



Introduction to Reliability Engineering
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Lecture 39
Maintainability and Availability (Continued)

Hello everyone. So, in previous class we were discussing about availability and steady state availability, in particular. Let us continue our discussion with the same what we were discussing last time.

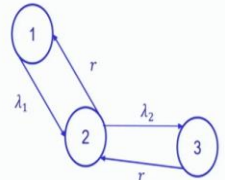
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Steady State Availability of Two Component Standby System



- For a system with single backup unit with repair permitted to either component (same repair rate, one unit at a time can be repaired) and no failures in standby mode, then steady state availability is evaluated as follows:



$$\frac{dP_1(t)}{dt} = -\lambda_1 P_1(t) + r P_2(t)$$

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) + r P_3(t) - (\lambda_2 + r) P_2(t)$$

$$P_1(t) + P_2(t) + P_3(t) = 1$$

In Steady State:

$$\lim_{t \rightarrow \infty} \frac{dP_i(t)}{dt} = 0; \checkmark$$


$$P_i(t) = P_i \checkmark$$

Therefore,

$$-\lambda_1 P_1 + r P_2 = 0 \checkmark$$

$$\lambda_1 P_1 + r P_3 - (\lambda_2 + r) P_2 = 0$$

$$P_1 + P_2 + P_3 = 1$$



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So, we were discussing this figure. In this figure, we discussed that how to evaluate this. Now, we discussed that in steady state availability that probabilities change in probability zero and probabilities are not a function of time, probabilities are individual probabilities only, constant qualities only. Now, let us see, if we want to make the Markov equations for this.

So, as we know that $\frac{dP_1(t)}{dt}$ as we discussed in Markov class $\frac{dP_1(t)}{dt}$ is whatever is incoming that is plus. So, and whatever is outgoing is minus. Outgoing is λ_1 . So, that is minus λ_1 from state P_1 , $P_1(t)$ and incoming is r from state number 2. So, plus $r P_2(t)$.

Similarly, I can write $\frac{dP_2(t)}{dt}$ equal to. Now, for state number 2, from state number 1, I am having the incoming link λ_1 . So, $\lambda_1 P_1(t)$. From state number 2 outgoing links are two, one is r here, 1 is λ_2 here. So, r plus λ_2 into $P_2(t)$. And from state

number 3, I have the incoming link plus r plus r P3 t. Third equation as we know; we will simply write P1 t plus P2 t plus P3 t equal to 1.

Now, if you want to solve this, as we discussed, this change probabilities are 0. So, I can put this value equal to 0, this value equal to 0. So, once I put lambda 1 P1 plus r P2 equal to 0, this becomes my equation lambda 1 P1 plus r. So, this t also I have removed and this has become r lambda 1 P1 plus r P3 minus lambda 2 plus r into P2 equal to 0 and P1 plus P2 plus P3 equal to 1. So, this becomes my set of equation. If I solve this set of equations, I will be able to get the probabilities which I want.

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$$\begin{aligned}
 -\lambda_1 P_1 + r P_2 &= 0 \\
 \lambda_1 P_1 - (\lambda_2 + r) P_2 + r P_3 &= 0 \\
 P_1 + P_2 + P_3 &= 1
 \end{aligned}$$

From Eq (1) $r P_2 = \lambda_1 P_1$
 $P_2 = \frac{\lambda_1}{r} P_1$

From Eq (2) $\lambda_1 P_1 - (\lambda_2 + r) P_2 + r P_3 = 0$
 $r P_3 = (\lambda_2 + r) P_2 - \lambda_1 P_1$
 $r P_3 = \left[\frac{\lambda_1 \lambda_2 + r \lambda_1 - \lambda_1^2}{r} \right] P_1$
 $r P_3 = \left[\frac{\lambda_1 \lambda_2 + r \lambda_1 - \lambda_1^2}{r} \right] P_1$
 $P_3 = \frac{\lambda_1 \lambda_2 + r \lambda_1 - \lambda_1^2}{r^2} P_1$

$$P_1 = \left[1 + \frac{\lambda_1}{r} + \frac{\lambda_1 \lambda_2}{r^2} \right]^{-1}$$

$$P_2 = \frac{\lambda_1}{r} P_1$$

$$P_3 = \frac{\lambda_1 \lambda_2}{r^2} P_1$$

$$P_0 = \frac{\lambda_1 \lambda_2}{r^2} P_1$$

$$P_1 \left[1 + \frac{\lambda_1}{r} + \frac{\lambda_1 \lambda_2}{r^2} \right] = 1$$

$$P_1 = \frac{1}{\left[1 + \frac{\lambda_1}{r} + \frac{\lambda_1 \lambda_2}{r^2} \right]}$$

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In Steady State:

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Therefore,

$$\left. \begin{aligned}
 -\lambda_1 P_1 + r P_2 &= 0 \checkmark \\
 \lambda_1 P_1 + r P_3 - (\lambda_2 + r) P_2 &= 0 \\
 P_1 + P_2 + P_3 &= 1
 \end{aligned} \right\}$$

$$0 = \frac{dP_1(t)}{dt} = -\lambda_1 P_1(t) + r P_2(t) = 0$$

$$0 = \frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - (\lambda_2 + r) P_2(t) + r P_3(t) = 0$$

$$P_1(t) + P_2(t) + P_3(t) = 1$$

Now, to solve these equations I have put it on next slide so that I can show you how to solve this the same set of equations are here. Now, let us say I call them equation number 1, 2 and 3. So, let us take for equation number 1. So, equation number 1, I will express P2 in terms of lambda P1. So, r P2 will be equal to lambda 1 P1. So, I can write P2 is equal to lambda 1 upon r P1.

Similarly, I can now, take down from equation number 2, I can write lambda into P1 lambda 1 into P1. Lambda 2, I will convert P2 I will convert in terms of P1 minus lambda 2 plus r divided by. So, P2 will be equal to lambda 1 divided by r, will become P1 plus r P3 equal to 0. Now, again I can take P1 terms on right hand side so this will become r P3 equal to lambda 2 plus r into lambda 1.

So, this will become lambda 1 lambda 2 and this is where r lambda 1 upon r minus lambda 1 into P1. This if I solve, I will take r common. So, this will become lambda 1 lambda 2 but plus r lambda 1 minus r lambda 1 into P1. This will become lambda 1. This will cut canceled, lambda 1 lambda 2 divided by r P1. Now, this P1, P2, we have got P3, we have got P2, all in terms of P1, I can replace all these terms from in equation number 3.

So, this will become P1 plus lambda 1 upon r P1 plus lambda 1 lambda 2 upon, I made some mistake somewhere. This P3, this is r P3. So, this will become r square. So, r when it goes here this will become r square that will be r square, P1 equal to 1. So, I can take P1 as common, 1 plus lambda 1 upon r plus lambda 1 lambda 2 divided by r square will be equal to 1. So, P1 can be calculated as 1 plus lambda 1 upon r plus lambda 1 lambda 2 divided by r square whole inverse.


Once I get P1 then P2 will be lambda 1 upon r into P1 and and my availability will be equal to probability of P1, probability P2. So, P1 plus P2 will give me the availability. This I have shown here by the calculation. So, generally, if you see here, one is for the P1 and for second state what we do?

This lambda departure divided by arrival, lambda 1 upon r. Then this same lambda 1 upon r remains same. And for second state, this get multiplied with lambda 2 divided by r lambda 2 divided by r. So, that becomes lambda 1 lambda 2 upon r square. This only happens if there is no reverse state. If there has been reverse state like this then there will be problem. If there is only one to one state transition qualities like this and there is no crossover states here then this formula can be fairly used there 1 plus.


So, P1 probability is nothing but 1 upon 1 plus r upon lambda 1 plus r upon lambda 1 into, sorry, lambda 1 upon r into lambda 2 upon r. So, that will become lambda 1 lambda 2 upon r square. If I had the third state, this would have become 1 plus lambda 1 upon r plus lambda 1 lambda 2 upon r square plus lambda 1 lambda 2 lambda 3 divided by r cube, whole to the power minus 1.

And P2 would be same as this factor multiplied by P1 lambda 1 upon r P1. P3 is equal to lambda 1 lambda 2 upon r square P1. And P4 would be equal to lambda 1 lambda 2 lambda 3 upon r cube into P1. So, we can use this formula in general, but care should be there, because there should not be two straight transitions happening, one to one state only, in that case only we can have this simplification. But better is you developed, do not use that kind of formula. You first develop a set of equations and then try to solve them using this formula.

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Example




- A two component standby system has the following parameters $\lambda_1 = 0.002$, $\lambda_2 = 0.001$, and $r = 0.01$. Find steady state availability.

$$P_1 = \left[1 + \frac{0.002}{0.01} + \frac{0.002 \times 0.001}{0.01^2} \right]^{-1} = 0.8196$$

$$P_2 = \frac{0.002}{0.01} P_1 = 0.1639$$

$$A = P_1 + P_2 = 0.9836$$

$\left[1 + \frac{0.002}{0.01} + \frac{0.002 \times 0.001}{0.01 \times 0.01} \right]^{-1}$



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$$P_1 + P_2 + P_3 = 1$$

Handwritten notes on the left:

$$0 = \frac{dP_1(t)}{dt} = -\lambda_1 P_1(t) + r P_2(t) = 0$$

$$0 = \frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - (\lambda_2 + r) P_2(t) + r P_3(t) = 0$$

$$P_1(t) + P_2(t) + P_3(t) = 1$$

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Handwritten derivations for steady state probabilities:

$$-\lambda_1 P_1 + r P_2 = 0 \quad \text{--- (1)}$$

$$\lambda_1 P_1 - (\lambda_2 + r) P_2 + r P_3 = 0 \quad \text{--- (2)}$$

$$P_1 + P_2 + P_3 = 1 \quad \text{--- (3)}$$

From Eq (1):

$$P_2 = \frac{\lambda_1}{r} P_1$$

From Eq (2):

$$\lambda_1 P_1 - \frac{(\lambda_2 + r) \lambda_1}{r} P_1 + r P_3 = 0$$

$$2x P_3 = \left[\frac{\lambda_1 \lambda_2 + r \lambda_1}{r} - \lambda_1 \right] P_1$$

$$2x P_3 = \left[\frac{\lambda_1 \lambda_2 + r \lambda_1 - \lambda_1 r}{r} \right] P_1$$

$$P_3 = \frac{\lambda_1 \lambda_2}{2r^2} P_1$$

From Eq (3):

$$P_1 + \frac{\lambda_1}{r} P_1 + \frac{\lambda_1 \lambda_2}{2r^2} P_1 = 1$$

$$P_1 \left[1 + \frac{\lambda_1}{r} + \frac{\lambda_1 \lambda_2}{2r^2} \right] = 1$$

$$P_1 = \frac{1}{\left[1 + \frac{\lambda_1}{r} + \frac{\lambda_1 \lambda_2}{2r^2} \right]}$$

Matrix form solution:

$$P_i = \left[1 + \frac{\lambda_1}{r} + \frac{\lambda_1 \lambda_2}{2r^2} \right]^{-1}$$

$$P_2 = \frac{\lambda_1}{r} P_1$$

$$P_3 = \frac{\lambda_1 \lambda_2}{2r^2} P_1$$

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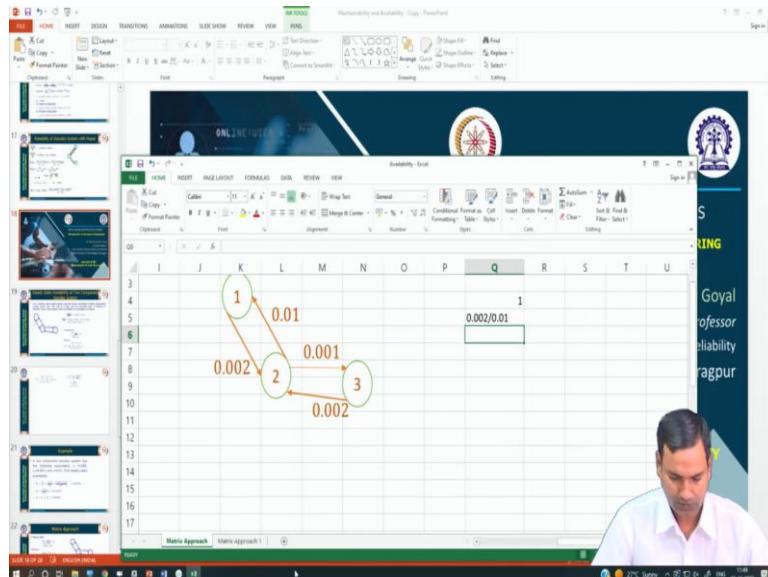
So, once I have developed this formula I can now solve this equation. Generally. So, this is the case when I am talking about the single repair person. So, in that case, the my P1 is equal to same formula I can use 0.002 divide, my failure rate is for lambda 1 is. So, I will use 1 plus 0.002 divided by r, that is 0.01 plus lambda 1 lambda 2 that means 0.002 into 0.001 divided by r square that is 0.01 into 0.01 and whole inverse. Whatever I calculate.

So, I take one upon of that. That will become inverse that comes out to be 0.8196. And P2 will be equal to this value multiplied by this. And P3 will be equal to this multiplied by this but I do not need P3. For calculation of availability, I need only P 1 and P2. P3 I can also calculate as 1 minus of P1 plus P2 because P1 plus P2 plus P3 is equal to 1.

Availability is summation of this 0.8196 plus 0.1639. And I will get 0.9836. So, these values can calculate, there are other methods which I am going to discuss which can be used for calculating the same value, the steady state availability. Before going for there, let us that also I will try to show that if I have used, let us say $2r$ here, rather than r if I have used $2r$ here.

The change would have been, this would have become $2r$ square. That would have been the only change $\lambda_1 \lambda_2$. Because this would have been here, as we calculated, this would have been $2r P_3$. This will be $2r P_3$. So, because of that this will become $2r P_3$ and this will become $2r$ square. So, my change would have been here. And in that case, if I want to calculate availability, I can calculate the same again, the only change would be this will become multiplied by 2. So, what I am trying to do?

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Example



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$$A = P_1 + P_2 = 0.9836$$

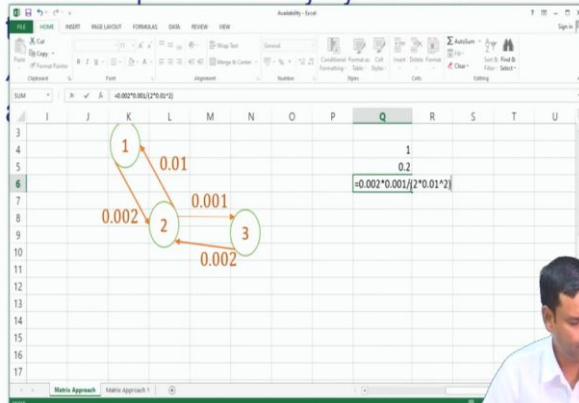
$$\left[1 + \frac{0.002}{0.01} + \frac{0.002 \times 0.001}{2 \times 0.01 \times 0.01} \right]^{-1}$$



Example



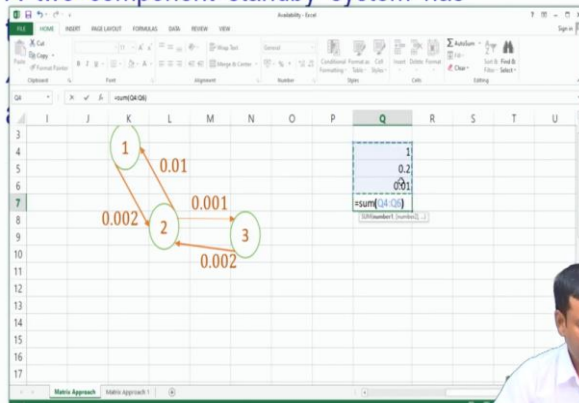
- A two component standby system has



Example



- A two component standby system has

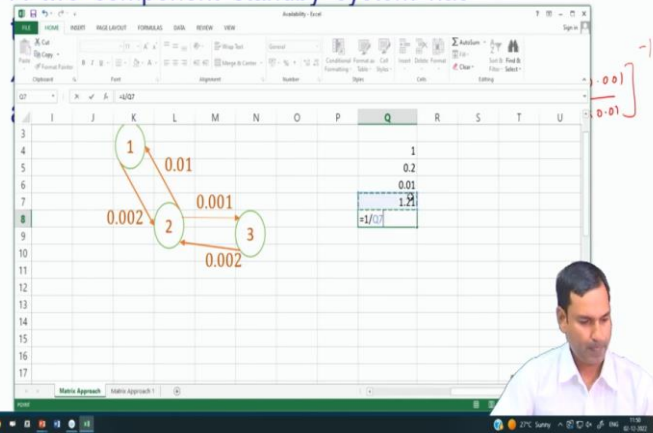




Example



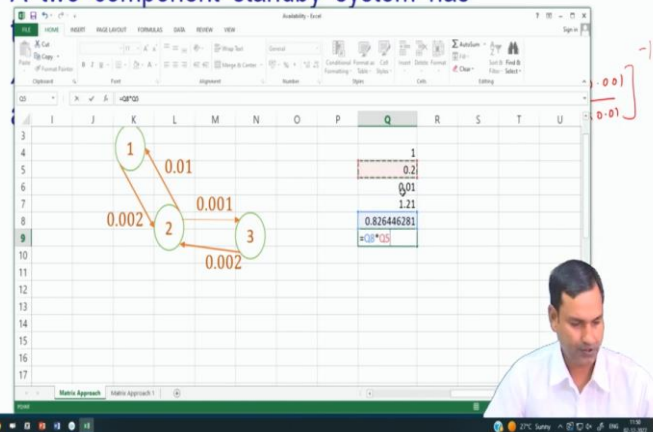
- A two component standby system has



Example



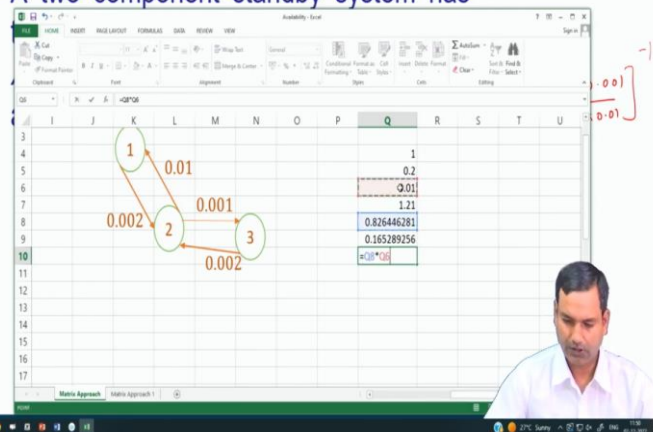
- A two component standby system has



Example



- A two component standby system has



Example

- A two component standby system has

P1	0.826446281
P2	0.165289256
P3	0.008264463
Availability	0.994735537

Example

- A two component standby system has

P1	0.826446281
P2	0.165289256
P3	0.008264463
Availability	0.994735537

I will just do this here for your reference only. So, I am showing that calculation here or I can use the, let us say I can use the calculator also. Generally, I am, let us do this here simple calculation. So, what I have to take? 1 for first for second term is 0.002 divided by 0.001 , I think. Let me just check, yeah. 0.002 divided to 0.01 and third term is 0.0012 into 0.001 .

Third term is equal to 0.002 multiply by 0.001 or divided by 2 into 0.01 square would be I can also do like this or I could directly write because 0.1 square is fairly easy to calculate, 0.01 . So, I have 0.01 . Now, what will be the sum of this and what will be the inverse of this, that will be 1 divided by this value. So, as you see, 0.8264 and this will be, second term will be equal to this, multiply by second term. And third term will be equal to this multiply by third term.

So, this becomes my P1, this becomes P2 and this becomes P3. If you see the summation has to be equal to 1, so, my three state probabilities are here and my availability will be some of this or I can say this is availability will be equal to, I do not only system state which is in which system is not available is the third state.

So, I can say 1 minus 0.13 or P1 plus P2, both will be giving me the same. As you here, the availability has become 99.17 but when we were seeing this, it was 0.98336. So, by ensuring that two units can be repaired together my probability has become higher, availability has become higher because the repair is has become faster.

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The same problem as we discussed that the set of equation which you developed here, these set of equations I can also solve using the matrix formula. So, for matrix formula I do not have to write these equations. Though I have written these equations but from these equations I can see that minus lambda 1 P1 r plus r P2 and P3 is not there, so, that will be 0. Second equation P1 multiplication is lambda 1 P2 is getting multiplied with minus lambda 2 plus r.

So, minus lambda 2 plus r and P3 is getting multiplied by r. And third equation is 1 plus 1 P1 P2 P3. So, that is 1 1 1. This if we multiply by P1 P2 P3, I will get the output, output is 0 0 1. So, this set of equation I can convert into the matrix equation also. This matrix though I have prepared from this but I can prepare from here also directly. Like how do I prepare?


As we discussed earlier, this rate matrix I can prepare for state number 1. For state number 1, whatever is outgoing that is the negative. So, from state number 1, I am having the outgoing

is lambda 1. From state number 1, whatever I am having incoming that is r from state number 2, nothing from state number 3, no connection with state number 3 so 0.


For state number 2, what is the relation with P1, state number 1? I have the incoming lambda 1 from state number 1, this. What is the state number 2 relation with itself? That is only the outgoing link; outgoing links are r and lambda 2. So, minus r plus minus r minus lambda 2. And what is the relation of second state with third state? That it is having the incoming link from third state, that is with r transition rate r. And third equation is 1 1 1 into P1 P2 P3 equal to 0 0 1.

Now, with this, if I want to calculate P1 P2 P3, this will be equal to inverse of this, multiplied by this. So, I will take minus lambda 1 r 0 lambda 1 minus r plus lambda 2 into r and plus this r 1 1 1. If I take matrix inverse and multiply by 0 0 1, I will get the metric result which will be same as P1 P2 P3. So, this I can do. I have done this in Excel. I will show it.

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Example



Example:

$$\begin{bmatrix} -0.002 & 0.01 & 0 \\ 0.002 & -0.01 & 0.01 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$T \times P = b$
 $P = T^{-1}b$

Matrix Inverse Approach

P1	✓	-172.131	-81.9672	0.819672	0
P2	=	65.57377	-16.3934	0.163934	0
P3		106.5574	98.36066	0.016393	1

Determinant Approach

-0.002	0.01	0	
0.002	-0.01	0.01	= 0.000122
1	1	1	


-0.002	0	0	
0.002	0	0.01	= 0.000002
1	1	1	

-0.002	0.01	0	
0.002	-0.01	0	= 0.0000002
1	1	1	

P1	=	0.8197
P2	=	0.1639
P3	=	0.0164

$A = P_1 + P_2 = 0.9836$

-0.002 x (



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Example

Example:

$$\begin{bmatrix} -0.002 & 0.01 & 0 \\ 0.002 & -0.011 & 0.01 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$T \times P = b$
 $P = T^{-1}b$

Determinant Approach

-0.002	0.01	0	
0.002	-0.011	0.01	= 0.000122
1	1	1	

P1	=	0.8197
P2	=	0.1639
P3	=	0.0164

Matrix Inverse Approach


P1		-172.131	-81.9672	0.819672		0
P2	=	65.57377	-16.3934	0.163934	x	0
P3		106.5574	98.36066	0.016393		1

$-0.002 \times (-0.011 \times 1 - 0.01 \times 1) - 0.01 \times (0.01 \times 1 - 0.01 \times 1) + 0 \times (0.002 \times 1 + 0.01 \times 1)$
 $A = P_1 + P_2 = 0.9836$

0	0.01	0	
0	-0.01	0.01	= 0.0001
1	1	1	

-0.002	0	0	
0.002	0	0.01	= 0.00002
1	1	1	

-0.002	0.01	0	
0.002	-0.01	0	= 0.000002
1	1	1	



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So, let me show you how I have done it here. I have copied it here also like if you see. So, here what I have done? Like my metric, here is my matrix is this. So, I put the values here. So, lambda 1 is 0.002, r is 0.01 and r plus lambda 2 is lambda 2 is 0.001. So, 0.001 plus 0.01 will be 0.011. And this again are 0.01 and 1 1 1. Now, this matrix solving, how can how can I do?

I can use a matrix inverse approach. In the matrix inverse approach, I have taken the inverse of this. So, inverse of this gives me this matrix. In this matrix, If I multiply 0 0 1, what will happen? For P1, this would be multiplied by 0, this will be multiplied by 0, this will be multiplied by 1.

So, this like we multiply this row with this column. So, 0 into this plus 0 into this plus 1 into this. So, P1 will be equal to this value. Similarly, this row multiplied by this will give the P2, that will again be this column. So, because only third column will come here because of the 0 0 1. So, we have the P1 P2 P3 which is coming from the third column.

Same values what we have calculated 0.8197 and 0.1639, 0.064, same values you are getting for P1 P2 P3 and availability will be P1 plus P2 that is 0.9836. So, this is another method which we can do. Matrix can also be solved using the determinant approach. What is the determinant approach?

The determinant approach is, let us say, if I want to calculate P1. So, to calculate P1, I will take the, whatever is my first column. Because P1 is the first entry here. So, the first column which I have here that will be replaced by the output column. So, that means I will write the

column as 0 0 1, 0 0 1 and second column will be 0.01, third will be same as, values are same. This is actually cartel because of the compression. And 0.0, 0.01, 1.

Now, this I take the determinant. How do I take the determinant? 0 into this will be 0. So, 0.01 into if I take this then that will be 0.01 into 0.01 again. So, that will be 0.0001. This is 1. Similarly, this matrix has to be divided by the determinant of this matrix, complete metric. And determinant of this matrix I have calculated here. Same metric which I have used here, I put here. And the determinant of this matrix is coming out to be 0.000122.

So, whatever value I have got here, this has to be divided by this value, that will give me the P1. Similarly, if I want to calculate P2, in this matrix second column will be replaced by this output column 0 0 1. So, this will remain same 0.0 minus 0.002, plus 0.002, 1. This will be replaced by 0 0 1. Third will remain as it is and we will take the determinant of this value.

Determinant of this value comes out to be this value. Then this will again be divided by the complete matrix determinant. Once I divide this by this, I get the P2 value which is this value. Similarly, for third what I will do? I will replace the third column with 0 0 1, first two will remain same and I will take the determinant of that. The same determinant I will again divide by the determinant of complete matrix and this will give me this value.

So, this is same formula like this is the use we can use the determinant rather than using the matrix inverse. Because matrix inverse can sometimes be tedious to calculate but it gives does the same thing. So, if we do not want to use matrix inverse, we can do the use a determinant approach.

And determinant approach is fairly easy, we know already how to calculate the determinant. Like like for this, if I want to calculate, the determinant is 0.002 minus multiply by, just let me remove this, I will do this here. So, let us say I calculate the determinant of this matrix. So, that will be equal to first term because this is the, so, that will come as it is multiplied by, I will take the cross multiplication here. So, that will be minus 0.01 multiplied by 1.

And this will be subtracted, the minus sign will come and this multiplication will be subtracted from here, 0.01 into 1. Then for second value, I will take the minus sign, minus of 0.01. This will be multiplied with again cross here. This column will not be considered; this row will not be considered. This will be cross multiplied. That is 0.01 into 0.002 into 0.02 into 1 minus 0.01 into 1.

And then I will take the third value. Third value will be plus, plus 0 into cross multiplication of these columns. So, that will be this one. That will be 0.002 multiplied by 1 minus, minus - minus will become plus because this is also minus. 0.01 into 1. This if I calculate, I will get the value equal to this. I have done this in Excel. I will show you. So, you will learn this how to use Excel for matrix operations.

(Refer Slide Time: 21:41)

The screenshot shows an Excel spreadsheet with the following data:

	-0	0.01	0	P1	=	0
	0.002	-0.011	0.01	P2	=	0
	1	1	1	P3	=	1

Matrix Inverse Approach

P2 ²	=	-172	-82	0.8197	=	0
P2	=	65.57	-16	0.1639	=	0
P3	=	106.6	98.4	0.0164	=	1

Determinant Approach

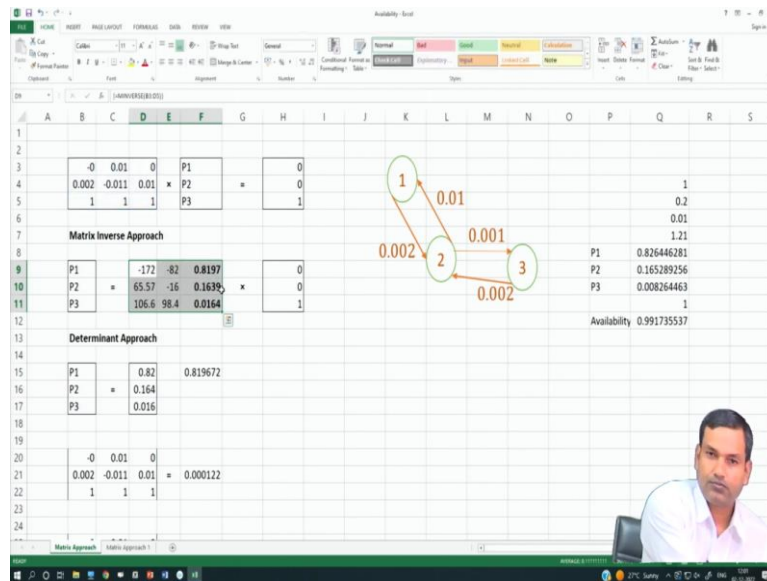
P1	=	0.82	0.819672
P2	=	0.164	
P3	=	0.016	

Final results:

P1	0.826446281
P2	0.165289256
P3	0.008264463
Availability	0.991735537

The screenshot shows the same Excel spreadsheet as above, but with the formula for the matrix inverse approach updated to use the MINVERSE function:

P1	=	=MINVERSE(B1:B3)	=	0
P2	=		=	0
P3	=		=	1



Now, let us say I have already put it here. Actually most of the things which I have copied there is taken from here. This is contacted, so, I am little expanding this. So, as you see here this is my metric which I have to solve. So, first is matrix inverse approach. For matrix inverse approach, what I have to do?

I will take the, there is a formula in here, matrix inverse, M inverse. So, I will take the M inverse of this matrix. But this formula will actually give me. What I will do? I will do this again because this is already output. So, this is equal to M inverse of this I want to calculate this. This will give me one value. But matrix inverse, I have to get the whole matrix. So, how to do that first I will select whole matrix.

And wherever my formula is there I will put either I will put here symbol or I can put f 2 also here for the selection of that. So, this column I will select. Now, I will do control shift enter. What I put? I put the Ctrl shift Enter. Ctrl Shift Enter helps us to make the array called calculations. So, here whenever I put Ctrl shift Enter, the same formula is applied for correspondingly modified and applied for all the entries here.

So, I have to select the metric then I have to go to the first entry where I have calculated and I have to put the Ctrl shift Enter and I get this value. So, this gives me the inverse directly. And from here like If I multiply I will get this but I am not trying to multiply here because I already know that because this is 0 0 1. So, only last column will give me the value. So, this becomes my P1 P2 P3.

(Refer Slide Time: 23:56)

The spreadsheet displays the following data:

P1	=	-172.1	-82	0.8197	x	0
P2	=	65.574	-16	0.1639	x	0
P3	=	106.56	98.4	0.0164	x	1

Matrix Inverse Approach

P1	=	0.8197	0.819672
P2	=	0.1639	
P3	=	0.0164	

Determinant Approach

P1	=	0.8197	0.819672
P2	=	0.1639	
P3	=	0.0164	

Availability: 0.991735537

Diagram values: Node 1 to Node 2: 0.01; Node 2 to Node 3: 0.001; Node 1 to Node 3: 0.002.

The spreadsheet shows the formula `=MDETERM(B20:B22)` in cell F21. The rest of the data and diagram are identical to the previous screenshot.

The spreadsheet shows the value 0.000122 in cell F22. The rest of the data and diagram are identical to the previous screenshots.

Availability - Excel

0.002	-0.011	0.01	x	P2	=	0
1	1	1		P3	=	1

Matrix Inverse Approach

P1	=	-172.1	-82	0.8197	x	0
P2	=	65.574	-16	0.1639	x	0
P3	=	106.56	98.4	0.0164	x	1

Determinant Approach

P1	=	0.8197	0.819672
P2	=	0.1639	
P3	=	0.0164	

-0	0.01	0	=	0.000122
0.002	-0.011	0.01	=	0.000122
1	1	1	=	0.0001

0	0.01	0	=	0.0001
0	-0.011	0.01	=	0.0001
1	1	1	=	0.0001

Availability 0.991735537

Availability - Excel

P2	=	65.574	-16	0.1639	x	0
P3	=	106.56	98.4	0.0164	x	1

Determinant Approach

P1	=	1	0.819672
P2	=	0.1639	
P3	=	0.0164	

-0	0.01	0	=	0.000122
0.002	-0.011	0.01	=	0.000122
1	1	1	=	0.00002

-0	0.01	0	=	0.00002
0.002	0	0.01	=	0.00002
1	1	1	=	0.00002

Availability 0.991735537

Availability - Excel

-0	0.01	0	x	P1	=	0
0.002	-0.011	0.01	x	P2	=	0
1	1	1		P3	=	1

Matrix Inverse Approach

P1	=	-172.1	-82	0.8197	x	0
P2	=	65.574	-16	0.1639	x	0
P3	=	106.56	98.4	0.0164	x	1

Determinant Approach

P1	=	1	0.819672
P2	=	0.1639	
P3	=	0.0164	

-0	0.01	0	=	0.000122
0.002	-0.011	0.01	=	0.000122
1	1	1	=	0.000122

Availability 0.991735537

Availability Score

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fx

A B C D E F G H I J K L M N O P Q R S

13 Determinant Approach

14

15 P1 = 0.8197 0.819672

16 P2 = 0.1639

17 P3 = 0.0164

18

19

20 -0 0.01 0

21 0.002 -0.011 0.01 = 0.000122

22 1 1 1

23

24

25 0 0.01 0

26 0 -0.011 0.01 = 0.0001

27 1 1 1

28

29

30 -0 0 0

31 0.002 0 0.01 = 0.00002

32 1 1 1


33

34

35 -0 0.01 0

36 0.002 -0.011 0 = 0.000002

Matrix Approach Matrix approach 1



Availability Score

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A B C D E F G H I J K L M N O P Q R S

13 Determinant Approach

14

15 P1 = 0.8197 0.819672

16 P2 = 0.1639

17 P3 = 0.0164

18

19

20 -0 0.01 0

21 0.002 -0.011 0.01 = 0.000122

22 1 1 1

23

24

25 0 0.01 0

26 0 -0.011 0.01 = 0.0001

27 1 1 1

28

29

30 -0 0 0

31 0.002 0 0.01 = 0.00002

32 1 1 1


33

34

35 -0 0.01 0

36 0.002 -0.011 0 = 0.000002

Matrix Approach Matrix approach 1



Availability Score

Home Insert Page Layout Formulas Data Review View

Clipboard Font Alignment Number Styles Cells Editing

fx

A B C D E F G H I J K L M N O P Q R S

13 Determinant Approach

14

15 P1 = 0.8197 0.819672

16 P2 = 0.1639

17 P3 = 0.0164

18

19

20 -0 0.01 0

21 0.002 -0.011 0.01 = 0.000122

22 1 1 1

23

24

25 0 0.01 0

26 0 -0.011 0.01 = 0.0001

27 1 1 1

28

29

30 -0 0 0

31 0.002 0 0.01 = 0.00002

32 1 1 1


33

34

35 -0 0.01 0

36 0.002 -0.011 0 = 0.000002

Matrix Approach Matrix approach 1



Now, let us see the determinant approach. For determinant approach, let us go ahead. So, what we do? We have to calculate determinant. So, first we have to calculate determinant of this which will be multiplying to or which will be divided division factor to all values of the $P_1 P_2 P_3$. So, let us do this. This I have calculated as $M D$ determinant. So, that is equal to m . There is a $M D E T E R M$, $M D E T E R M I N A N T$ is written as $D E T E R M$. So, $M D E T E R M I N A N T$ of this.

So, whatever matrix I take, I will take the determinant of that. That will give me the value. So, it becomes simple. Now, what I will do? I will take this and I will paste only values here. Once I paste the values here. Now, I have to convert, copy this and I will change the first column, because I want to calculate P_1 .

So, for calculating P_1 , I will change the first column to the output column and I will get this value. And for calculating P_2 , I have changed the second column here, rest of the values are same. And for P_3 , I change the third column only. I have got the. And to calculate P_1 , what I will do?

I will do this. This is equal to this value divided by my original determinant, this is equal to this. After column replacement divided by this value. This is equal to this divided by this value. So, I get all the values here, $P_1 P_2 P_3$. So, as we have seen, the equations which we have got, you can solve them directly also, you can solve using matrix inverse approach also or you can use this determinant approach also. Generally, determinant approach I find a little bit easier and directly straightforward, only calculation has to be made and you can use it.

(Refer Slide Time: 26:10)

Matrix Inverse Approach

$$\begin{bmatrix} -0 & 0.01 & 0 \\ 0.002 & -0.011 & 0.02 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} P1 \\ P2 \\ P3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P1 \\ P2 \\ P3 \end{bmatrix} = \begin{bmatrix} -172.1 & -82 & 0.8197 \\ 65.574 & -16 & 0.1639 \\ 106.56 & 98.4 & 0.0164 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Determinant Approach

$$\begin{bmatrix} P1 \\ P2 \\ P3 \end{bmatrix} = \begin{bmatrix} 0.8197 & 0.819672 \\ 0.1639 \\ 0.0164 \end{bmatrix}$$

Availability 0.991735537

Matrix Inverse Approach

$$\begin{bmatrix} -0 & 0.01 & 0 \\ 0.002 & -0.011 & 0.02 \\ 1 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} P1 \\ P2 \\ P3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P1 \\ P2 \\ P3 \end{bmatrix} = \begin{bmatrix} -128.1 & -41 & 0.8264 \\ 74.38 & -8.3 & 0.1653 \\ 53.719 & 49.6 & 0.0083 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Determinant Approach

$$\begin{bmatrix} P1 \\ P2 \\ P3 \end{bmatrix} = \begin{bmatrix} 0.8197 & 0.819672 \\ 0.1639 \\ 0.0164 \end{bmatrix}$$

Availability 0.991735537

Determinant Approach

$$\begin{bmatrix} P1 \\ P2 \\ P3 \end{bmatrix} = \begin{bmatrix} 0.4132 & 0.4132 & 0.819672 \\ 0.0826 & 0.0826 & 0.0083 \end{bmatrix}$$

Availability 0.991735537

Determinant Approach

P1	=	0.8264		0.819672
P2	=	0.0826		
P3	=	0.0083		

-0	0.01	0		
0.002	-0.011	0.02	=	0.000242
1	1	1		

0	0.01	0		
0	-0.011	0.02	=	0.0002
1	1	1		

-0	0	0		
0.002	0	0.01	=	0.000002
1	1	1		

Matrix Approach

-0	0.01	0		
0.002	-0.011	0.02	=	0.000242
1	1	1		

0	0.01	0		
0	-0.011	0.02	=	0.0002
1	1	1		

-0	0	0		
0.002	0	0.02	=	0.000004
1	1	1		

-0	0.01	0		
0.002	-0.011	0	=	0.000002
1	1	1		

Now, let us say if I had used r rather than $2r$, if I had used the $2r$ here, then this would have become 0.02 . That means the repair rate which I am taking here is 0.02 . If I do this I will just do this change here. Because this value is only changed or I will just copy this because this is only change. So, wherever it is I will just paste it. I get the new values here.

(Refer Slide Time: 26:58)

Matrix Inverse Approach

P_1	-128.1	-41	0.8264	\times	$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	
P_2	74.38	-8.3	0.1653	\times	$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	
P_3	53.719	49.6	0.0083	\times	$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	

Determinant Approach

P_1	0.8264	= 0.15 * 0.16
P_2	0.1653	
P_3	0.0083	

$\begin{bmatrix} -0 & 0.01 & 0 \\ 0.002 & -0.011 & 0.02 \\ 1 & 1 & 1 \end{bmatrix} = 0.000242$
 $\begin{bmatrix} 0 & 0.01 & 0 \\ 0 & -0.011 & 0.02 \\ 1 & 1 & 1 \end{bmatrix} = 0.0002$
 $\begin{bmatrix} -0 & 0.01 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Diagram: A diagram shows three nodes labeled 1, 2, and 3. Node 1 is connected to node 2 with a weight of 0.002. Node 2 is connected to node 3 with a weight of 0.001. Node 3 is connected to node 2 with a weight of 0.02.

Results:

P_1	0.826446281
P_2	0.165289256
P_3	0.008264463
Availability	0.991735537

Determinant Approach

P_1	0.8264	0.991736
P_2	0.1653	
P_3	0.0083	

Diagram: A diagram shows three nodes labeled 1, 2, and 3. Node 1 is connected to node 2 with a weight of 0.002. Node 2 is connected to node 3 with a weight of 0.001. Node 3 is connected to node 2 with a weight of 0.02.

Results:

P_1	0.826446281
P_2	0.165289256
P_3	0.008264463
Availability	0.991735537

Matrix Inverse Approach

P_1	-128.1	-41	0.8264	\times	$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	
P_2	74.38	-8.3	0.1653	\times	$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	
P_3	53.719	49.6	0.0083	\times	$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$	

Determinant Approach

P_1	0.8264	0.991736
P_2	0.1653	
P_3	0.0083	

$\begin{bmatrix} -0 & 0.01 & 0 \\ 0.002 & -0.011 & 0.02 \\ 1 & 1 & 1 \end{bmatrix} = 0.000242$
 $\begin{bmatrix} 0 & 0.01 & 0 \\ 0 & -0.011 & 0.02 \\ 1 & 1 & 1 \end{bmatrix} = 0.0002$

Diagram: A diagram shows three nodes labeled 1, 2, and 3. Node 1 is connected to node 2 with a weight of 0.002. Node 2 is connected to node 3 with a weight of 0.001. Node 3 is connected to node 2 with a weight of 0.02.

Results:

P_1	0.826446281
P_2	0.165289256
P_3	0.008264463
Availability	0.991735537

The screenshot shows an Excel spreadsheet with the following data:

1	0	0.01	0	P1	=	0
2	0.002	-0.011	0.02	P2	=	0
3	1	1	1	P3	=	1

Matrix Inverse Approach

P1	=	-128.1	-41	0.8264	=	0
P2	=	74.38	-8.3	0.1653	=	0
P3	=	53.719	49.6	0.0083	=	1

Determinant Approach

P1	=	0.8264	Availability	0.991736
P2	=	0.1653		
P3	=	0.0083		

Markov Diagram:

```

graph TD
    1((1)) -- 0.01 --> 2((2))
    2 -- 0.001 --> 1
    2 -- 0.02 --> 3((3))
    3 -- 0.002 --> 2
  
```

And my availability will be equal to this plus second, 0.99176 as we earlier evaluated. I think it was here. No, I think the same yeah, this one. This we have solved by equation solving and this this is what we are getting here. So, we have seen that if we are having this kind of systems and we are able to prepare a Markov diagram here, we can solve this problem and we can get the answer. Now, let us take another system here. This is given in problem here also in next problem.

(Refer Slide Time: 27:42)

Example

- A system will be in one of three states. In state 1 the system is fully operational. In state 2 it operates in degraded mode, and in state 3 it is in failed mode.
- The system can be repaired to a fully operational status only once it is failed.
- Refer Rate Diagram.
- Given, $\lambda_1 = 2$, $\lambda_2 = 3$, $\lambda_3 = 1$, $r = 10$

Rate Diagram:

```

graph TD
    1((1 Normal)) -- λ1=2 --> 2((2 Degraded))
    2 -- λ2=3 --> 1
    2 -- λ3=1 --> 3((3 Failed))
    3 -- r=10 --> 1
  
```

Matrix Inverse Approach

P1	=	-0.051	0.17	0.5085	=	0
P2	=	-0.034	-0.2	0.3390	=	0
P3	=	0.0847	0.05	0.1525	=	1

Availability: 0.847458

Let us say this is our system. Like we discussed about the degraded system. In degraded system what we consider that we have the system state 1, where system is in good state. But system can degrade to state number 2 or system can also fail directly. So, we have two state,

three state system here. We have this is the normal operating and this is degraded and this is failed, completely failed.

Now, here we are considering that the system is fully operational, state two is degraded and state three is in failed mode. The system can be repaired only when it is failed. That means from degraded state we are not repairing it. So, that means repair is only happening here. So, this becomes our diagram.

Now, for this diagram, let us say λ_1 is 2 λ_2 is 3 and λ_3 is equal to 1 and repair is equal to 10. So, I can prepare the rate diagram here. Rate means from 1, how much is outgoing? That two link outgoing, 2 and 1. So, this will become minus 3. What is coming from 2? Nothing coming from 2? So, 0. From 3 anything coming? Yes, r is coming. So, this becomes 10.

For state number 2, anything coming from 1? Yes, 2 is coming. Whatever outgoing from 2, that is only λ_2 minus 3. And what is coming from state number 3 to 2? Nothing is coming. So, no link. And then third equation remains same 1 1 1. This, if we do the matrix inverse, we get this values directly. So, last column becomes the answer. This I have done here also.

(Refer Slide Time: 29:37)

The screenshot shows an Excel spreadsheet with the following content:

Transition Matrix:

	-3	0	10	P1	= 0
	2	-3	0	P2	= 0
	1	1	1	P3	= 1

Rate Diagram:

```

    graph TD
      1((1)) -- 2 --> 2((2))
      1 -- 1 --> 3((3))
      3 -- 10 --> 2
  
```

Matrix Inverse Approach:

P1	=	-0.051	0.17	0.5085	x	0
P2	=	-0.034	-0.2	0.3390	x	0
P3	=	0.085	0.05	0.1525	x	1

Determinant Approach:

P1	=	0.508	Availability
P2	=	0.339	0.847458
P3	=	0.153	

Summary:

-3	0	10	
2	-3	0	= 59
1	1	1	

Availability - Excel

Matrix Inverse Approach

$$\begin{bmatrix} P1 \\ P2 \\ P3 \end{bmatrix} = \begin{bmatrix} -0.051 & 0.17 & 0.5085 \\ -0.034 & 0.2 & 0.3390 \\ 0.085 & 0.05 & 0.1525 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Determinant Approach

P1	=	0.508	Availability
P2	=	0.339	0.847458
P3	=	0.153	

Augmented matrix for P1:

$$\begin{bmatrix} -3 & 0 & 10 \\ 2 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 10$$

Augmented matrix for P2:

$$\begin{bmatrix} 0 & 0 & 10 \\ 0 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 30$$

Augmented matrix for P3:

$$\begin{bmatrix} -3 & 0 & 10 \\ 2 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 20$$

Augmented matrix for P4:

$$\begin{bmatrix} -3 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 9$$

Availability - Excel

Matrix Inverse Approach

P2	=	0.339	0.847458
P3	=	0.153	

Augmented matrix for P1:

$$\begin{bmatrix} -3 & 0 & 10 \\ 2 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 59$$

Augmented matrix for P2:

$$\begin{bmatrix} 0 & 0 & 10 \\ 0 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 30$$

Augmented matrix for P3:

$$\begin{bmatrix} -3 & 0 & 10 \\ 2 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 20$$

Augmented matrix for P4:

$$\begin{bmatrix} -3 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 9$$

Availability - Excel

Matrix Inverse Approach

P2	=	0.339	0.847458
P3	=	0.153	

Augmented matrix for P1:

$$\begin{bmatrix} -3 & 0 & 10 \\ 2 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 59$$

Augmented matrix for P2:

$$\begin{bmatrix} 0 & 0 & 10 \\ 0 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 30$$

Augmented matrix for P3:

$$\begin{bmatrix} -3 & 0 & 10 \\ 2 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 20$$

Augmented matrix for P4:

$$\begin{bmatrix} -3 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix} = 9$$

The screenshot shows an Excel spreadsheet with the following content:

- Transition Matrix (Rows 3-5):**

-3	0	10	P1	=	0
2	-3	0	P2	=	0
1	1	1	P3	=	1
- Diagram (Rows 4-6):** A state transition diagram with three states: 1, 2, and 3. Transitions are: 1 to 2 (prob 2), 1 to 3 (prob 10), 2 to 1 (prob 1), and 3 to 2 (prob 3).
- Matrix Inverse Approach (Rows 9-11):**

P1	=	-0.051	0.17	0.5085	x	0
P2	=	-0.034	-0.2	0.3390	x	0
P3	=	0.085	0.05	0.1525	x	1
- Determinant Approach (Rows 15-17):**

P1	=	0.508	Availability
P2	=	0.339	0.847458
P3	=	0.153	
- Other Calculations (Rows 20-22):**

-3	0	10	=	
2	-3	0	=	59
1	1	1	=	

Like here same value minus 3, 0, 10, 2 minus 3 0 1 1 1 have taken. And same formula what I have used in previous, I have used here and this value gives me the answer. I can use the same thing using the determinant approach also. So, in determinant, what I did? First, I will take the determinant of this matrix that will be the division factor to all.

Then for P1 when I am calculating, I will replace first one with 0 0 1. This first column will be 0 0 1. When I calculate P2 then second column will be 0 0 1 and when I call calculate for P3 third column will be 0 0 1. So, P1 will be 30 divided by 59, P2 will be 20 divided by 59 and P3 will be 9 divided by 59. Same I have calculated already here.

And if you see my availability coming to be 0.8474. So, as you see here that whatever the diagram we use based on our assessment, how system is working, what repair it will be there, what failure rate it will be there, we just mentioned it here and we are able to solve this using this.

But the assumption here is that distributions are exponential, repair distribution is also exponential and failure distribution is also exponential. Because then only we can use this Markov approach which we are using here. This link is I think it is there it is somehow not shown here. Now, 10 is the link from here to here. This is my r_{10} . So, it will stop here today. And next lecture would be the last lecture, where we will try to see little bit more about maintainability, availability and have some general discussions. Thank you.