Introduction to Reliability Engineering

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Lecture 04

Probability Basics

Hello, everyone. So, this is lecture number 4 of our series on Introduction to Reliability Engineering, We earlier discussed about various terms and definitions and their formulas, which are used for; used in reliability engineering. As we have seen that most of the terms are measured in terms of probability.

Therefore, we have to understand and refresh our probability concepts. So, here, we will be going through basic probability concepts which you might have studied your secondary education or your B.Tech also. Same thing will be refreshed here because these are the terms which we will be using when we are discussing other things.

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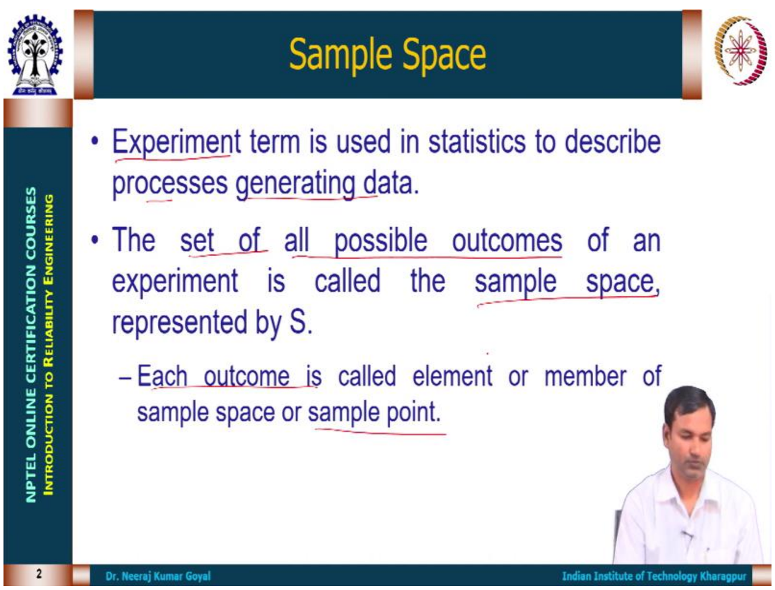

We will start with sample space. Sample space, if you want to understand sample space, first is a, there is a common term, which is used as experiment. So what is experiment? Experiment is used in statistics, which is describing the process which is useful generating the data. So, experiment is a process by which we are generating the data.

Now, this data is like a various outcomes, possible outcomes from the experiment. And all pos, set of all possible outcomes is called sample space. That means anything which is possible when we are doing the experiment is covered in sample space. Every outcome in the sample space itself is called the sample point.

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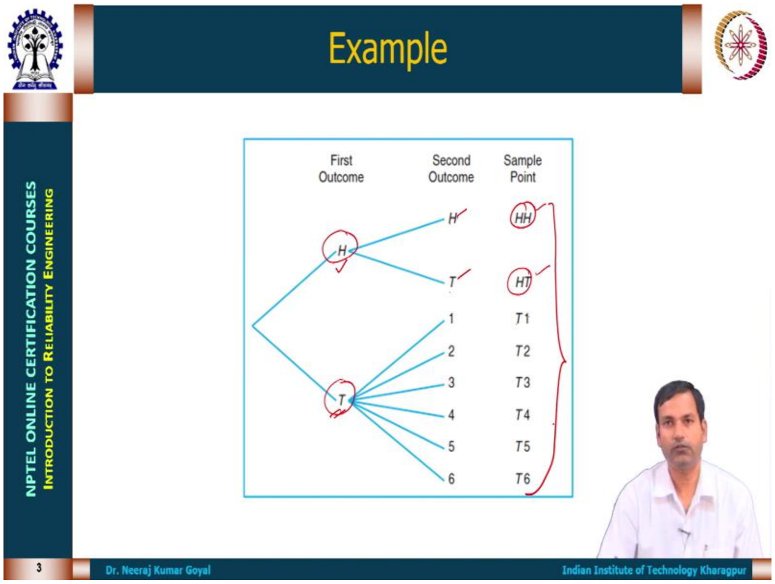

Like, if we take an example, we are tossing a coin and we are rolling a dice, sorry, yeah. This is throwing a dice, this is tossing a coin. So when we toss a coin, there are two possible outcomes, head or tail. Similarly, when we roll a dice, we have six possible outcomes, 1 to 6. Now, we are doing this together.

Since, we are doing this together, now we have the two outcomes. I, one outcome is on coin, one outcome is on dice. So, possible outcomes can be head, sorry, the sample points here can be head and sorry, I am sorry here. Here, actually this is an experiment which we are doing in a way where we are first tossing a coin.

If coin toss gives a head, then we tossed the coin again. If coin toss gives tail, then we throw the dice. So we have possible outcomes here. So the possible outcome is that first time we get head, second time we get head. First time we get head, second time we get tail. Second case is that first time we get tail.

If we get first time tail, then we have dice. So, we will have tail. Now, on the dice we have six possibilities, 1, 2, 3, 4, 5, 6. So this describes our sample space where each outcome is sample point, each outcome is head and head, head and tail, tail and 1. All these outcomes are sample point, and set of all these points is called sample space.

Similarly, whatever experiment we are doing, we will have all possibilities. All possibilities when we list out, each possibility we are calling as a sample point, and all, set of all possibilities we are calling as sample space.

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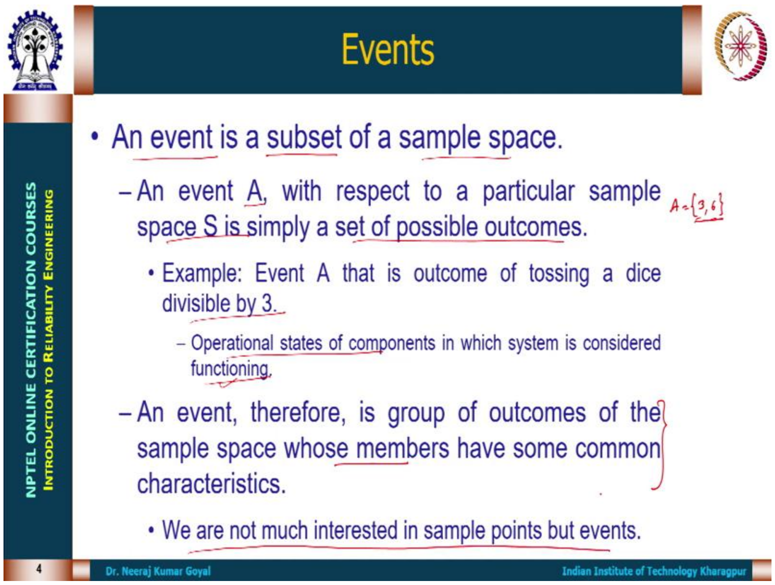

But we are more interested in event. We are not, less interested in sample points, we are interested in events. What is the event? Event is set, subset of sample space. So, it may be complete sample space, it may be null or it may be a partial set of the sample space. Any event A with respect to a particular sample space is simply set of possible outcomes.

Like, if we say, event A, that is outcome of tossing a dice which is divisible by 2. So, we have two possible outcomes here, 3 and 6. So here, event A means 3 and 6, two outcomes are there. So two sample points are there, but they together have a meaning. What is the meaning? Meaning is, they are divisible by 2. So we have the event A, which is a set of two events, 3 and 6.

And operational is, like we can have another example that operational states of component in which system is considered functioning. So, there can be various states of the components in which the system, in some states the system may be functioning, in some states system maybe considered as a failure state.

So, the states where, states of components where system is considered to be functioning, that can be one event. Another events can be failure events. Therefore, what is an event? It is a group of outcome of the sample space. And members of these, like these sample points have some common characteristics.

So, here, whenever, we are saying an event, at that moment, we have to see that there is some meaning to it. We cannot call anything as an event. To call it an event, we should have some purpose, some meaning assigned to it. So, we are interested in events, not in samples, sample points.

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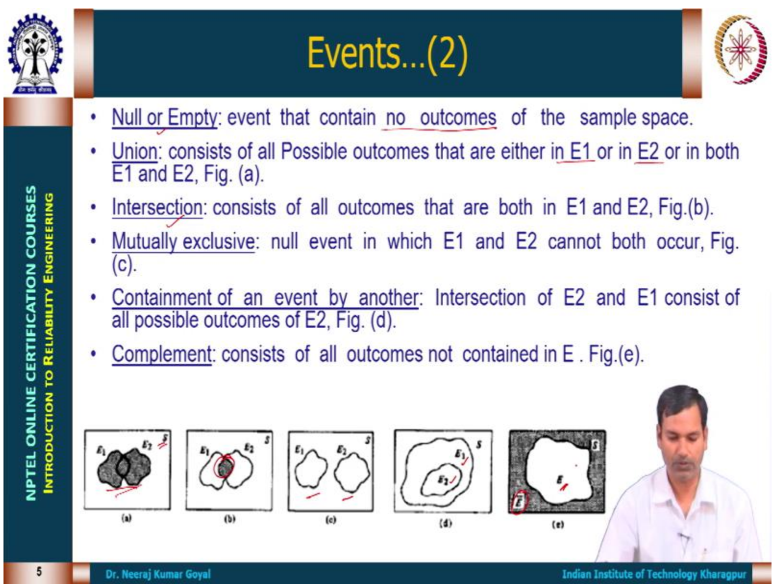

Now, events when we have, as we see that events will have different sets. So they, but they are subset of sample space. So, in sample space S, like we have event E1 here, E2 here, like same way. So, there are various possibilities. Like, a set can be null set or empty set. So an event which is not having any outcome belonging to the sample space, then it is having the null set.

Then, we have the union set. Union set for two events or more events. So for two events, we are showing it here, like two events, E1 and E2 are there. If we take union of this, then all possibilities covered in E1 and E2, all together we are calling us union. So here, the outcomes which are there in E1 outcomes which are there in E2 or outcomes which are common to both, all are covered here.

While if you talk about intersection, then only the outcomes which are common to both events, they are covered in the intersection. In case of mutually exclusive, the two events are mutually exclusive when they have nothing common. So, intersection of the two event will result in a null set.

When we say containment, so like here, E2 is contained in E1. So all events or all outcomes given in E1, E2 are already part of E1. So E2 is contained in E1. Complement event is in sample space, everything outside the event is the complement event. So event E is here, event E bar is everything in sample space which is not there in E. That is our complement event.

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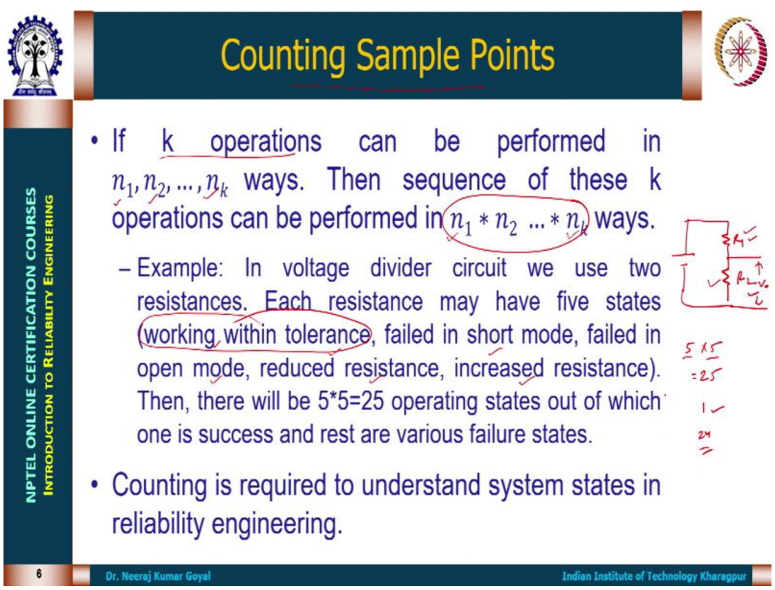

Here, as we go through, we will see that we have various states, we have various possibilities. Now, counting of these possibilities and states is required whenever we are going to study reliability engineering. So, that is why we will be briefly discuss various methods which are used to understand how can we count the total sample points or assess the sample space.

So, here, let us say if we have key operations, which can be, and each operation can be performed in different, different ways. Like, first operation can be performed in n 1 ways, second can be in n 2 ways, like kth operation can be in n k ways. Then, if we do these operations one by one, then how many ways the operation, total operation can be performed is, first can be n 1, second can be in n 2, and k can be in n k.

So, if we multiply these possibilities, then the possible ways or this sequence of event operations can be performed is n 1 into n 2 into till n k. Like, if we take an example, in voltage divider circuit, we use two resistances. Like voltage divider look like generally, we may have like this, r 1, r 2.

So, here we have two resistances. Each resistance, let us say, have the five work, five states. In five states, we have one working estate. Then working with tolerance. Within tolerance, that is our working estate, failed in short mode, failed in open mode, reduced resistance, increased resistance.

Now, as we see here, this is the only state where our system will work properly. In all other cases, the system will not be able to give us the proper output here. The output will not be having the proper division as we require. So, how many states are there? Both resistance can have 5-5 states. So, total possibilities are 25.

Like, working with first one working second one working, first one working second, failed in short mode, first one working second failed in open mode. Like this. So, we have 5 into 5 possibilities. But out of which, there is only one possibility in which the system will work. And that possibility is when r 1 is working within tolerance and r 2 is also working with intolerance.

When any one of them is in work, is in any other state, it will result in a failure. So it has one success state and it has 24 failure states. So, we need this counting process to understand that how many states will be there and in what ways we will come to know which are the failures cases and which other success cases.

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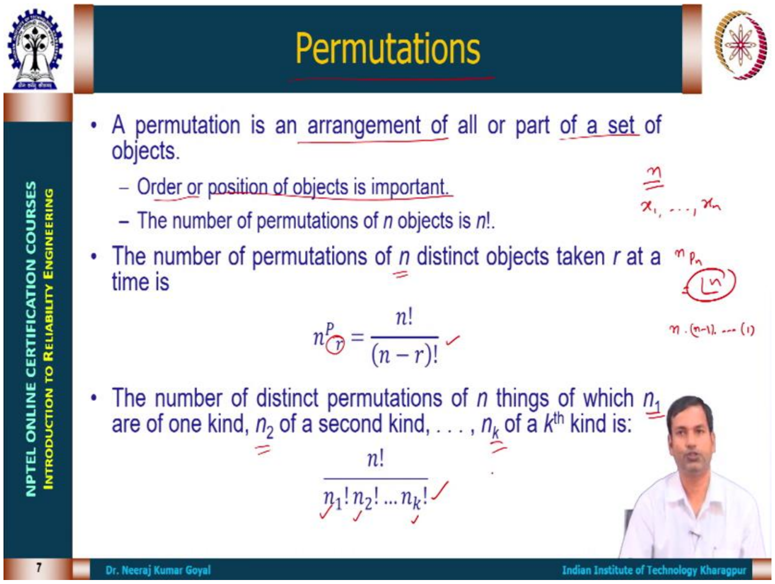

The way of doing counting, first one is permutation. Permutation is used for arranging, that whenever either we have all possibilities arranged in a sequence or we have the part of the set we are arranging in sequence. Here, the order of positioning is important. So which happens first, which happens second, like that, that is important.

So, we have n different objects here, like we can have x 1 and x n, different items. Now, all are different, they are all distinct. So if you want to know the permutation of this, then permutation of this, n out of n if we take, then this is going to be factorial n. Like, first place can be filled n ways, second filled, can be filled in n minus 1 ways. Same way, till 1. That is our factorial n.

Similarly, but if we want to pick up some items from here, let us say we pick up r number of items from here, then those r number of items, we want to arrange. So, such arrangements will be out of factorial n, n minus r combinations will be gone. Only factorial n, so factorial n divided by factorial n minus r will be the permutations of selecting r and arranging them from n number of items.

Similarly, but these are distinct items. Sometimes, the items may not be distinct. Sometimes, items can be similar. So if n 1 is of one kind, n 2 is of second kind and n k of nth kind, and if you are arranging all of them, then the total number of possibilities will be factorial n divided by factorial n 1, factorial n 2. So, these are the number of combinations which we are losing because of the similarity.

So, total combinations, which if they were distinct it was factorial n, but since they were sim, there were groups similars, similar groups, those similar groups we divide. If there is no group, then it will become 1. So factorial 1 will be 1, and this will make no change.

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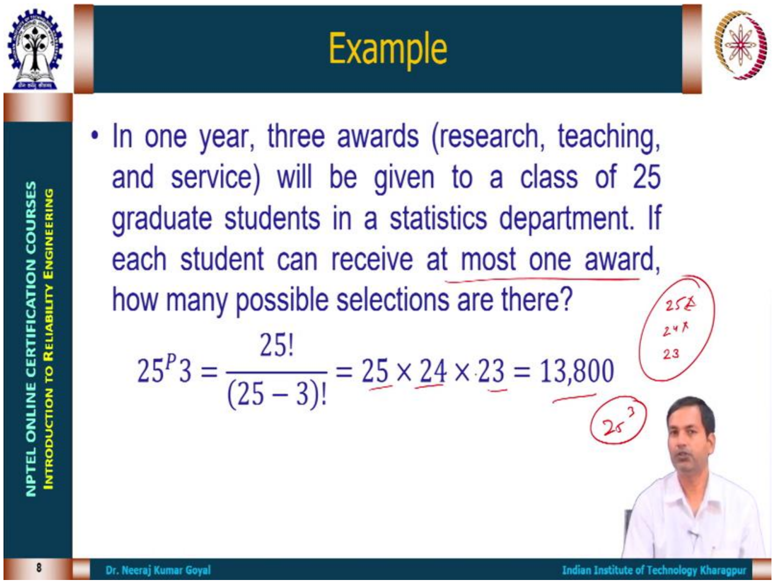

If we take an example that in one year, three awards, one for research, one for teaching, and one for service is given to a class of 25 graduate students in a statistics department. If each student can receive at most one award, then how many possible selections are there? Now, since one person can receive only one award, so there are three pos, three sequence, 1, 2, 3. So, we have 25 P 3 possible permutation.

Like, first award can be given to 25 pupil, but one of them will receive the award. So, second award can be given only to 24 people, and third award can be only given from one of the third 23 people. And this gives us the number of per mutations, 25 into 24 into 23. That is 13,000.

If there was no condition that we cannot give it to, one award to one person, if this condition was not valid, then it would have been 25 raised to power 3 because anyone can, could be given any award. So same person can be given, then there will be 25, 25, 25 possibilities.

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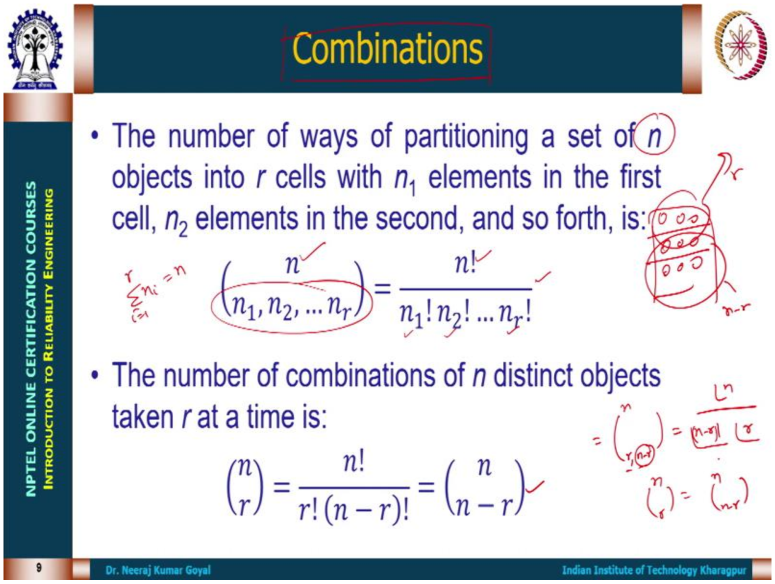

But we, many times and more frequently used combination. Combinations, what we are trying to do, we have n set of elements. Like, we have n elements here. Now, out of these, n elements, we want to group them in separate groups. We want to partition them into groups. So, let us say we are partitioning them in n 1, n 2, n r groups.

And if you see that submission of n i here, i equal to 1 to r will be equal to n. The whole group is, the n number of items are divided into different, different groups. Then, how many possible ways it can be divided into groups, that is factorial n divided by factorial n 1, factorial n 2 till factorial n r. If you see, this is the same formula as we have used for the permutation, because in a group, items are considered to be same.

So, here, this comes the answer. But most of the time, we are dealing with two states or we are dealing with two possible groups, like failure and success. So, in that case, we can say, if we have two groups, then we have n C, if let us say we take r item out of it. So, if we take r items out of it, either take it out or separate it out, then how many items are remaining? That is n minus r. So we have r n minus r.

This will be equal to factorial n divided by factorial n minus r. I will use this factorial sign, factorial r. Same formula is here. So, this is given, n C, but here writing the two things is redundant because if we write n C r or if we write n C n minus r, that is enough because second item becomes obvious. So n C r is only written.

Rather than writing two terms, we write only one term. If you know, n C r is equal to n C n minus r, because this is dividing in two group. Whether we take r item or whether we take n minus r item, it will give you the same two groups.

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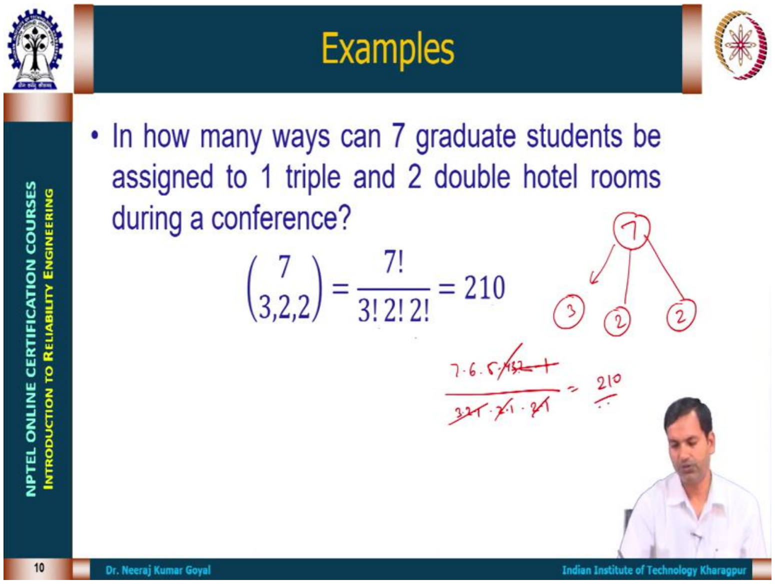

So how many ways or 7 graduate students can be assigned to 1 triple 2 double hostel rooms? So we have 7 pupil, out of 7 people, we have to choose 3 persons for triple room, 2 persons for double room, first double room, 2 persons for second double room.

So, we are dividing the 7 into three parts, 3, 2, 2. So 7, 3, 2, 2. So, 7, 3, 2, 2, factorial 7 divided by factorial 3, factorial 2, factorial 2. This comes out to be 7, 6, 5, 4, 2, 1. And this is 3, 2, 1. This is 2 1, this is 2 1. So we have 3 2 1 5 4 3, I, and then 2 into 2 is 4, so 7 6 5, 5 sixes are 30, 7 threes are 21, 2 1 0. We get this.

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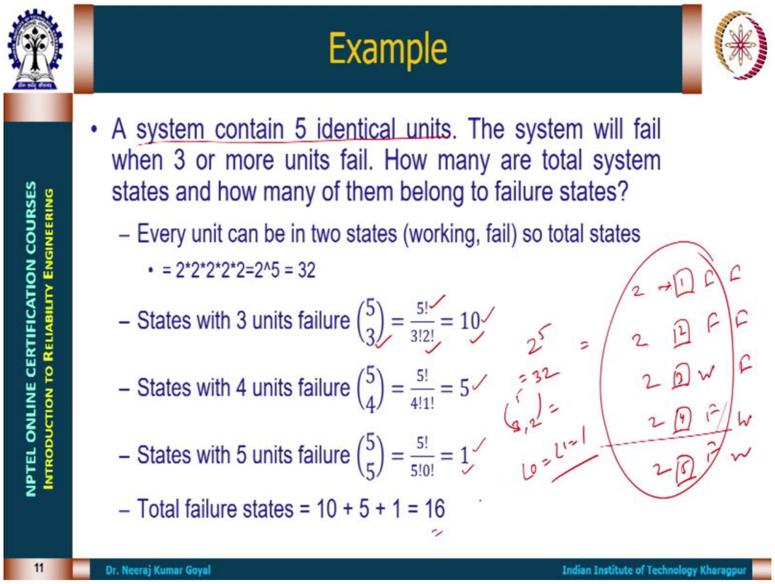

Now, if we fit it in another example, let us say a system is there, which is having 5 identical units. Unit 1, Unit 2, Unit 3, Unit 4, Unit 5. All are identical. Identical means they are coming from same process or they are looking, or they are similar in that nature. Now, the system will fail when 3 or more unit fail.

Now, any 3 units, if it fails or 4 units fails or 5 unit fails, in that case, the system will be in failed condition. So the question is how many total system states are there? How many total system streets are there? Each system can be 2 states, either fail or success. So 2 2 2 2 2, so that is equal to 2 raised to the power 5, that is 32.

That is my total system states. What are the system states with, what are the system states? System is all 5. So what are the system states in which 3 units are failing, exactly 3 units are failing? That means 3 failing and 2 working, that means 3, like this is in failed, this is in failed, this is in failed, this is working, this is working. Another can be fail, fail, this can be working, this can be failed.

So, possible combination is that means out of the 5 device I am choosing 3 for failure, I am choosing 2 for success. So I can say 5 C 3 or 5 C 2. That will be called to factorial 5 divided by factorial 3 factorial 2. This comes out to be 10. Similarly, how many devices states will be there with 4 unit failure? That means I am partitioning this into 4 and 1. So, I select 4 out of the 5, so 5 C 4. That means factorial 5 factorial 4 4 1, that is 5.

And states with 5 units failure, so 5 units means all failed. So, that is only 1 case in which all failed. That is 5 C 5, 5 factorial 5 divided by factorial 5, that gives 1. We know factorial 0 and factorial 1, both are equal to 1. And so, total failure states becomes, in this also it is fail, in this also it is fail, in this also it is fail. So, we get 10 plus 5 plus 1, that is 16. So, this is how we are able to understand that in reliability, how we get the failure states and success states.

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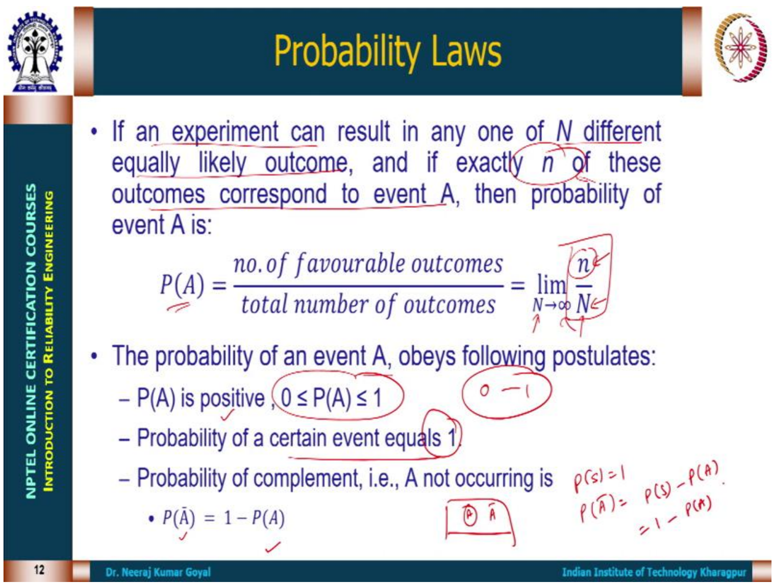

Now, we try to understand the basic probability. Whenever we are doing an experiment, we know that there are N different likely outcome, sample space we can say. But we are interested in an event, and that event is having n posi, n outcomes. And these n outcomes are corresponding to the event A. So we are interested in that. What is the probability?

So, the probability is number of times event A occurs divided by total number of outcomes. So, and the same thing when we take it into the data point, that is observations point of view. In that case, if we say N is the total observations and small n is the observations in which event A is true.

In that case, the probability of event A is n, n is the number of times event A occurred divided by total number of events or total number of trials which are done, that is capital N. But here, this will be a biased estimate if we are not able to do this experiment or trial sufficient number of time. So, this number of trials should be done to a very, very large amount of time, that is N intending to infinity.

In that case, this gives us a true assessment or true estimate of the probability of event A. The probability of event A obeys following postulates. Probability is always a positive quantity, and probability is always a value from 0 to 1. It is a proportion. So, proportion is 0 to 1, it can be maximum 1, minimum 0.

Probability of certain event equals 0. If any event is certain, we are sure about it, there is no doubt about it, this has happened already, then this property is equal to 1. Probability of complement event, that is Probability of A bar is equal to 1 minus P(A). As we saw earlier, this is A, and this is A bar.

So, Probability of sample space is 1. So, probability of A bar will be called to, now, total space is S, that is Probability of space, sample space minus probability of A. And we know Probability of sample space is 1. So, this will become Probability 1 minus P(A).

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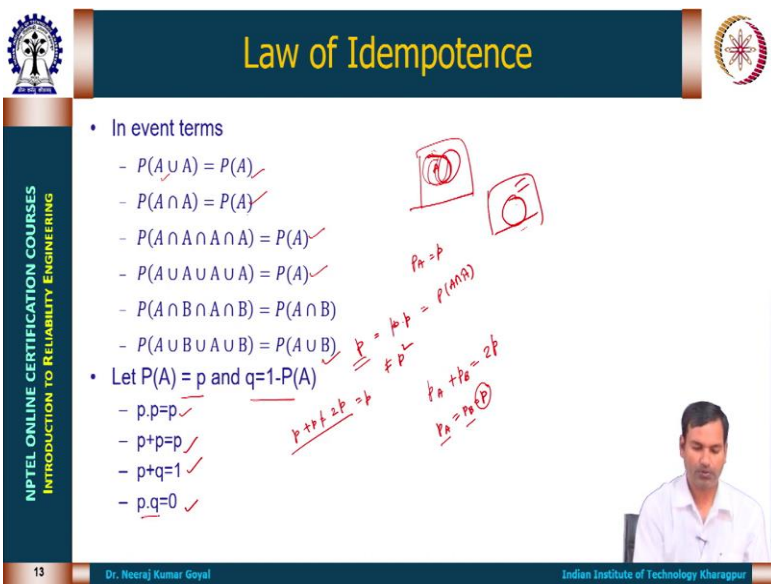

If we look into Idempotence Law, Idempotence Law is also widely used. We know if we have an event A, so if we take union of A with A, we get only A. There is no change in the A. So, if we take union of A with A, the event does not change, event remains same. Similarly, if we take intersection of A with A, the event does not change, it remains A.

If you take A intersection multiple times also it will remain A, union multiple times also it will remain A. If I take A intersection B, then again if I intersect with A or again intersect with B, it is not going to change. It will remain A intersect B only. So intersect B intersect A intersect B will remain A intersect B. We can say A intersect A is A, B intersect B is B. So, this remains B A intersection B. Similar is applicable to the union.

Now, if we say that probability of A is p and probability of A bar is q. In that case, if we say P(A), that is we are saying p, so p into p, p into p means probability of A intersection with A, this is always going to be p, it is not going to be, it is not equal to p square. Why? Because this does not get shortened. It remains A only. So p into p does not give p square, p into p gives p only.

Similarly, if we take union of that P(A) again and again, then p plus p will not give, will not be equal to 2P. It will be equal to p only. Many times, there can be confusion. Many times, let us say there are two events, P(A) and P(B). But P(A) is equal to P(B) is equal to P. Both are, let us say, we are using two register, both are same. So, probability of suc, reliability or probability of success will be same for both, that is, let us say, p.

In that case, this will become 2 p because this is another event, this is another event. This is not same event. So, we have to be cautious here that same event, whenever it is there, we have to solve first and we have to ensure that event level, Idempotence Law is applied. So p plus p is p, p plus q is 1, p dot q is 0. p plus q means A happens and A bar.

So, that means complete sample space. So, it is 1. p dot q, there is nothing common in p and q, A and A bar. So, intersection probability is always going to be 0.

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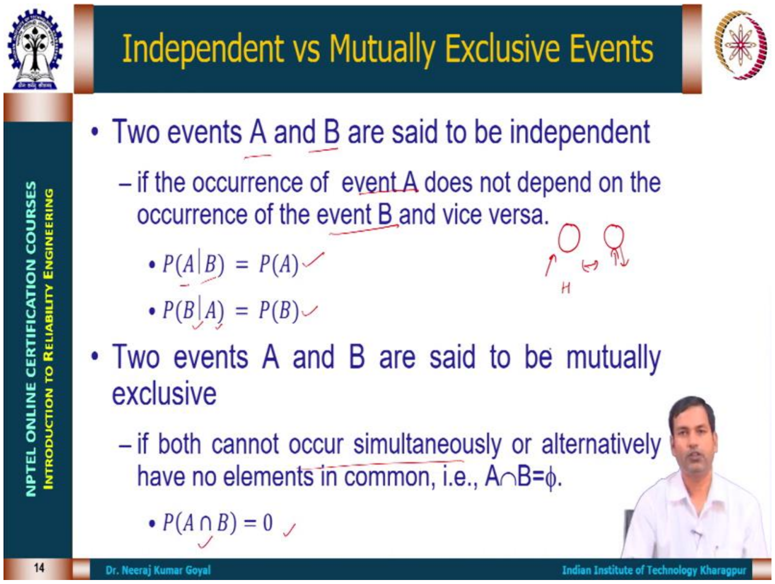

There are two types of concepts which comes into the picture, one is independent events and others is mutually exclusive event. Two events A and B are set to be independent if they, if occurrence of event A does not depend on the occurrence of B. That means, event A and B do not influence each other. They are independent of each other.

In that case, probability of occurrence of A given B has occurred is P(A). Similarly, priority of occurrence of B given A has occurred is P(B). They do not change, they remain same. Two events are set to be mutually exclusive if both cannot occur simultaneously. So like tossing a coin, so both head and tail cannot come together, but if you toss two coins, then they are independent.

What comes on this, like if head comes on this, it is not going to influence whether head will be here or tail will be here. So, the two events are independent. But for the same coin, the outcome of head or tail is mutually exclusive because on the same coin, if head comes, tail will not come, if tail comes, then head will not come. So, intersection of two or mutually exclusive event is 0 because they do not have anything in common.

Mutually exclusive events are generally dependent events because they are influencing each other. If head is there then tail will not come. So, they are independent, they are not considered independent, they are dependent.

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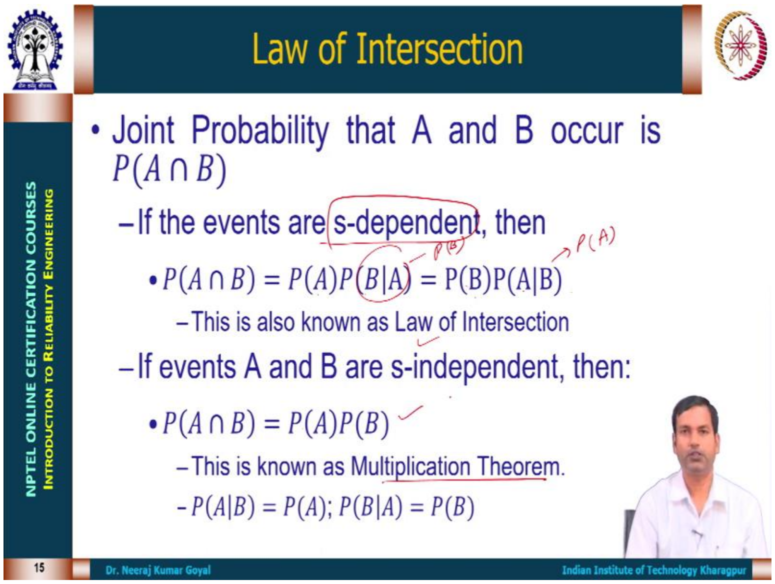

Joint probability that A and B occur, so probability of A intersection B is nothing but probability of A multiplied with probability of B given that A or probability of B multiplied with probability of A given that B, that B happens and A happens given B happens, A happens into B happens given A happens.

This is called as Law of Intersection. This is the formula when this is dependent events. If they are independent, in that case we know, probability of B given A is P(B) and probability of A given, A given B is P(A). So, in that case, this simply becomes P(A)P(B), this becomes P(B)P(A), which is same as P(A)P(B).

This is called, if there are, if we have, this is called Multiplication Theorem. So, for independent cases, the intersection is simply leading to the multiplication of probabilities.

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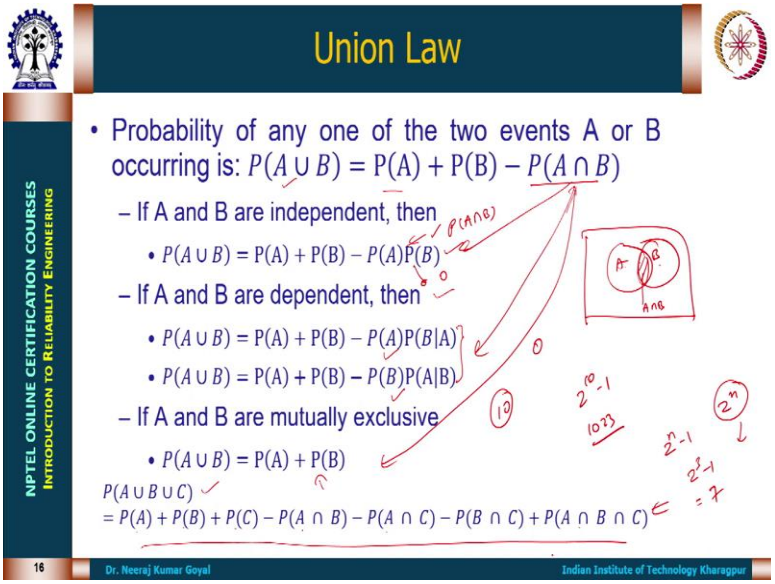

Union Law, like union of two. In union of two, there is a little difficulty here. What happens, if we have two events here, A and B, then the intersection area, A intersection B, this is counted in event A also and event B also. So, when we say probability of event A, then same outcomes are covered in A, which are common as A intersection B, and same outcomes are also part of B.

So, what will happen, same outcome is counted twice. So the improbability space, this area is counted twice, in A also and B also. So, but we have to count it only once, so we have to reduce this once. So this intersection probability is reduced from P(A) plus P(B) so that A intersection B is not counted twice.

So, this is, so in case of independent, probability of A union B is P(A) plus P(B) minus P(A) into P(B). If they are dependent, then this will be, this is actually probability of A intersection B. So, we can either calculate A, P(A) into P(B) given A or calculate as P(B) into P(A) given that B.

If both are mutually exclusive, then this will be equal to 0. So, this is for depend, independent, this is for independent, dependent, and this is for mutually exclusive. In mutually exclusive, this will be 0. And probability of A union B will be summation of the probabilities.

If I want to take three events, now in case of three events, the same formula, if I use, for independent events, then this will become P(A) plus P(B) plus P(C) minus 2 event interaction, AB, AC, BC plus three event interaction, ABC. Now, as we see here, these number of, this is, if I am having n number of events here, so that will, the combinations would be 2 to the power n minus 1.

So, like here, we have 3, so 2 to the power 3 minus 1, that is 7. If you say 1, 2, 3, 4, 5, 6, 7, because all pos, all possibilities are counted 1, 2, 3, 1 out of 3, 2 out of 3 here, only possibility which is not counted is absence of all. So, possible com, total possible combinations is 2 to the power n, out of which one, only possible combination we are not counting here.

This creates a problem. This formula creates a problem because if we, when we are having, let us 10 items also, then this will become 2 to the power 10 minus 1, almost 1023. These many combinations, we have to evaluate. This is becoming cumbersome. We will see that how that can be addressed little later.

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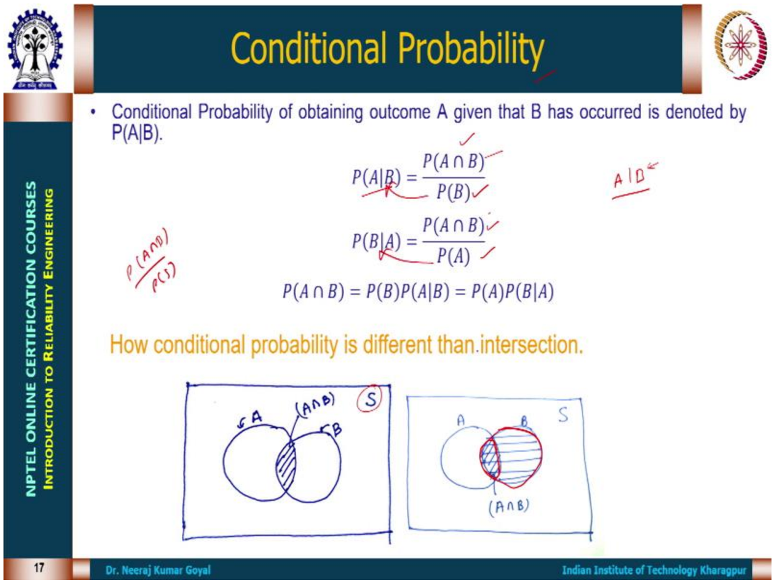

Then there is a concept of conditional probability. Conditional probability is little different than intersection. In case of conditional probability also, both the events are happening. A happening given B. That means given that event B has happened, what is the probability that B will hap, A will happen?

Now, this property, when we are evaluating, the problem is in this case A intersection B also, both the events are happening. But their reference point is changed. Whenever we say intersection, in that case, out of the sample space, that is P A intersection B divided by probability of sample space.

But when we say this is conditional probability, then B event has already happened. So, our sample space is reduced to only B. So, our sample space probability was 1, but now our samples space has changed to probability B. So, we have to divide by probability of B, because out of the B, what is the priority that A intersection B happens? That is the probability of A given B.

So, given that B has happened, the probability of A is only limited to this area. That is A intersection B. So probability of A intersection B given divided by probability B gives A given B. Similarly, B given A means given that A has happened, out of that, what is the priority B will happen? That is the intersection area only, intersection divided by P(A). So, P A intersection B, we can calculate as from here like, multiplication of this or this, multiplied by this.

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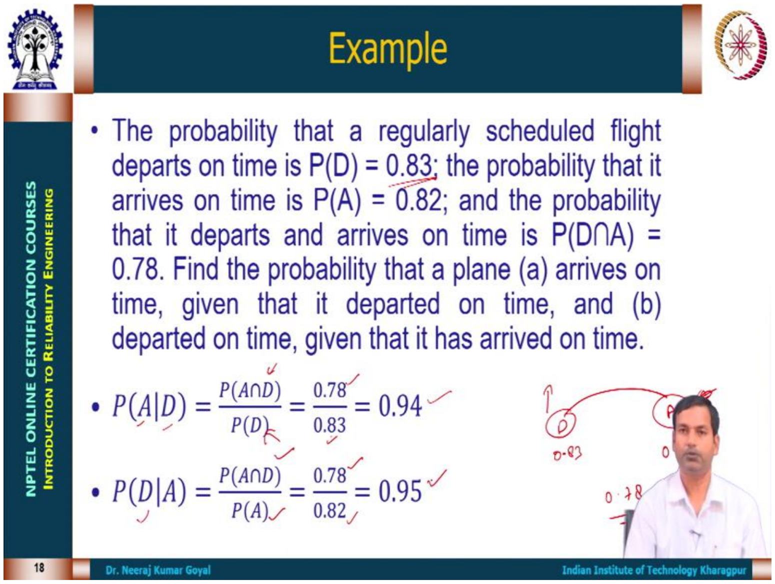

For an example, the probability that a regularly scheduled flight, like we have a flight, we have departure and arrival. So, flight departure, probability that flight depart as per the scheduled correct time is 0.83. Flight arrives at the same time, or correct time, is 0.82. Probability that flight is departing and arriving on time is 0.78. That means, it is departing as well as arriving on the same time, that is 0.78.

Now, I am interested to know that the plane which is arriving on time, like I am here, my flight has arrived on time. I want to know what is the probability that it is arriving on time. That means, I know it has departed on time. I want to know what is the probability it is arriving on time.

So, probability of arriving on time given it has departed on time, that means out of all correct departure, how many times it has arrived correctly. So, correct departures has been 0.83, and how many times it has arrived when correct departure is there, that is 0.78. So 0.78 divided by 0.83 gives me, that means 94% possibility is there that if a flight which has departed on time, it will arrive on time.

Similarly, I can evaluate the probability that what is the probability it will, it has been departed on time given that it has arrived on time. That is, probability of A intersection D divided by probability of A, that is 0.78 divided by 0.82. That comes out to be 0.95.

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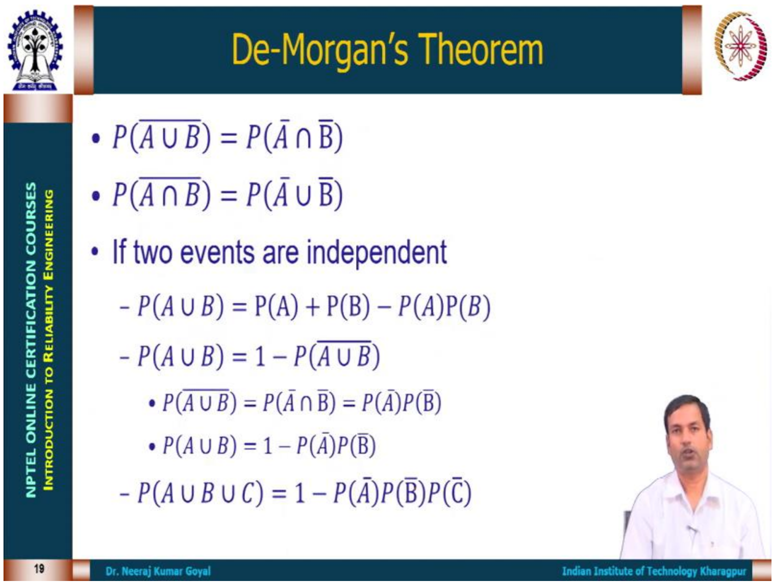

I think we will stop it here, and we will continue this discussion to the next lecture.