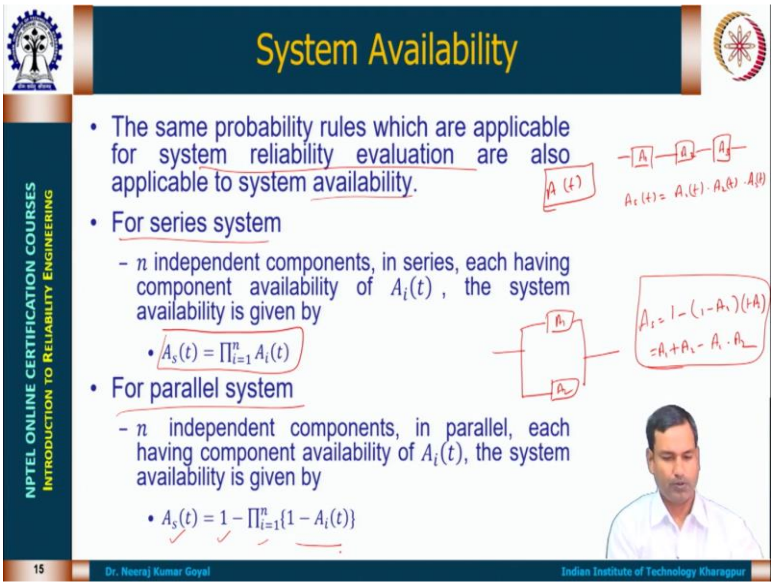
Introduction to Reliability Engineering
Professor Neeraj Kumar Goyal
Subir Chowdhury School of Quality and Reliability
Indian Institute of Technology Kharagpur
Lecture 38
Maintainability and Availability (Continued)

Hello everyone. So, we will start discussing about Availability today. We already started discussion and we discussed a few things about it already. So, we will continue our discussion. Our focus would be mostly on steady state availability evaluation. So, in previous class, as we discussed, we found the system availability or component availability.

(Refer Slide Time: 00:53)



Now, if you remember, we found the component availability A t. So, we get this availability for one of the components. Now, assuming that all components are independent as well as repair is also independent, that means all components are independently getting repaired. One repair is not influencing other repair, one failure is not influencing other failures, in that case, we can evaluate system availability without doing complex modeling.

We can do the system availability evaluation by assuming, by using the same formulas which we have used for the reliability. Like whatever formulas we use for reliability evaluation same we can use for availability also. Now, only difference would be that in system reliability evaluation, we considered component reliability then when we consider system availability calculation, we will use the component availabilities to get the system availability. That is the only difference. Because the probability concept is same.

If component is available then system will be available. So, whatever models we discuss series, parallel, key out of M, all models which we discussed earlier, the same way whatever we have calculated reliability, the same way, using the same RBD, we can also calculate the availability.

So, let us say for series system. We know that for series system, our series system is given like this. We have data set, three components here. So, earlier we were using r1, r2, r3, now, we can use a1, a2, a3. So, when we use a1, a2, a3, our availability of system A S t would be equal to a1 into a2 into a3 which is same as what we discussed for reliability evaluation of system. So, system availability is nothing but multiplication of component availability formulas.

But remember, this formula is only applicable when we are considering that repairs, each component whenever it fails will be getting repaired immediately and one repair of component will not affect another component of repair, similarly, one failure will not affect another failure or one of one component failure will not affect repair of another component.

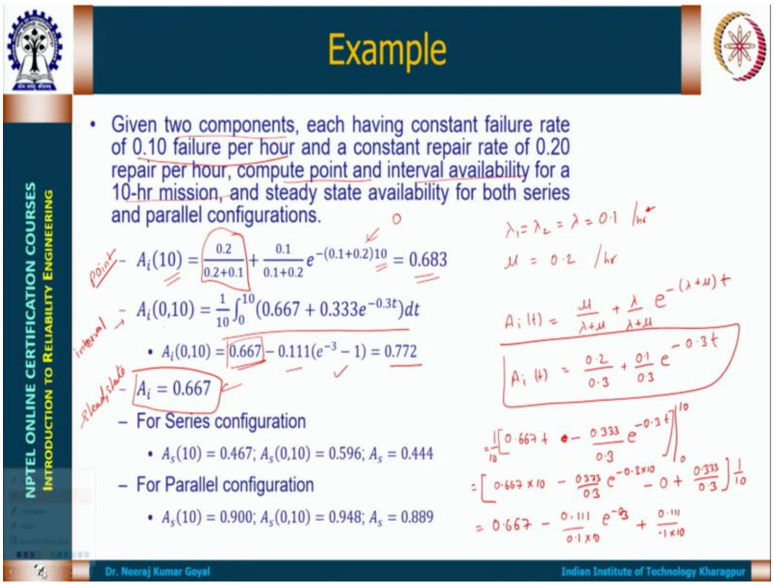
So, so many things are there which may not be always justifiable, but still when we are dealing with small downtime, this still can be giving you the good analysis and good result. But when your down times are comparable which is influencing the operating time, in that case, that availability calculation should be done by using the Markov diagram properly, which we will discuss next.

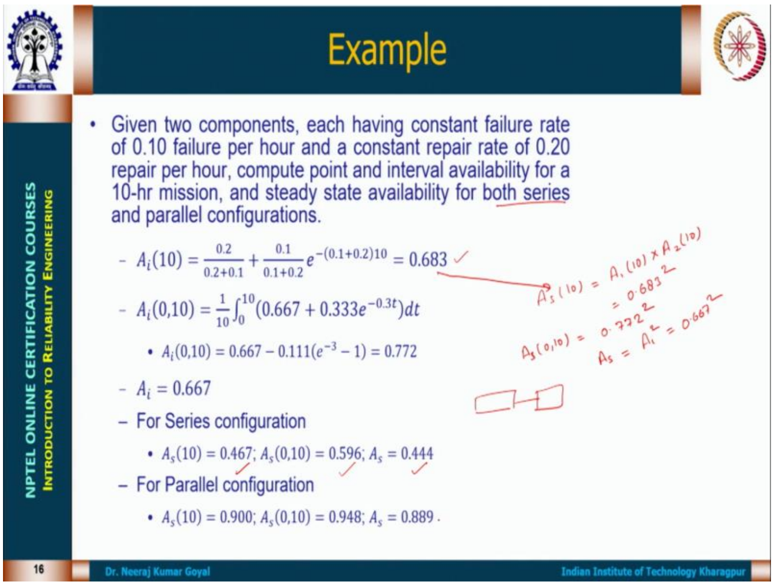
However, as I discussed for most of the calculation, in general, especially in the project phase, when you are a design phase, when you are having the system, you may be able to use this formulas. But if you have the correct evaluation that will help you to get the availability values in a proper way.

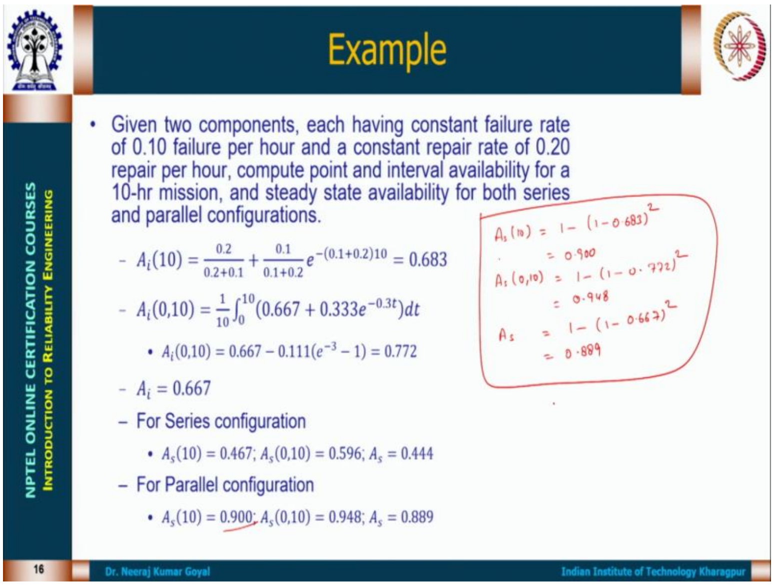
For parallel system, again, we know that if we have a parallel system, let us say two component parallel system, A1, A2, then if I want to know the system availability then system availability is 1 minus 1 minus A1 into 1 minus A2 or we can say A1 plus A2 minus A1 A2.

Here, we are writing availability as a function of time, this may be steady state availability also. Same way we can use this formula. So, system availability is nothing but 1 minus multiplication of 1 minus A i t for all the components. So, same formulas we are able to use it.

(Refer Slide Time: 05:04)







$$A_i(10) = \frac{0.2}{0.2 + 0.1} + \frac{0.1}{0.1 + 0.2} e^{-(0.1+0.2)10} = 0.683$$

$$A_i(0{,}10) = \frac{1}{10}\int_0^{10} (0.667 + 0.333e^{-0.3t})dt$$

$$A_i(0{,}10) = 0.667 - 0.111(e^{-3} - 1) = 0.772$$

$$A_i = 0.667$$

For Series configuration

- $A_s(10) = 0.467; A_s(0{,}10) = 0.596; A_s = 0.444$

For Parallel configuration

- $A_s(10) = 0.900; A_s(0{,}10) = 0.948; A_s = 0.889$

Now, if we take an example. Let us say there are two components and each having constant failure rate of 0.10 failures per hour. So, that means lambda 1 is equal to lambda 2 that is equal to lambda is 0.1 failure per hour. Now, let us say repair rate, mu, is equal to 0.2 repair per hour.

So, generally, as we see that repair rate is supposed to be higher than the failure rate or we can say the MTTF is supposed to be higher than the MTTR, the repair time supposed to be lesser than the operating time. Similarly, the repair rate has to be faster compared to the failure rate.

Now, we want to compute the point availability and interval availability for 10-hour mission. So, first let us find out what is the any availability of the component. So, availability of any component as we know it is given as mu upon lambda plus mu plus lambda upon lambda plus mu e to the power minus lambda plus mu into t.

So, this will be equal to, now, mu is 0.2 divided by 0.2 plus 0.13 plus 0.1 divided by 0.3 e to the power minus 0.3 and t, I am currently writing only t. Now, from here I can calculate the 10 hour mission availability. So, that means I want to know the availability for 10 hours. So, that will be t will be equal to 10. So, at that time 10, how much will be availability?

Availability is 0.683. So, this is the point availability. We can also calculate them interval availability for 10-hour mission. So, that means 0 to 10. So, that means if we integrate this from 0 to 10, how much would this wiil be. This similar example we did it in last class where we integrated this.

So, when we integrated this, this will become 0.667 t minus 0.333 divided by 0.3 e to the power minus 0.3 t where t values I have to take from 0 and 10. So, once I put 10 and this whole has to be divided by 10 minus 0, that will be 10, 1 by 10. So, once I do this that you will get 0.667 into 10 minus 0.333 divided by 0.3 e to the power minus 0.3 into 10 minus.

When I put t equal to 0 this will be 0 and minus minus will become plus. When I put t equal to 0 here, this will become 1. So, my value will be 0.333 divided by 0.3 whole divided by 10. So, this will turn out to be 0.667 minus 0.33 divided by 0.3 would be 0.111 will be divided by 0.1 because 0.1 into 10 will become 1 e to the power minus 0.3. So, 0.3 multiplied by 10 will give you 3 only minus 3 plus 0.33 that is 0.1 multiplied by 10.

So, as we can see this becomes 0.667, 0.111, if it is common then e to the power minus 3 minus 1 and this value turns out to be 0.772. So, my availability, A i, is the steady state availability. So, steady state availability is not a function of time because it is the long term availability. So, this will remain there which will not be changing with time.

So, this value is nothing but the first item here, first entry here, constant value which is independent of t and that value is 0.667. So, 0.667 becomes a steady state availability. That means if I put t equal to infinity then this term would be 0. Because e to the power minus infinity is 0.

So, the only term which is remaining is 0.2 by 0.3 that is 0.667. So, my availability, steady state availability is 0.667, interval availability is 0.772 and my point availability is 0.683. Now, let us see I want to calculate the same. I will remove this. I want to calculate the same for series configuration. Because there are two components in series.

So, I have component 1 and component 2 in series. So, same thing I can do for all three configurations. Like I want to know the point availability for this series configuration. So, point availability A S 10 will be equal to A1 10 into A2 10. Both are same, that will be 0.683 square, this value. When I calculate interval availability that is A S 0 to 10 that will be equal to 0.772 square.

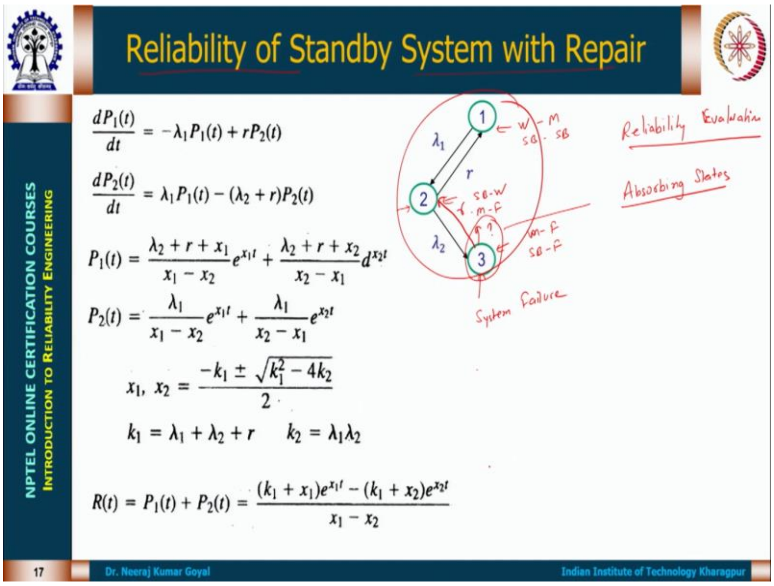
Similarly, A S that is for system will be equal to A i square that will be equal to 0.667 square which comes out to be 0.444, 0.596, 0.467. Similarly, if we want to calculate the same for parallel configuration. For parallel configuration, the same way as we calculated for series, same way we have to calculate for parallel. That means A S 10, point availability will be equal to 1 minus 1 minus 0.683 square.

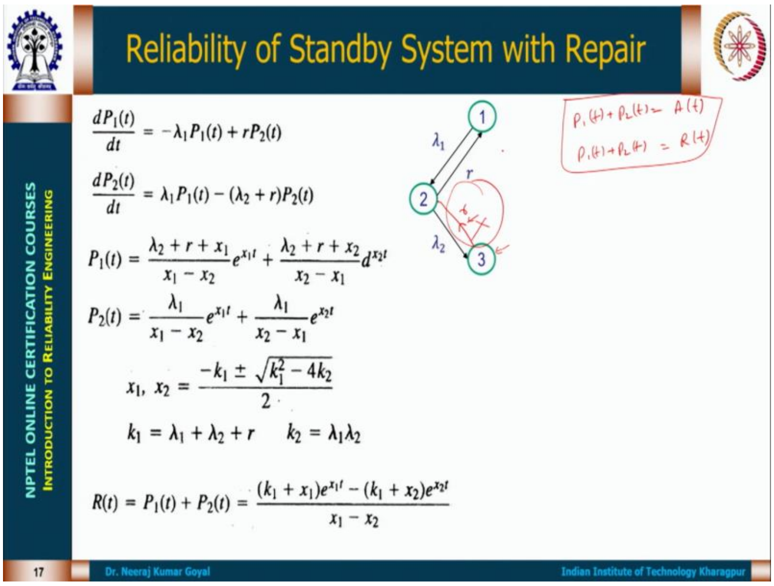
Similarly, this comes out to be 0.9. Similarly, A S 0 to 10, for interval that is equal to 1 minus 1 minus 0.772 whole square. This turns out to be 0.948. Similarly, A S if I want to calculate steady state availability, that will be equal to 1 minus 1 minus steady state individual availability 0.667 whole square. This is equal to 0.8989.

So, we are able to calculate, if we know the individual component availability, we are able to calculate the availability for any configuration, series, parallel, if I say k out of M that also I can evaluate, any other configuration which we have, we can use the same formula which we developed for the RBD for reliability evaluation.

The only thing is we have to, the only problem here is that the assumption is there that the component repair and failure distributions are unaffected by another component failure and appear. So, that means they are all independent. So, when a component gets failed, it will be independently repaired.

(Refer Slide Time: 13:42)





$$\frac{dP_1(t)}{dt} = -\lambda_1 P_1(t) + rP_2(t)$$
$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - (\lambda_2 + r)P_2(t)$$
$$P_1(t) = \frac{\lambda_2 + r + x_1}{x_1 - x_2} e^{x_1 t} + \frac{\lambda_2 + r + x_2}{x_2 - x_1} d^{x_2 t}$$
$$P_2(t) = \frac{\lambda_1}{x_1 - x_2} e^{x_1 t} + \frac{\lambda_1}{x_2 - x_1} e^{x_2 t}$$
$$x_1, x_2 = \frac{-k_1 \pm \sqrt{k_1^2 - 4k_2}}{2}$$
$$k_1 = \lambda_1 + \lambda_2 + r \; k_2 = \lambda_1 \lambda_2$$
$$R(t) = P_1(t) + P_2(t) = \frac{(k_1 + x_1)e^{x_1 t} - (k_1 + x_2)e^{x_2 t}}{x_1 - x_2}$$

Now, let us see we have discussed this problem earlier that when we wanted to know the reliability of a standby system with repair. So, this is the system we considered system 1 is the working state, system 2 is the standby is working, here the main unit is working and stand by unit is in stand by. And here both unit failed, work, main unit is failed and strength by unit is also failed.

Here, main unit is failed. This we have already evaluated, we have already discussed in detail, done one example also. Now, here why I mentioned this again here? To help you understand the difference between availability and reliability concept. Here, in whenever we are seeing reliability evaluation, in reliability evaluation, the system states which are marked as the system failure state not the component failure, system failure.

So, though here one, in state number 2, state number 2, one component is failed but one component is working. So, system is not completely failed. That means our objective or function is still continuing. So, therefore reliability objective is not failed here. But when we talk about state number 3, in state number 3, the primary unit as well as the standby unit, both have been failed.

Since both have been failed, so, our mission is influenced here, mission is failed. We are no longer able to have the continuous use of the system. So, our system action mission is failed. So, therefore our reliability is, here, we have the failure. So, whenever we are evaluating reliability, the failure states which we have in the failure states, we will not have any outgoing link, these will be called absorbing states or terminating states. Why these are absorbing states?

Because in these states when the moment system enters it cannot come out. Why it cannot come out? Because system is failed, there is no future after that. The moment it leads to system, system failure after that the mission is failed and we are no longer concerned about the system.

However, in case of repairable system, so, this state is also not considered to be the failure state, even from the failure state, means this is a failure state but from failure state also repair is considered. Even though system failed but after the system failure, we do the repair and again make the system up again. So, in this case, we have the reverse repair path from this state also.

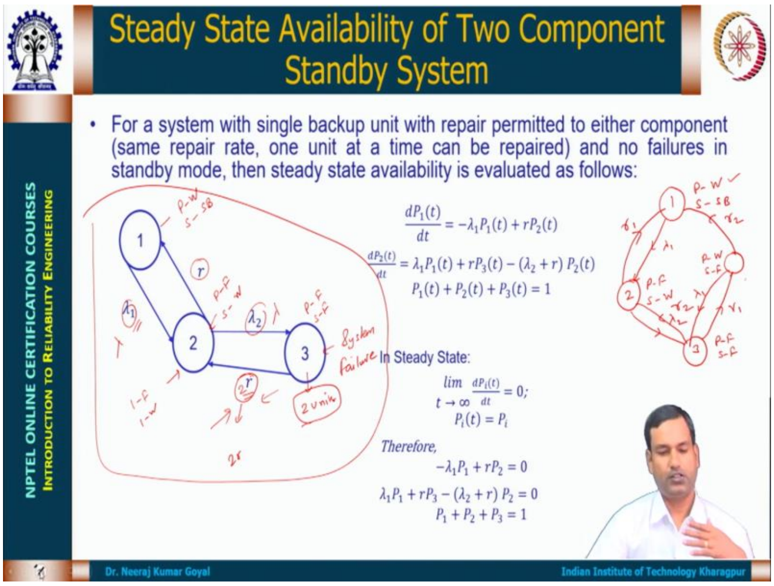
So, in availability calculations, whenever we do, generally, you will not have any state which is the absorbing state. Now, if you see no state is absorbing, in absorbing state, you have only incoming link, there is no outgoing link. What does it mean? That once you fall in this state, there is no way out. It is absorbing or terminating state.

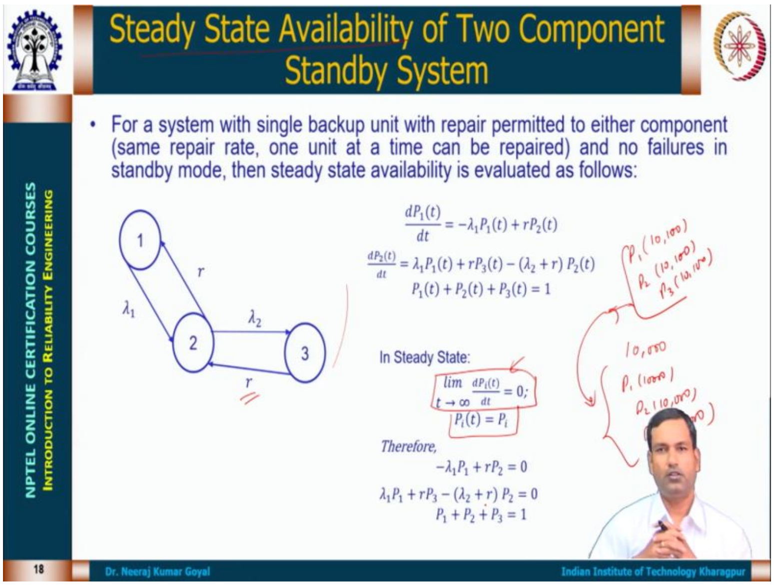
But in case of repairable system, no state is absorbable, absorbing state because from average failure, you can do the repair and you can once again start functioning the system. So, that is where the difference comes in reliability and availability. So, if I include this link here and with the repair rate r then my probability of state 1 plus probability of state 2 gives me the availability.

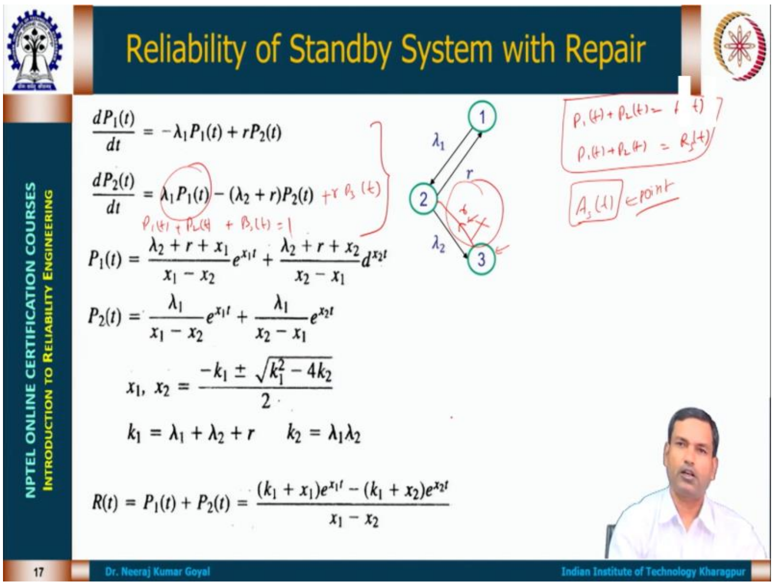
But if I do not use this link, if this link is removed, then probability of state 1 plus probability of state 2 gives me the reliability. What is the difference between the two? The difference between the two is the existence of absorbing state. So, this concept of absorbing state should be understood that is why I have included this slide from the previous discussion which we already made where we calculated the reliability.

In case of reliability calculation, the failure state, system failure states were the absorbing state, the system once fall in the failure state, it cannot be considered to be repaired again to operating state again. But in case of availability concept, you are repairing the system even if it is failed. So, that is where the difference in definition, difference in calculation. So, just to explain this concept I have taken it here.

(Refer Slide Time: 18:56)







Now, let us say. So, the figure which I showed earlier, the figure would look like this that of system state 1 is primary unit working and second the unit is in standby mode. Then what will happen? You will have the failure rate for primary unit. So, after that your primary unit may fail and stand by unit will be coming in the working condition here.

After that again, the secondary unit may fail and because secondary unit may fail then primary unit will also be failed and second unit will also be failed. But here, we have the system failure. But when I want to evaluate availability, I will be considering a repair from here also.

So, repair here is also considered. So, that means from here the repair rate is r by which the system can go into this state. From third state also you have the repair possibility which can bring you bring the system to the second state. So, this becomes our Markov diagram. Generally, this is given but in this case, there is a inherent assumption.

Because here, if you have the two units in field condition, both primary unit is also failed, secondary unit is also failed. But repair rate is same. What does it mean? That means same as here. Because here, one unit is in failed condition, one unit is working. So, only one unit is under repair.

So, when one unit is under repair, repair rate is r. When two unit is under repair, the repair rate will be r, if the repair crew is only one. That means you have only one set of facility, one repair channel, means one set of equipment, or one person only or one group of person which is, so, that means at one time you can do only one repair.

If that is the case, then repair it would be same. But if you had let us see two repair channels that means you can do the repair on two systems simultaneously, in that case, this repair rate would become 2 r, because you are able to repair two systems simultaneously. So, your repair rate has become double.

So, that will be 2r. So, that means if you have infinite resources, repair resources then the repair rate would be equal to the number of equipment failed multiplied by the repair rate. If all are equally repairable or all are same or identical component then they repair it would be same. If there are two different then you may have to take the summation of the repair MTTR 1 plus MTTR 2.

But then that will be little bit more complicated, we have to do this diagram in a different way. So, let us say means whenever we have a certain situation, we have to prepare a diagram accordingly. Like the example which I stated that let us see in state number 1, we have the primary unit is working and secondary unit is in standby.

Now, from here, I can go to state number 2. In state number 2, there is two possibility that we have the primary unit is failed and secondary unit is working. So, from here, if the repair is on and the possibility is that while secondary unit is working, primary unit is getting repaired and that will become working again. So, if primary unit is repaired then what will happen?

Primary unit will again take place and it will start working and secondary unit will again be made as stand by unit. Now, from here, we have the third state, that means there is a failure of second item or the standby system. In that case, primary unit is also failed and second unit is also failed. From here the repair is carried out.

So, if we do repair here, then this is let us say, if we say this is r 1, repair of primary unit, if you say this is r2, repair of secondary unit, so, then what will happen? Secondary unit will be repaired but primary unit will remain failed. But there is also possibility that if we are doing repair of primary unit here that means r1 if we do then what will happen?

We will have the state in which primary unit is in working state and second unit is in failed state. So, this may also be another working situation. From here again, you may have two possibilities that again primary unit can fail and you may have the again both failure or from here, you may have a secondary unit repair, that is r2, in that case, it will again become the working state 1 in which secondary unit is also repaired, first unit is already working.

So, that will become the first state where first unit is working and second unit is under stand by. So, this state diagram we have to prepare as per the system condition. So, whatever is our system condition, how we are operating the system, the system state diagram can be prepared.

And in each system state diagram, whenever we consider, there is a transition of only one item. There is a change of system state is only one item, the change of system state not in two items. So, this condition is only possible when we have the two different repair channels for say primary unit and secondary unit. But if we have only one channel then this would be the only case that r will be there.

And if we say that both repair are same r1 and r2 both are same, in that case, what will happen? This will become 2 r. So, first here, we are considering that one of the unit will get repaired and that will be under stand by, that will start operating, and meantime if second unit get repaired then it will become another unit will become stand by and one unit will be in operating condition.

Again, this diagram looks very good, this kind of diagram you will be able to make if both units are identical. That means both units are same. If both units are same, their failure rate is same; their repair rate is same then it becomes a simpler diagram. Then it will be lambda and it will be also lambda.

And while this will be two r and this will be r. 2 r only in case when two repair channels are there. If it is a single repair channel is only available then at a time only one equipment can be repaired. So, that will be r. So, I am explaining this for the diagram which we have taken only with only r.

The same discussion which we do, we can do the same thing with, once we make this diagram, whatever diagram we make, if you are able to make this diagram, and if you are able to put rates over there, then any diagram can be solved, to calculate the steady state availability. As we discussed here, if you want to keep calculate availability here that is the A t, A t is the system availability A S t, I can say or I can say R S t. This is the system availability but point, point availability of the system.

Generally, point availability of system, we can calculate this but we have already seen how difficult it is to calculate the reliability. Now, if I want to use this link also then this equation will become more complex. As you see here or this will not change but this equation d P 2 over d t will have more changes like this will have plus P 3 state will also come plus r into P 3 t.

So, when I am solving this equation, like earlier when I solved, an another equation is P 1 t plus P 2 t plus P 3 t is equal to 1. Again, last system equation, we generally do not try one system equation because that is imperative. So, here now, to solve this I have to again do the Laplace transformation and again do then first Lplace transformation.

So, we can solve this but it will require more efforts compared to what a force were required to solve the this equation. However, many times, for our decision making, we are more concerned about the steady state values rather than the point values. Since, we are more interested in steady state values, so, we do not want to put these kind of efforts which are required for calculation of point availabilities.

So, we steady state availability, because steady state availability calculation as we will see, the method we will discuss, these methods allow steady state system availability for calculation for even complex systems. So, whatever systems we are using, whatever processes we are using, once we are able to develop the rate diagram, we can calculate the probabilities fairly easy compared to the point availabilities.

Here, we will not require lot of the differential equations. Because what happens here? The moment we say steady state availability, in case of steady state availability, that means time is long time, time tedning to infinity. When time is tending to infinity, system is achieving steady state. That means the change in probability of any state is 0. What does it mean?

Change in probability of any state is 0. So, change in probability of state 1 is 0. So, that means whatever is this as, let us see if I spend or 10000 hours then whatever is the probability P 1 at 10000, P 2 at 10000. Let us say p 3 at 10000. Now, these probabilities which I am taking, these probabilities would be not, now, I want to calculate probability as at 10100, P1, P2 10100, P3 10100.

These P1, P2, P3 which I am calculating here and these P1, P2, P3 which I am calculating here, these probability values will not change much, that changes very very minor almost inobservable. So, when time is high then the probabilities are not changing, as we have seen earlier in our diagram that availability tends to become constant after a certain period of the time.

Generally, around five times or we can say mean five times of mean MTBF etcetera, if you see this will tend to become the constant. So, because of that this change in probability with time becomes 0. That means probability 10000 is equal to probability 10100 or 10000 to 11000, both are same. Because probability has become almost same, they are not changing.

The state probabilities are becoming same because there is a interchange, the repair and of failure are having a too many iterations and because of the iterations we have lost the effect of time. So, it does not matter what time point we are talking about, the chances of failure and chances of recovery are same.

So, because of that average availability or the availability which we are seeing at that time is almost same. Because the system newness is lost and system has iterated many times in failure and repair. So, when failure and repair process have been iterated few times then systems reach the steady state where the chances that system will be in repair of our operating instead would remain same irrespective of which time, we are talking about whether it is 10000 hours or 10100 hours.

Therefore, in these cases, the probability of state we are not writing as a function of time, we are writing the P i t as P i only. So, because this is no longer a function of time. So, P i t, steady state probability is written as independent of time. It is directly expressed in terms of time. So, here these diagrams we will discuss in more detail in next class. We will stop it here today. Thank you.