane. So, if the potassium ion had a mind of its own, it will prefer to have the membrane potential at this value because it does not have to make any movements. So, this is there it would like to have it, but where is it, where is the membrane potential relative to its equilibrium potential will determine to what extent the movement is happening that we will reserve for future classes, right.

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Equilibrium potential of Cl⁻

- Intracellular concentration of Cl⁻, C_i = 10 *mEq/L*
- Extracellular concentration of Cl⁻, C_e = 150 *mEq/L*
- Charge in Cl⁻, Z = -1 *4E 310K*

*Na⁺ = +1
K⁺ = +1*

$$E_{Cl^-} = \frac{-60mV}{Z} \log_{10} \frac{C_i}{C_e}$$

Z = -1



Similarly, for chloride once again I know the inside concentration as about 10 milliequivalent per litre. The external concentration is about 150 milliequivalents per litre and the problem is to be solved at 310 Kelvin say for example.

Then, I could substitute for C_i and C_e and Z , note Z has a sign. So, here I will have to substitute for Z as minus 1. So, this is crucial. So, when it comes to sodium and potassium, since it is plus 1, we did not really care to substitute that because anything divided by plus 1 is the same. It does not really matter, however when it is chloride, it is important for you to substitute this as minus 1, not as 1 because then the answer will be very different, right.

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Equilibrium potential of Cl^-

- Intracellular concentration of Cl^- , $C_i = 10$
- Extracellular concentration of Cl^- , $C_e = 150$
- Charge in Cl^- , $Z = -1$

$$E_{\text{Cl}^-} = \frac{-60\text{mV}}{-1} \log_{10} \frac{10}{150}$$



So, I am substituting this by minus 1 and I am solving this problem, proceeding to solve these problems and substituting 10 there and 150 there, right.

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Equilibrium potential of Cl⁻

- Intracellular concentration of Cl⁻, C_i = 10
- Extracellular concentration of Cl⁻, C_e = 150
- Charge in Cl⁻, Z = -1

$$E_{Cl^-} = 60 \log_{10}(0.066) mV$$



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Equilibrium potential of Cl⁻

- Intracellular concentration of Cl⁻, C_i = 10
- Extracellular concentration of Cl⁻, C_e = 150
- Charge in Cl⁻, Z = -1

$$\underline{E_{Cl^-}} \approx \underline{-70mV}$$



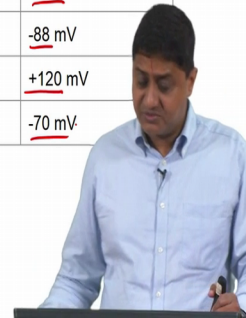
So, that will basically give me the equilibrium potential for chloride minus 70 millivolts. Once again this is the value, right. So, this is the potential that the chloride ion would like the membrane to have because at this potential, the chloride ion does not have to make any movement. So, it can be lazy it does not have the net movement of ions across the membrane will be 0, right. So, this is ideally where the chloride ion would like to have, but once again the difference between the actual potential and the equilibrium potential will determine the movement, ok.

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Equilibrium potential of various Ions

Ion	ICF	ECF	E ^x
Na ⁺	<u>14</u> mEq/L	<u>140</u> mEq/L	<u>+60</u> mV
K ⁺	<u>120</u> mEq/L	<u>4</u> mEq/L	<u>-88</u> mV
Ca ²⁺	<u>0.0001</u> mEq/L	<u>2.5</u> mEq/L	<u>+120</u> mV
Cl ⁻	<u>10</u> mEq/L	<u>150</u> mEq/L	<u>-70</u> mV

→ Z = +2.



So, let us remember the concentration, so 14 and 140 between inside and outside. So, the equilibrium potential is plus 60 millivolts for 120 and 4 equilibrium potential. So, minus 88 millivolts once again note this is the calcium concentration inside and outside. So, the equilibrium potential is plus 120 millivolts. So far calcium for calcium Z is equal to plus 2 remember.

So, also note that in the exam problems, the actual numbers may vary. So, you have to substitute up properly. These are some examples that I am giving. So, this is 10 and 150. So, the equilibrium potential minus 70 millivolts.

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Driving Force $E_m = -80 \text{ mV}$

Net driving Force (mV) = $E_m - E_x$

Where

E_m = Actual membrane potential (mV) ✓

E_x = Equilibrium potential for X (mV) ✓



So, what is the driving force? What constitutes, what causes the movement of ions across the biological membrane? That is as I said the difference between the membrane potential, between the actual membrane potential which is E_m and the equilibrium potential which is E_x . So, the larger this value is, higher is going to be a movement of that particular ion. Suppose let say minus 80 millivolts, right. Let us go back to the previous and see suppose E_m is equal to minus 80 millivolts, then in that case which of these ions will have large moments, which ones will have small moments? This one, right will have small moments. Why? It is already close to, it is already the membrane potential is close to the equilibrium potential whereas, these two will have large moments, right or in other words, if there is possibility of an opening, if there is possibility for moment to happen, then a large amount of movement will happen, right so something to remember. So, relatively this will also have a small amount of moment, ok. So, with this we come to the end of this lecture. We will continue the discussion in future classes.

Thank you very much.