Introduction to Reliability Engineering
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Lecture 25
Markov Analysis (Contd.)

Hello everyone, so we will continue our discussion from our previous lecture. In previous lecture, we discussed that when a repair is considered and then, with the effect of repair how it we can consider it in the Markov diagram by taking a repair path also. Once we take the repair path then our reliability equations can be again calculated and we can get the system reliability.

We will continue our discussion with one more system that in earlier case we considered that there are two components which are working and for that we consider that only one system is enough to work for the purpose. So, that whenever one out of the two, one system fails, it can be repaired by another system continues to work and our system is still working.

Now, in this case we will see the standby case, that there is one unit which is working and one unit is in standby. Again, we will consider identical units again here and because of that what will happen if one unit fails then another unit will be plugged in. So, that will be in standby earlier. So, it will not fail during the standby condition. So, when it is plugged in, the field condition field unit will go to the repair and the plugged-in equipment will continue to work.

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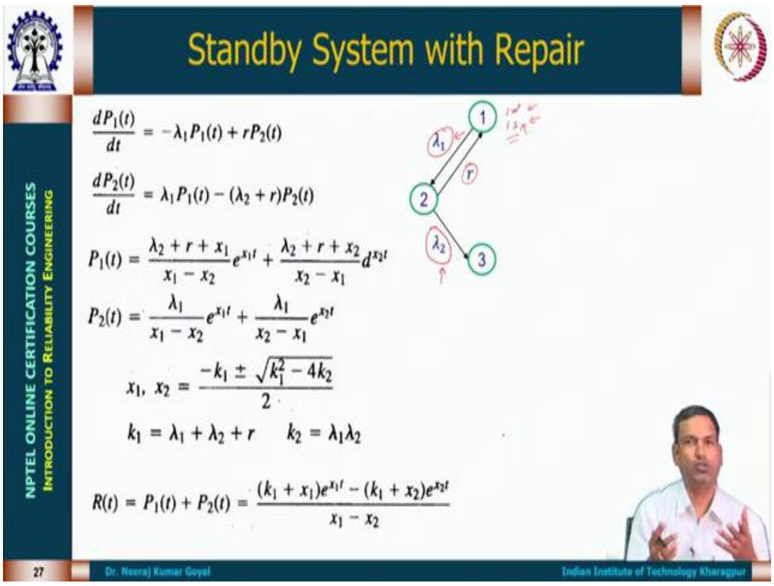

So, this system can be represented by Markov diagram as we have made it here, that here we are considering that the 2 units are different the unit which we are considering for standby is different and our unit which is considered for primary unit that is different. They may be same also, if they are same, you can consider both as the Lambda.

But what happens in earlier case 2 units were working but here 1 unit is working and 1 unit is in standby. So, only the working unit will have the failure rate. So, the failure rate of working unit that is unit 1 primary unit the failure rate is Lambda 1 and the for the unit 2 the failure rate is Lambda 2.

So, now what happens if 1 unit fails then another unit will be replacing the function, while the second unit is functioning the first primary unit can be repaired and it can be going to the state number 1. If it goes state number 1 again, then what will happen? Lambda 2 will the second unit will again become standby and first primary unit will be replacing the standby unit again, that is what generally happens if we take our ups system in case. So, here we have the main power supply which is working and this UPS is acting as this standby.

So, when main Supply fails then what will happen our secondary unit which is the UPS that will take care of the power, mean time if the main Supply is to be restored, the rate of restoration is r. So, if main Supply is restored then again the UPS we will switch Supply from UPS to again to the main Supply, that will be our again the same system will be there that we are working on Main Supply and UPS becomes the standby. So, here whatever we consider the same situation here is also depicted.

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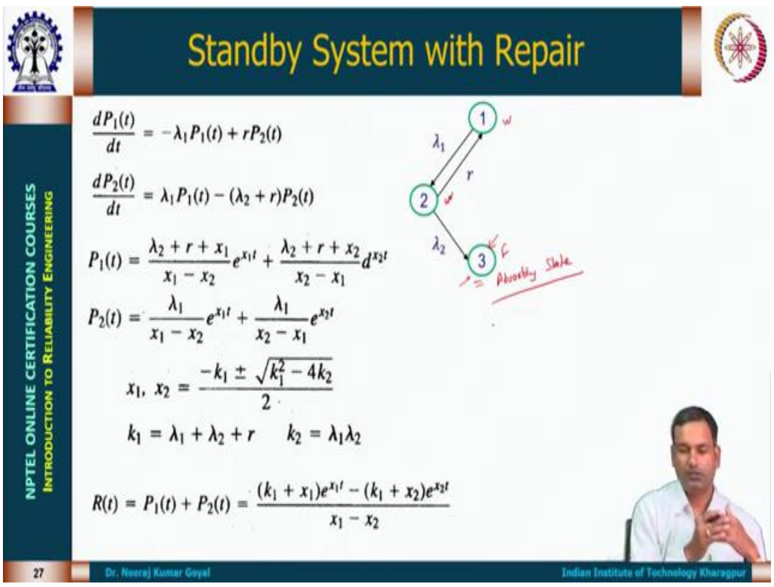

$$\frac{dP_1(t)}{dt} = -\lambda_1 P_1(t) + rP_2(t)$$
$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - (\lambda_2 + r)P_2(t)$$
$$P_1(t) = \frac{\lambda_2 + r + x_1}{x_1 - x_2} e^{x_1 t} + \frac{\lambda_2 + r + x_2}{x_2 - x_1} d^{x_2 t}$$
$$P_2(t) = \frac{\lambda_1}{x_1 - x_2} e^{x_1 t} + \frac{\lambda_1}{x_2 - x_1} e^{x_2 t}$$
$$x_1, x_2 = \frac{-k_1 \pm \sqrt{k_1^2 - 4k_2}}{2}$$
$$k_1 = \lambda_1 + \lambda_2 + r$$
$$R(t) = P_1(t) + P_2(t) = \frac{(k_1 + x_1)e^{x_1 t} - (k_1 + x_2)e^{x_2 t}}{x_1 - x_2}$$

So, in that case if we want to calculate reliability for this kind of system, how can we calculate? So, this is what we have done earlier similar is applicable here. Same thing the way we solve the earlier problem for the 2 unit working and 1 unit in, 1 one unit can get repaired whenever it fails then system will be working. So, here again this is a working condition, this is also working for the system, this is also working and this is failed. In reliability analysis whenever we do, we do not consider this this also called is absorbing state.

The failure state is generally considered the absorbing state. So, whenever we reach the failure, then our reliability goal is diff is not fulfilled. So, whenever a system failure happens then it is considered to be a failure, and from that system we do not consider the repair again. So, there is a difference in the concept of availability and reliability, whenever we consider availability at that time, we consider that repair is possible from complete failure state also.

So, in that case we will take the repair path from here also, but in case of reliability we do not consider repair from the failure state. So, that system continues to work, the moment system is failed our system objective is not completed and we will be having the consequences of failure.

So, because of that, that last state or the failure State whenever we want to calculate reliability, they will not have any repair, they will be absorbing state, only incoming States will be there, there will be no outgoing State from here and that is what we are calling absorbing state. So, what happens if time becomes infinite?

If time is more and more as time is progressing, at the end the system will always land up in this state. So, because from once it reaches to this state, then it cannot change, the state once it is failed, then it is permanently failed, then there is no possibility of going back into the working States again. So, that is what is when we consider the reliability.

Actually, there may be a possibility of repairing, but in that case, we will be considering availability calculation, we will not be calculating that as a reliability. The moment system fails, our relative objective is defined and because of that system our prob re-labbed is the probability of successful continuous operation. So, here that becomes unreliability.

So, probability of being in state 3 is unreliability. And/or logically if we see whatever happens here, it will continue but the moment it reaches to state 3 then system objective is failed. So, and if I take time t equal to Infinity some day or some sometime it is going to lead to the state number 3, that will be my failure probability.

Now, to evaluate this, earlier we considered like if we compare this diagram with earlier diagram it will look similar not much your difference will be there, the only thing is the values and representation is only difference. So, let us try to solve this time this diagram with use of values. So, we will try to solve these equations are already given here, rather than solving them in terms of Lambda upon Lambda 2r, which you can do by following the same steps which we followed for the 2-component system, let us try to do this directly with the numerical value.

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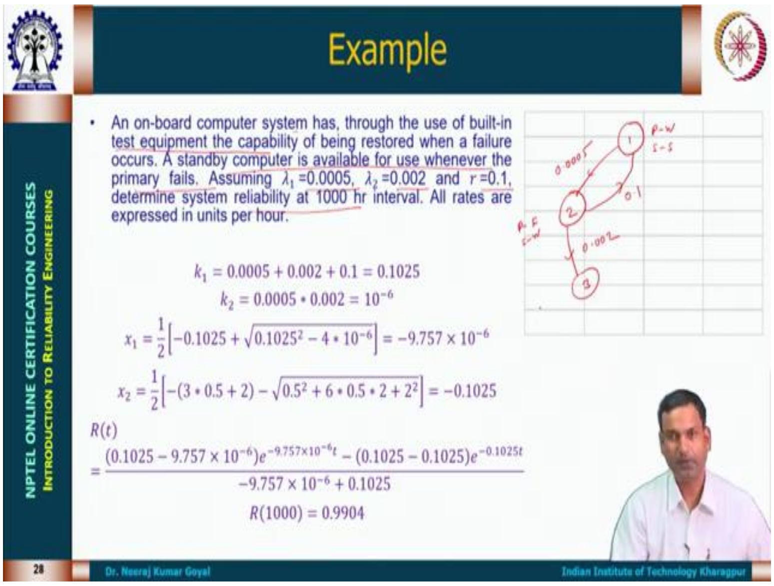

So, let us take one example here. So, let us take that we have on board computer system which has built in test equipment capability. Built-in test equipment is generally critical systems which on failure can lead to disasters, like airplane systems, missile systems. So, they what they have they have self-checking capability.

So, they have a built-in test equipment. So, like if you have seen many times various plays, etcetera that whenever even if you have gone somewhere then initially before taking the flight or before the any rocket is launched, like if you have seen ISRO launch, if you have seen NASA launch, they have a reverse counting, reverse counting that means actually that test is running for almost 1 week or 4 days where they are testing each and every parameter of your subsystems and identifying back other all parameters are in range or not.

So, that is the built-in test equipment, which is testing, its own circuit, its own components and verifying whether they are in good state or not whenever everything is verified and found in the good State then only the launch happens. So, this is the built-in test equipment and of being restored when failure occurs.

So, built-in test equipment can identify the fillers, can also correct the fields. So, we have a standby computer here whenever the primary fails. Assuming, Lambda 1 as 0.005, Lambda 2 is 0.002 and r is 0.1, determine the system reliability for 1000 hour interval? So, I can make this diagram here, I have the, here the primary system is working and secondary is in standby.

So, my failure rate from State 1 is Lambda 1, that is 0.0005, then it may reach to the second stage, in second state what happens primary is in failed condition, I can say it is in failed condition or I can say that primer is under repair, because the field unit is getting repaired, immediately when it fails, it will go to the repair.

So, it can get repaired and again you can reach to the first stage where primary is working and secondaries in standby. The repair rate is 0.1 and how much is the failure rate for unit 2? Because now in this case, secondary, primaries field and secondary is working. So, the failure rate of secondary is Lambda 2 that is 0.002, and my third state is the failure state, if it reaches to this state I cannot recover the system, my complete system failure happens here. So, for this system now we want to calculate the reliability.

So, to calculate the reliability as we have seen earlier, I can develop the equations here again. So, I will make this in a separate slide, so, that we can follow the same. So, my values are 0.0005, 0.002, 0.1.

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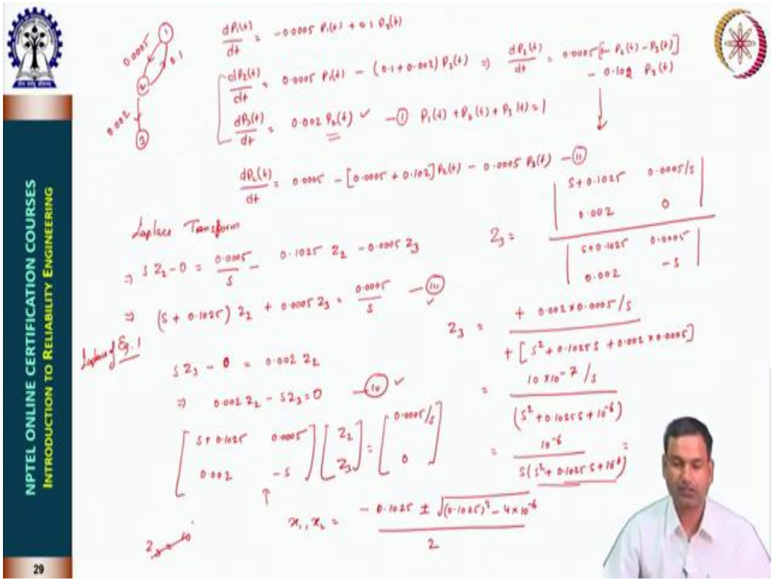

I will make the same system here 0.0005 and repair it as 0.1 and this is 0.002. So, let us follow the same steps as we have followed earlier. So, dp1 t over dt will be equal to minus 0.0005 P1t and plus 0.1, P2 t incoming positive outgoing negative. Similarly, dp2 t over dt is equal to for this state incoming is 0.0005 from P1, State 1 and outgoing is 2, outgoings are there 0.1, here 0.002 here, minus 0.1 plus 0.00 2 into P2 t and similarly dp3 t over dt, there is only incoming no outgoing this is of absorbing state, that is 0.002 into d2 t.

As we discussed earlier, we can solve these equations, to solve these equations we will be using 1 more equation that at any time system has to be in 1 of the 3 states, and, the summation of probability of the 3 state is always 1. So, as we discussed earlier, we can solve these 2 equations by replacing P1 here, this second equation I can again write it as P2 t differentiation versus dt is equal to 0.0005.

Now, P1 t I can write it as 1 minus P2 t. So, minus P 2 T minus P3 t, minus this is 0.102, 0.102 into P2 t. This equation let us solve further, here my dp2 differentiation of P2 t over dt will be 0.0005 minus 0.0005. Now, here 0.102 is also 0, 0.102 into P2 t and for p3 t, only 1 term is there that is 0.0005 P3 t.

Now, this equation and another equation is this. So, if let us say this is equation number 1, this is equation number 2 for solving this. Now, let us take the Laplace transform. So, if I take Laplace transform, this will become Z2 into S and P2 at 0 is 0, so minus 0, this is equal to for constant term Laplace transform is 1 upon S minus…

Now, this if we sum up this will become 1.102, 0.0005. So, that will become a 0.1025 and P2 team is Z2 minus 0.0005 Z3. If I take it on 1 side, then S Plus 0.1025 into Z 2 plus 0.0005 Z3 is equal to 0.0005 divided by S, this becomes my lattice equation number 3. Now, from this equation number 1, if I take Laplace transform, then my equation will become Z3 into S minus 0 P3 at 0, 0 will be equal to 0.002 into Z2. So, this I can write it as 0.002 Z2 minus S, Z3 is equal to 0.

So, now I have set of 2 equation third and fourth, I can convert this into Matrix form, if I convert this into Matrix form this will become S Plus 0.1025, 0.0005 and for this equation this is 0.002 and this is minus S multiplied with Z2 Z3 is equal to, this equation is 0.0005 divided by S and this is 0. Same equation as we developed here, same equation has come here.

As we know that my objective is to calculate system reliability, why will I calculate? So, many values I will simply calculate the Z3 only, I will calculate the Z3 here. So, Z3 will be equal to replacing the second column with this, this will be S Plus 0.1025 First Column will remain same and second column will replace by the side of the values 0.0005 divided by S and 0, divided by this Matrix or determinant S Plus 0.1025, 0.0005 0.002 minus S. If I solve this, then Z3, if I solve it this is 0 and minus of this will be minus of 0.002 into 0.0005 divided by S, whole divided by this.

If I multiply then minus of S into this, that will become S Square Plus 0.1025 into S and minus of this because minus sign is outside. So, this will become plus plus 0.002 into 0.0005. This if I solve, then 5 into 2 will become 10 and or minus 3, 10 to the power minus 3 10 to the power minus 4, 10 into 10, 10 into 10 to the power minus 3 minus 4, 7 divided by S, now, minus has become plus divided by S square plus 0.1025 S this 10 into 1 10 to the power minus 7, I can write it as 10 to the power minus 6 this becomes 10 to the power minus 6 divided by S into S square plus 0.1025 as plus 10 to the power minus 6.

Now, this equation if I want to solve, then I will get the X1 X2, for X1 X2, I have to solve this equation that is minus B, minus 0.1025 plus minus square root of B Square, that is 0.1025 Square minus 4, A is 1 and C is 10 to the power minus 6 divided by 2a, these values which I have got I can calculate I will show that in using the excel, I can calculate and I will show you. X1 X2 I have got, so I can write this as, sorry, I can write this Z3 now as 10 to the power minus, I will take this on next slide.

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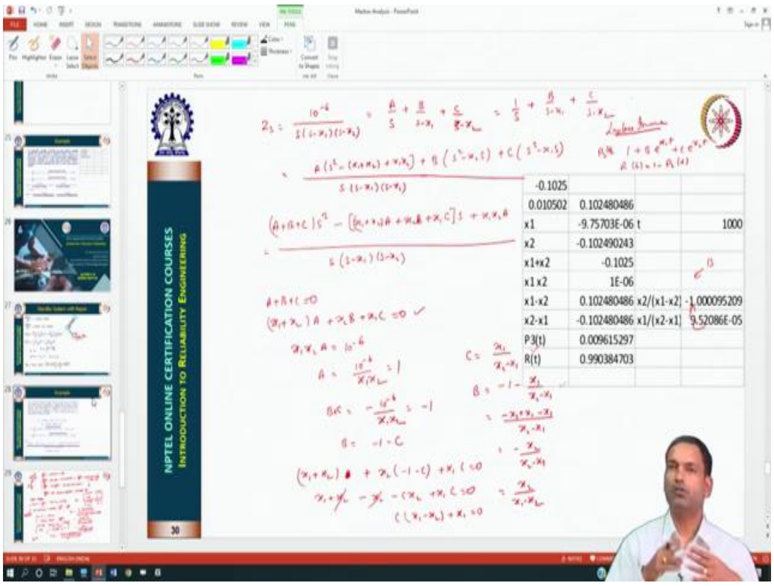

So, I will say Z 3 is equal to 10 to the power minus 6 divided by S into S Square S into S minus X1 into S minus X2, X1 X2 I have already seen how to calculate and this I can again put as a upon S Plus B upon S minus X1, plus C upon S minus X2. And as we have solved earlier that is S into S minus X1 into S minus X2, this will become A into S Square minus X1 Plus X2.

If you wish, we can calculate directly first and can put the values here, that will be even easier. So, let me calculate the X1 and X2 here, I will use this I will keep the notation here, so that we can follow it later on. I will remove this from here, let us see my values were 1.1025 and 4 into 10 to the power minus 6.

So, I will place it here, minus 0.1025 and second value was minus 1.025, 0.025, 0.1025 square and, 0.1025 Square minus 4E minus 6, I will put equal sign here and I have to now take the square root of this this is equal to this power 1 by 2 or 0.5. So, this is my first term this is my second term.

So, my X1, if I calculate X1 is equal to this term minus, I will take the plus here, plus square root of this, that is my square root term and whole divided by 2, so I will divide this by 2. Same way, I can calculate X2, for X2, I will take the negative term. So, that means I will take this term minus square root term divided by 2.

So, my X1 X2 is known here, same X1 X2, I can use directly here and X1 plus X2 is also required. So, I will take the X1 Plus X2. So, X1 Plus X2 is equal to this Plus this, that is Point similar to because 0.97 K of 0.102 B4 plus B3, sorry, there is a mistake here, so minus 1.1025.

So, same thing let us see here rather than writing. So, I can write here this is equal to A into, X1 X2 is known to me, if I want, first let us solve here that A into S Square minus. So, minus will become plus, 0.1025 plus X1 X2. So, X1 X2 again, we have to calculate or that will be little problem. So, what we do? We first solve this in X1 X2 term and let us see how we go forward.

So, plus X1 X2 will follow the same as we did earlier B into S Square minus X2 into S plus C into S Square minus X1S and when we compare the term then A plus B plus C into S Square minus X1 plus X2 into a plus X2 into S X2 into B Plus X1 into C whole multiplied by S plus X1 X2 a divided by S into S minus X1, S minus X2, and solving this will give A plus B plus C is equal to 0, and X1 Plus X2 into A plus X2 B plus X1 C equal to 0 and third is X1 X2 a equal to what we have 10 to the power minus 6. So, A will be equal to 10 to the power minus 6 and X1 X2.

Now, here, and B will be equal to, A plus B plus C will be equal to minus A. So, minus 10 to the power minus 6 divided by X1 X2. Here, we will try to calculate this X1 X2 here, X1 into X2 will be equal to this multiply by X1. So, that is 10 to the power minus 6, this is almost similar to what we solved earlier, this becomes X1 X2 is 10 to the power minus 6, that will become minus 1 and. So, A is equal to 1, B plus C is minus 1, B is equal to minus 1 minus C if I put that in here, then this will become X1 Plus X2 into A, Ais 1 plus B is minus 1 minus C, plus X2 into minus 1 minus C plus X1 C equal to 0.

Now, X1 Plus X2 minus X2 minus C X2 plus X1 C equal to 0. So, C into X1 minus X2 plus X1 equal to 0. So, C will be equal to X1 upon reverse of sine will happen, X2 minus X1, and B will be equal to minus 1 minus C, minus 1 minus X1 upon X2 minus X1. If I take X2 minus X1 as common minus X2 plus X1 minus X1. So, minus X2 upon X2 minus X1, that will be equal to X2 upon X1 minus X2.

Now, all the values I have got and I can calculate this here. So, this will be equal to 1 upon S and how much is B? value of B is X2 upon X1 minus x 2. So, let me calculate that again here, I can calculate X1 minus X2 here, that is equal to X1 minus X2. Similarly, I can calculate X2 minus X1 that will be nothing but minus of this 2 minus X1.

So, minus 1.025 approximately you can take. So, this will become plus 1 plus B is how much? B is now we can take X2 divided by X1 minus X2 again, we have to, we can again do this. So, this will be equal to X2 divided by X1 minus X2, if I calculate this, this will be equal to X2 divided by X1 minus X2.

Similarly, we can calculate X1 divided by X2 minus X1, this is equal to X1 divided by X2 minus X1. Now here, our equation will become simpler, we have got all the values here, I can take the values directly from here. So, my B values here, A value is here, I have calculated all I can put them directly here.

Now, I want to calculate this reliability. So, reliability will be equal to 1 minus P3. So, I will write B upon S minus X1, plus C upon S minus X2, I will know I know B value I know C value directly which is here. So, X2 upon X1 minus X2 that is my B, and this is my C value, I can take directly from here and put it.

So, since Z3 is here, from here I can calculate P3. So, P 3 will be taking the LaPlace inverse of this. If I take LaPlace inverse then P3 will become 1, plus B,, E to the power X1 t, plus C E to the power X2 t, X1 t X2 t all are available with me, I can calculate, and R t will be equal to 1 minus P3 t. So, I can calculate that directly here, let me just calculate the P3 t here.

So, this is, let us calculate P3 I will take value of t here somewhere, value of T was thousand. So, P3 t is equal to 1 Plus as we discussed B is X2 upon X1 minus X2, this one, B multiply by exponential of X1 t this is my X1 multiply by t Plus C, C is X1 upon X2 minus X1, C into exponential of X2 t X2 is this multiply by t. So, this becomes my P3 t and how much will be Rt, Rt will be equal to 1 minus P3 t, 1 minus of P3 t.

So, this becomes my reliability 0.99038. So, here I can solve this using the using this or Lambda and X1 X2 etcetera or I can put the values continuously and try to solve and that those values also, I can use directly in the equation. So, as we see let us compare whether we are getting the similar values or not, we have the 0.90399038, let us see how much we are having, yeah, 99038, if you make this approximate 38 will become 4.

So, same equation what we have got here we can solve our equations numerically also and then also using the same steps we can use the Laplace transform and inverse Laplace transform and we can again get the values here. So, this I am, here like Excel sheet, we have used for this purpose, for calculation because calculator I will it will be difficult for me to show you, how to calculate using calculator, but I can use the Excel sheet in much easy way and that can be replacing the calculator work. Because here I can store the values easily for you to later on C.

So, here as we have seen or we have discussed Marco methods and we are able to solve various equations, various systems, and various configurations we have taken, and, here as we see that actually for repair case the Markov is considered to be the way to calc, evaluate the reliability. So, to calculate the reliability we can develop the Markov diagram, then repair is there, and we can solve using this Laplace transform and inverse Laplace transform, all the systems can be solved, and once you solve them you will be able to get the state probabilities.

Now, from State probabilities, we can get the system relative system to have t is the summation of probability of those states which are considered to be working States, and/or we can say reliability is 1 minus probability of failed State. Like in this case, we did not calculate all State probabilities, we calculate only the probability of failure state, and from there we calculate the reliability 1 minus probability of the failure State, P third stage, first the state was the failure state. So, sometimes that can also be done to save the calculation efforts.

So, with this our discussion on Markov analysis, whatever we wanted to cover in this course is completed. So, in next week we will move to the next topic. Thank you.