Introduction to Reliability Engineering
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Lecture 07
Constant Failure Rate Model-II

Hello everyone, now, we move to lecture number 7, which is continuation of lecture number 6 on exponential distribution or constant failure rate models.

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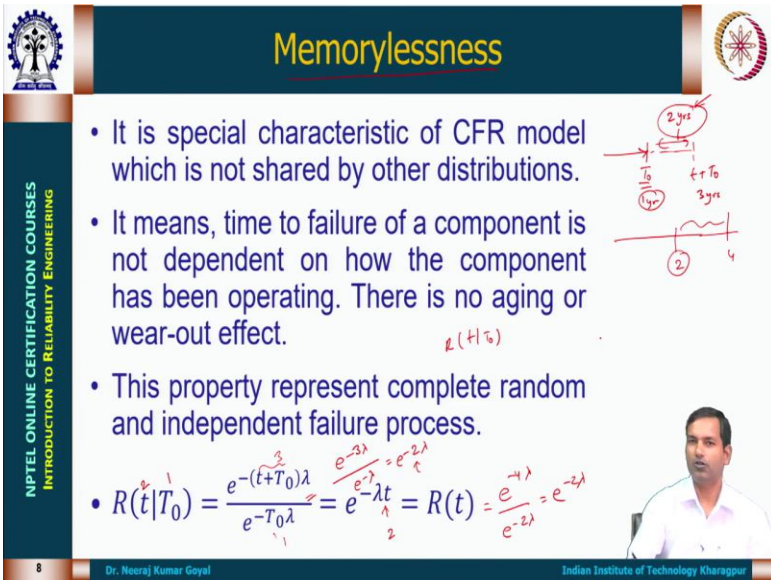

Constant failure rate models show a very important and a distinct property that this is a memorylessness. So, this memorylessness property what does it mean, means memorylessness we can also say it is a forgetfulness. So, what is this forgetfulness which means in terms of this exponential distribution of CFR model. Now, here what does it mean that it is for what it is forgetting, it is for getting the age, like someone like us whatever as we age, we have a failure rate will increase our hazarded or we can say the fatality rate or we can say the mortality rate will increase.

Similarly, the components as they age we expect them they will wear out they will have degradation and because of that, later on the chances of failures will increase or failure rate will also increase. But here, CFR has a constant failure rate has a specific meaning that per probability, if the component is surviving up to a certain time, then probability of failure per unit time remains same. It does not change.

What does it mean? That means, in terms of aging, if you see that component is not aging, component is forgetting its age. It does not remember that it has aged it does not remember how long it has worked. So, let us say if we say so, this property I can explain using this conditional reliability formula, conditional reliability formula we have already discussed in previous class. So, condition reliability formula RT given t0 means that RT plus t0, R t given t0 that means, what does it mean? I am here at t0 time, and I want to know reliability in future of t time.

$$R(t \mid T_0) = \frac{e^{-(t+T_0)\lambda}}{e^{-T_0\lambda}} = e^{-\lambda t} = R(t)$$

That means, t plus t0 time, as we see here, at t0 time I am sitting at t0 time that means, let us say if I take an example of any let us say let us take example for let us say watch. So, for watch, I have used it for let us see one year. So, my t0 is one year and it is working right now, that after one year of uses currently it is working. Now, let us say watch follows the exponential distribution.

So in that case, I am interested to know that if I use this watch for further 2 years, what will be the reliability that means today it is working that is one year and that means on completion of 3 years, but from today it is 2 years, what is going to be the reliability. So reliability for 2 years given that it is worked for one year reliability for 2 years, given that it is worked for one year. Now, it is following the constant failure rate lambda.

This if I calculate, then this will be equal to e to the power minus total time is 3. So 3 lambda divided by how long it is worked, that is e to the power minus lambda t0 is 1. So this will become 1, this is 1 plus 2 will become 3. So I get e to the power minus 2 lambda. That is the same time 2 years, as we see this time is 2 years, as you see here, that whatever is my failure probability for 2 years, it remains same.

Let us say if I have done I have used a watch for 2 years. And now I am talking about 2 more years that means up to 4, in that case, this value will be equal to e to the power minus 4 lambda divided by e to the power minus 2 lambda, again, I get e to the power minus2 lambda. That means probability of working for 2 years here is same irrespective of the age whether I have worked used it for one year or whether I have used it for 2 years. If it is working today, then for next 2 years the probability of failure of or probability of success is going to be same, it is not going to change it change only with amount of time.

If I make it 3 years this will become e to the power minus 3 lambda t, irrespective of the time it has already spent. So, in a way, we are saying that this is a memoryless property that it does not remember that it has worked for so many time. So, whatever it has worked for, it does not cumulate any damage, it remains same as when it was new, the chances of failure does not change with the age or with the life, this is a memoryless property, which becomes very useful here, because here the chances of failures per unit times are not changing with the time.

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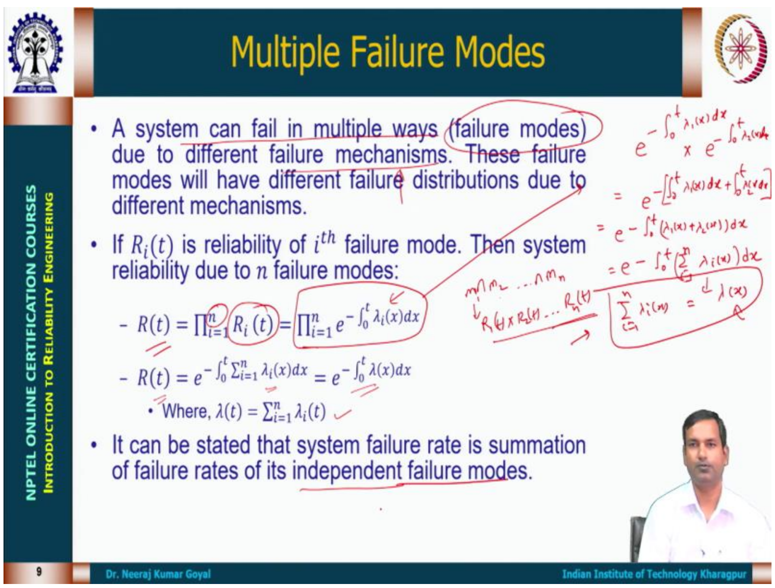

In case of multiple failure modes, so, generally what happens our devices are having complex systems, which is where multiple components are there, multiple subsystems are there that let us say, if I talk about a communication system, so, in a communication system the failure can happen either due to the antenna failure or it can happen due to the failure of receiver or failure of the transmitter.

If there are multiple challenges multiple elements over there, it can also be due to the power failure. Now, as we see that the system can fail in multiple ways, why it is failing in multiple ways, because, there are multiple components which can fail in different, different ways. So, system is also can fail in different ways, these different ways we are calling as the failure modes. So, these are the failure modes of the system.

Now, these failure modes now, what is happening, the failure mechanism, failure mechanism tells us that, why they are failing, how they are failing, what is changing. See, if everything remains same, then there is no chance that there will be a failure. If things stays same, then and the uses also remains stay same, then there will be no chance of failure it will remain as it is if it is working today, it will work next time also, but what happens over the time the way we are using that is our load that is changing as well as the components or the system which are using they are also changing because nothing remains same with the time everything changes with the time and uses.

So, as they are changing what happens there is a degradation or there is some sort of development of fault or there is excessive stress coming to this. So, different failure mechanisms are involve, like antenna failure causes would be different, failure pattern would be different, failure distribution would be different compared to the receiver failure distribution or transmitter failure distribution or power system failure power source distribution. So, different failure modes generally are involving different failure mechanism.

For antenna failure, it may be the dimension property or material problem or there may be binding or there may be some other issues, but if I am talking about a power failure, it may be that battery failure battery cell is not working properly or some wiring failure has happened or some it is not retaining the charge. So, various different reasons different failure mechanisms will be there.

So, due to which we they are tend to having the different failure distributions I want to know the reliability. So, for knowing the reliability I know I need to know the reliability against each failure mode, because for my system to work, the system should not fail in any failure mode. So, that means, if for my system to work, if let us say there are n number of failure modes that means, there are n possible components or n possible ways the system can fail each. So, the reliability of the system is if any of the failure mode happens the system will fail.

$$R(t) = \prod_{i=1}^{n} R_i(t) = \prod_{i=1}^{n} e^{-\int_0^t \lambda_i(x)dx}$$
$$R(t) = e^{-\int_0^t \sum_{i=1}^{n} \lambda_i(x)dx} = e^{-\int_0^t \lambda(x)dx}$$

So, that means, for each failure mode it should be reliable, it should not fail in that failure mode. So, 1 minus failure probability for each failure mode that is my RIT. So, probability that it is not failing in ith failure mode that is we are calling as RIT. So, for system reliability, what will be the probability it does not fail in any of, so, that means, it does not fail in any means, intersection. Neither it fails in failure mode 1 nor it fails in failure mode 2, like nor it fails in failure mode n.

So, reliability will be equal to reliability, let us say we get reliability for this as R1T for this R2T. So, as we know as we have discussed earlier, we follow the multiplication principle here. So, whenever all has to work that means, we have to use some multiplication because all have to work. So, if there is an intersection involved this is working and this is working and this is working and relationship, we use the multiplication here.

So, once we multiply here we get the system reliability. So, system reliability is that all our reliable or we can say none of the failure occurs. Now, RIT as we know that is equal to e to the power minus lambda I x dx, this is in the case of general model where lambda i can be the time dependent. So, first we will do it for the time dependent and then we will see if this is time independent or constant failure rate how the change will happen.

So, the same formula that we developed for time dependent same formula will be applicable for the time independent also. So, as we see here, if we do this now, here if you see that we get for lambda 1 lambda 2 lambda 3 x I will get it. Now, in this case if I want to know the reliability here to see that how this can be solved. Now, we are multiplying it, so, for an example, let us say if I had the 2 failure modes. So, for 2 failure modes this will be 0 to t lambda 1x dx multiply with e to the power minus 0 to t lambda 2x dx.

Now, the same thing I can write it as e to the power minus integration from 0 to t lambda 1x dx plus 0 to t lambda 2x dx, same way I can do it further I can write it as e to the power minus integration from 0 to t lambda 1 plus lambda 1x plus lambda 2x into dx. So, same way if I have n number of components this will become lambda 1x plus lambda 2x plus lambda 3x till lambda nx. So, this I can write it as e to the power minus integration from 0 to t summation i equal to 1 to n lambda I x dx same thing we have written it here.

So, this way what does it mean now, if I call this as a lambda x. So, lambda x will be equal to summation i equal to 1 to n lambda i x, what does it mean that if I sum up individual failure rates for given time t then I will get the failure rate at time t for the system. So, failure rates are when system is having multiple failure modes, then system failure rate is summation of the failure rates of the each failure mode.

So, this is generic this is independent whether this is time dependent failure distribution or failure rate or it is time independent failure rate whatever is the failure rate at that time t we take it for all failure modes and we sum it up and we get the system failure rate. So, system failure rates as we see when reliabilities are multiply cable here, system failure rates are summable here.

So, this becomes an easy formula to evaluate that RT system reliability is equal to e to the power minus integration from 0 to t lambda x dx, where lambda x is nothing but the summation of individual failure rates of the all failure modes. This helps us this formula will help us to evaluate, once we know the system failure rate, we simply sum it up and we get the individual failure rate we sum it up together system failure rate. Here the inherent assumption is that failure modes are independent that means, one failure mode does not influence the another failure mode.

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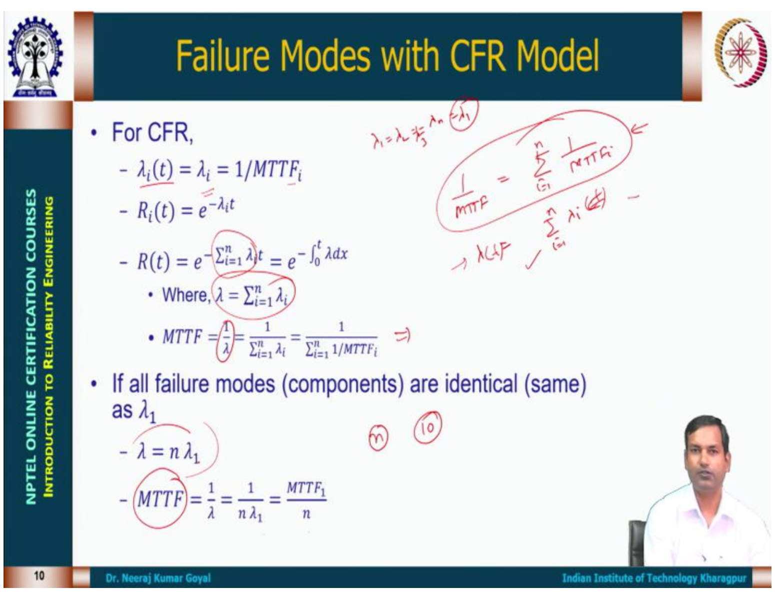

Now, let us say if we consider that failure modes are following the constant failure rate distribution. So, if there are constant failure rate then what will happen each failure rate lambda it will become independent of time t, it will become lambda i, which we can write it and we know the failure rate is 1 upon MTTF. So, this will be one upon MTTF i, and reliability for each failure mode will be e to the power minus lambda it.

$$\lambda_i(t) = \lambda_i = 1/MTTF_i$$
$$R_i(t) = e^{-\lambda_i t}$$
$$R(t) = e^{-\sum_{i=1}^{n} \lambda_i t} = e^{-\int_0^t \lambda dx}$$

Where, $\lambda = \sum_{i=1}^{n} \lambda_i$

$$MTTF = \frac{1}{\lambda} = \frac{1}{\sum_{i=1}^{n} \lambda_i} = \frac{1}{\sum_{i=1}^{n} 1/MTTF_i}$$

So, we can get the system reliability, system reliability will be summation of individual failure rate into t, e to the power minus lambda t, where lambda here is the summation of individual lambda i, this gives us now from here, if I want to know MTTF, MTTF will be equal to 1 upon lambda 1 upon lambda and what is lambda? Lambda here is 1 upon lambda here is submission of lambda i.

So, MTTF if I replaced lambda i with 1 upon MTTF i, this will be in a way I can say that 1 upon MTTF for a system will be equal to summation of i equal to 1 to n 1 upon MTTFi or we can say lambda is equal to summation of i equal to 1 to n lambda i. That means either we take the MTTF inverse in sum, I will get the inverse of MTTF or we can simply sum up the failure rate I will get the failure rate of the system.

But this property is true only for CFR model this is true for this is lambda t is equal to lambda it this is true for all distributions, but this is true only for CFR model because, in case of CFR only lambda i becomes 1 upon MTTF for other distributions lambda i may not be 1 upon MTTF. If all failure modes are identical, if we assume that lambda one is equal to lambda 2 all r is equal to lambda n and this is all equal to lambda 1.

$$\lambda = n\lambda_1$$

$$MTTF = \frac{1}{\lambda} = \frac{1}{n\lambda_1} = \frac{MTTF_1}{n}$$

So, if I replace this then what will happen summation of lambda i n number of times will give me the n lambda 1. So, my system failure rate will be n times of component failure rate and MTTF will be 1 upon lambda that is 1 upon lambda 1 lambda 1 is 1 upon MTTF 1. So that will become MTTF 1 upon n, and that means, my MTT if I am having n components, let us say if I am having 10 components or 10 Failure Modes, what will happen, my MTTF and all are having same MTTF, then system MTTF will be 10 times less than the component MTTF or failure rate will be 10 times of failure rate of each component.

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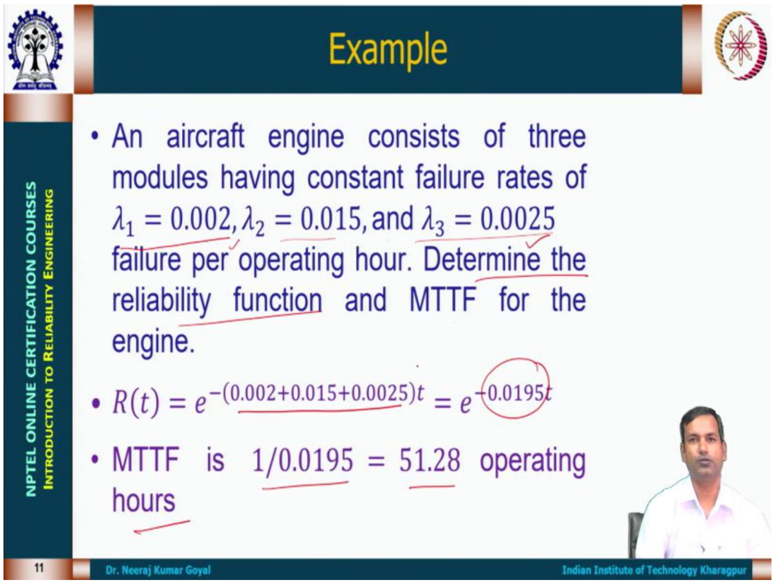

Let us take one example. An aircraft engine consists of 3 modules which is having constant failure rates, lambda 1, lambda 2 lambda 3. So this also means that we are having 3 different failure modes of components, so, how much will be the failure rate for the system? System failure rate will be this plus this plus this, this comes out to be this. So my system failure rate will be this. And my reliability will be so we want to know the reliability functions.

$$R(t) = e^{-(0.002+0.015+0.0025)t} = e^{-0.0195t}$$

MTTF is $1/0.0195 = 51.28$ operating hours

So R(T), R(T) will be equal to e to the power minus summation of lambda into T, that comes out to be to the power minus if we sum it up, we get the 0.0195 multiply by T and what is MTTF? MTTF is one upon of this lambda. So, we get one upon of this that comes out to be around 51.28 operating hours. So, we can easily calculate MTTF for the system, we can calculate failure rate of the system we can calculate reliability of the system, unreliability of the system, PDF everything we are able to calculate if we have the data for the individual failed component or individual failure modes of the system.

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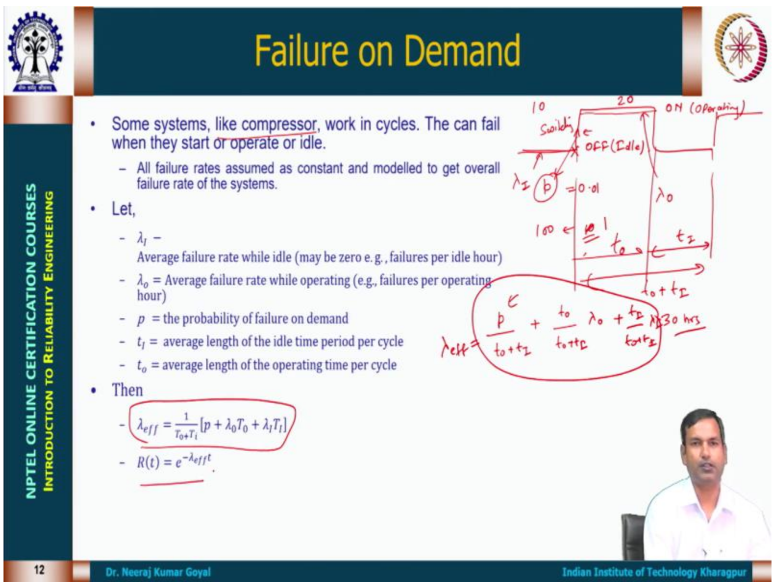

There is another kind of system which is failure on demand, like some system like compressor is there or we can say this compressor or we can say sometimes like bulb is there. So, like electric bulb which you are using at home or some other similar devices, what they happen, they follow a pattern like this. This is off, let us say and this is let us say on. So, what will happen here switching will be carried out. Generally, what happens similarly, motors may be there, generators may be there.

So, and because of the nature, nature of these devices, electrical nature of these devices, at the time of switching there is sometimes surge current happening there is a state change happens.

The components which we are not having any load suddenly start having the load. Now, because of this state change, what happens? There is another failure mechanism involved.

So, there are different failure mechanisms involved, there are actually 3 failure mechanisms we are going to discuss here. One failure mechanism is let this is off or we are saying, idle state, on means operating state and here it is switching. So, there is a possibility it can fail during the switching this fail during the switching the probability we are calling as p, p is the probability that it can fail when we are switching.

But here it is continuously either operating here it is switching is one time instant phenomenon. So, for instant formula this is not a function of time this is a probability that if we let us say p is equal to 0.1, what does it mean? If I am making 100 switching actions, I am expecting that 10 times it will fail, is it proportion of the time if I am saying 0.01, I am expecting one failure out of 100 switching. So, every time it is switching it is having a different failure mechanism because parts are stress differently when there is a switching.

Then there are 2 modes this is idle mode, in idle mode let us say the failure rate is lambda i and there is another on mode that is the working mode let us say failure rate is lambda o, failure rate means probability of failure per unit time. Now, here let us say in general we can assume that on an average how many how much time it is going to be on and how much time it is going to be off. Let us say this remains, I will use black red pen only.

So, let us say the duration here for this is t o and off duration is let us say t idle, t o means operating time this is idle time. So, how much is total time here my total time is t o plus t i. So, one cycle time is t o plus t i, in this 1 cycle, I am having proportion of operation and proportion of idle time. Now, I want to know the failure rate. So, to calculate the failure rate, we can calculate it in a manner like probability of failure in one cycle there is one switching.

So, per unit time how much switching will be there we will have p divided by t 0 plus t i, due to switching let us say if I let us say that idle is 10 hours and it is working for let us say or 20 hours. So, my cycle time is 30 hours, in 30 hours 10 hours it works in idle mode 20 hours it works in the operating mode. So, in 30 hours, I will have only one switching only one time it will be switched on, during off generally there is no stress coming. So, during off it is generally not counted most of the time we are considering the switching during the on.

So, what is the now failure rate for this operation of switching one for one cycle probability of failure is p. So, p divided by t 0 plus t i will give me the failure rate expected number of failures per unit time due to the switching. What is the switching failure probability due to due to the operating hours? I have t 0 time it is operating out of t 0 plus t i. So, it will have contribution only up to the t 0 and the failure rate is lambda 0.

Similarly, for idle time I will have this t i divided by t 0 plus t i multiply by lambda i. This gives me the failure rate total failure rate for the or effective failure rate for the this kind of system where we have the switching and where we have the idle, and operating state of the systems. This gives me the same formula is given here that 1 upon t 0 plus t i, P plus lambda 0, t 0 plus lambda t i. If I want to know the reliability here, I can easily calculate it by RT equals to e to the power minus lambda effective into t.

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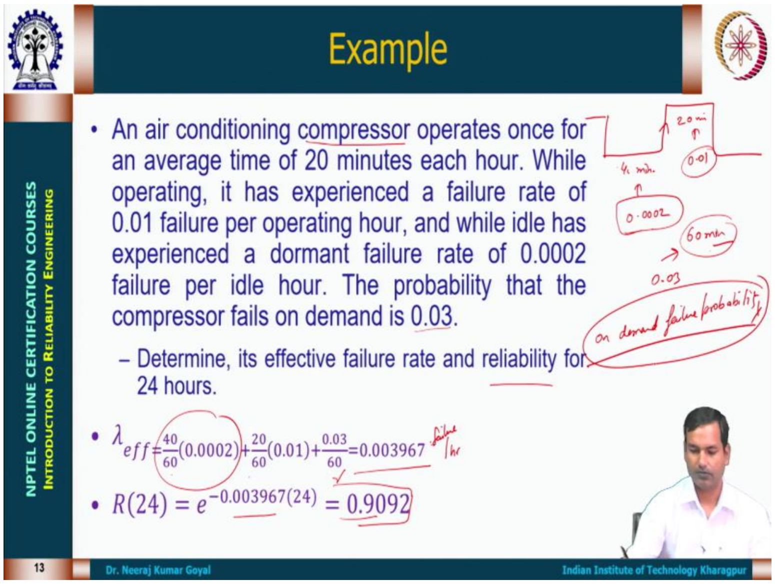

Let us take an example for this. Let us say one air conditioner compressor is there, as we know like conditioner, the compressor whether it is in AC, whether it is refrigerator, it does not operate continuously it operates in the cycle. So, depending on the temperature requirements it will start then it will run for some time when temperature required temperature, threshold temperature is achieved, then it will shut down and wait for some time till temperature increases again and reaches to a certain value.

So, this operates now, this operation is let us say if we denote it, like this as we see here this is working on an average 20 minutes for each hour that means 20 minutes is the uptime in one hour. So, that means, out of 60 minutes 20 minutes it is operating 40 minutes it is idle, it has experienced failure rate of 0.01 failures per operating hour. So, failure rate in operating case is 0.01 failure rate for idle a dormant is generally dormant failure rate is very, very low, it is so low many times we can ignore it, if we drop it to 0 then from our calculation, this portion which we are having for the idle failure rate, if it is 0 failure rate is 0 this will become 0.

Now, similarly, since one switching is happening in 60 minutes. So, now, my lambda effective I can calculate that is 40 divided by 60 from the 60 minutes cycle 40 divided by 60 into 0.002 plus 20 divided by 60 into 0.01 and the failure probability for each switching is how much 0.03, so, 0.03 divided by 60 will give me the probability of failure rate due to the switching operation. So, my effective failure rate comes out to be 0.003967 I want to know the probability of failure that compression works reliability for 24 hours.

So, per hour failure rate is this is per hour failure per hour I want to know the reliability for 24 hours so, this will become e to the power minus this effective failure rate which we got multiply by 24 and my reliability comes out to be 0.9092. So, similarly, for any other device like bulb electric bulb or any other device which you are having you can calculate if you know there is a failure reason which is for switching failure regions are different and for normal operation is different and for non-operating region.

There is different reasons for the failure and if you know the failure rates and if you know the probability of switching failure, you will be able to calculate this generally this switching failure is also called on demand failure probability. This is very much applicable for these compressors, motors or even our engines even bikes etcetera, whenever we start them.

So, what happens they do not start there is a starting failure why because for starting they need some excessive current they need some excessive force and because of that, there is a chance of failure, some other circuitry is also involved some time for the starting purpose which is different than the usual circuitry. So, starting failure probabilities may be different than the usual working probability. The same way is applicable for our motor this pumps etcetera,

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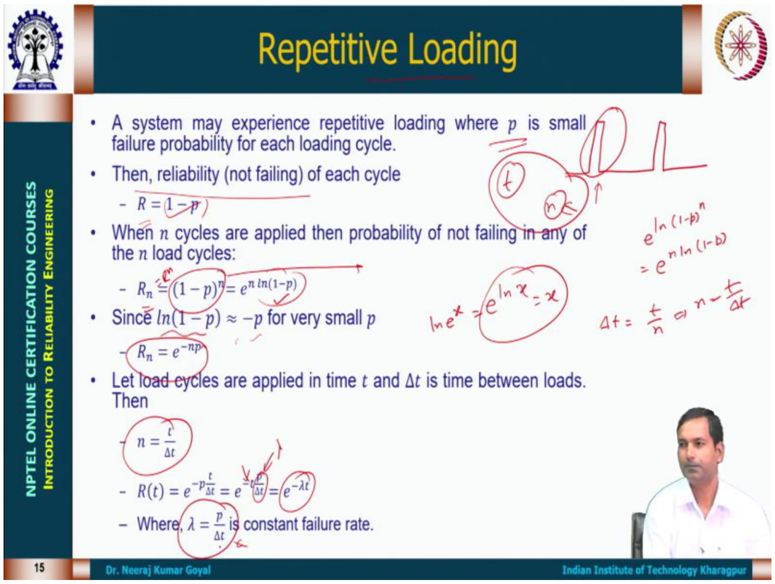

You will see this repetitive loading, repetitive loading, what does it mean? That means, there is some operation which is carried out like instantaneously let us say punch machine is there. So, it will just punch and go punch and go. So, each time one load cycle will be applied. So, here let us say, but at some moment it may fail to punch because of the load cycle. So, a system may experience repetitive loading.

And let us say the probability of failure in each cycle, each load cycle here it does not matter, during non-operating time does nothing matter and it is not like switching and working for a longer period. It is just one time operation. So it is kind of switching just it just goes and do the work and goes off. Let us see the probability of one cycle, the cycle may be longer also, but generally it will be not so high, in one cycle the failure probability is p. So, if I am interested in reliability, then the probability that it will not be failing in one cycle that is reliability that is equal to 1 minus p probability of failure is p.

So for reliability is 1 minus p. Now, let us say n such cycles are applied in a given time t in small time T I am applying n number of cycles. So, what does it have what will happen in this case, in none of the cycles that for the system to work and sustained n number of cycles, it should not fail in any n number of cycle. So, reliability for n number of cycle will be that it is not failing in any, so, multiplication of this value or that is equal to r to the power n and what is r? That is 1 minus p raised to the power n.

Now, this I can write this value if I take exponential and log, exponential and log of x is equal to x for any value x if I take exponential log or similarly, if I take ln of e to the power x both are equal to x. Here I am taking the this part that x is equal to e to the power ln of x, what is x here? 1 minus p to the power n. So, e to the power ln of 1 minus p to the power n, this I can write it as e to the power n can come out here ln 1 minus p. Now, we know that in one cycle the failure chance in such cases are very, very small the probability of failure is very, very small.

So, in that case because p is very small this ln of 1 minus p if we expand it comes out to be we can assume it equal to minus p and for minus p if I replace ln of 1 minus p with minus p if this RN will become e to the power minus np, if you see this now looks like same formula as e to the power minus lambda t, but there is no t here. So, t how we can get we can get from here, we know that n number of cycles are applied in t time. So, for each cycle how much time is there that is t divided by m.

So, delta t is the each cycle time on an average that will become t divided by n or we can say number of cycles is equal to t divided by delta t from there I can get n is equal to t divided by delta t. So, reliability will be equal to e to the power minus, I replace n with t divided by delta t multiplied by the p. Now, here what I will do t I will takeout and I will write it as p upon lambda t.

Now, if you see here t has come here and this I can write it as the lambda probability per unit time is lambda this becomes e to the minus lambda t. So, as we see that repetitive loading case can also be converted into a exponential distribution where the failure rate is the parameter is nothing but the probability of failure per unit cycle time.

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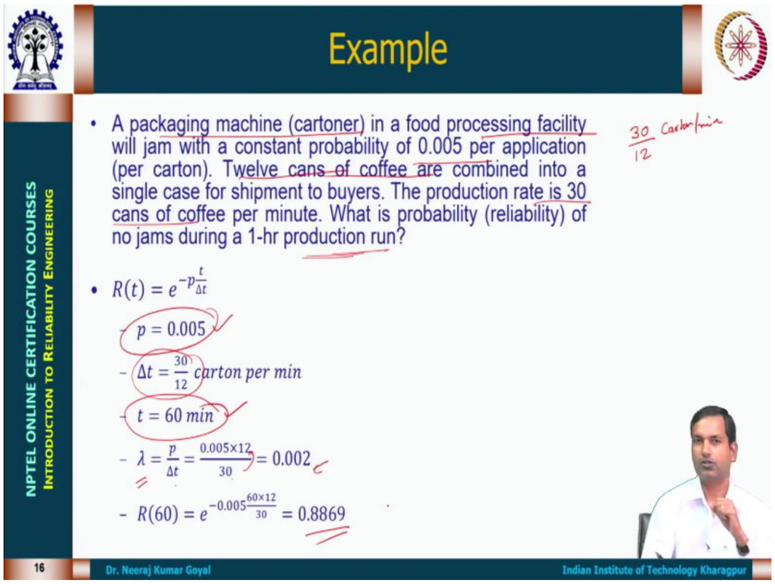

Let us take one example that there is a packaging machine or carton or is it is doing the carton packing box in a in a food processing facility. Let us say we have the chips packet or we have the cake packet or we have apple packet like that. So, in one carton we are having 12 cans of coffee it is already given coffee cans are there, do you know this cartooning machine can fail with a probability of 0.005 per application that means in making one carton, the failure probabilities 0.005.

This carton is having 12 cans so, and production rate is 30 cans. So, that means, how many cartons will be there in 1 minute that is 13 divided by 12, in 1 minute 30 cans are produced and 12 goes to the carton so, that number of cartons will be 30 divided by 12. So, that is my rate of carton production. And so here my p probability of failure for each carton is 0.005 and my frequencies 30 divided by carton per minute and I am talking about 1 hour period run, production run.

$$R(t) = e^{-p\frac{t}{\Delta t}}$$
$$p = 0.005$$
$$\Delta t = \frac{30}{12} \text{ carton per min}$$
$$t = 60 \text{ min}$$
$$\lambda = \frac{p}{\Delta t} = \frac{0.005 \times 12}{30} = 0.002$$
$$R(60) = e^{-0.005\frac{60 \times 12}{30}} = 0.8869$$

So, in 1 hour production, how many times carton will be made that is 60 divided by so, T becomes 60. So, my lambda will be equal to p divided by delta t that is 0.005 divided by delta t that is 30 by 12, so, divided by 30 then divided by 12, that becomes 0.002. I want to know the reliability for 60 minutes, so, reliability for 60 minutes will be e to the power minus 0.005 into 12 by 30 into 60 minutes that comes out to be 0.8869.

So, using this as we have seen that with the approximation to exponential distribution, we are able to solve problems for these kinds of systems which are not directly following exponential distribution, but they have the instant failure rates and these instant failure rates we are able to convert into a exponential distribution, and it helps us to calculate the values very easily. So, we will stop it here and continue our discussion in the next lecture. Thank you.