Introduction to Reliability Engineering
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Lecture 23
Markov Analysis (Contd)

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Hello everyone, we have been discussing about Markov Analysis. And in previous 3 lectures we discussed how we can use Markov analysis to determine the system reliability considering two component considering shared load system considering standby system. So, today we will consider few more system configurations today, we will also try to see that if repair is also possible, then how will it impact the reliability that analysis becomes little tedious, but we will try to do it with one example or two examples and hopefully, you will learn that.

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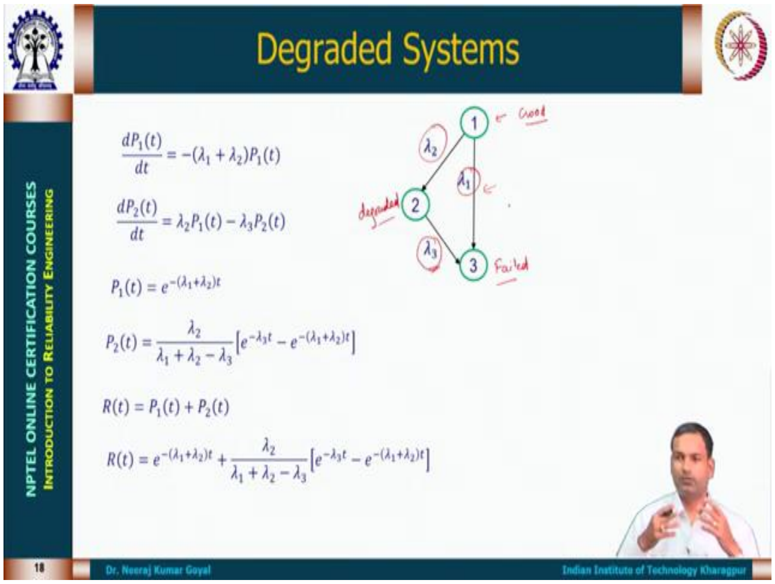

First let us discuss about degraded systems. First to understand the degraded system, we need to understand what is mean by this. So, most of the systems when we use initially they will be in good state good state means capacity is full, they can take all the load and the reliability is also good, but as the system becomes older or system has sometimes sudden experience experiencing sudden stresses because of which they become weaker.

So, system will be in degraded state so, degraded state means that system is having little bit loss in health the health is not as good as new. And because of that the chances of failures becomes high. So, here the system can be in three state. So, this our system or we can say component this is applicable to component also system also.

So, at the component level or the system level, we are experiencing three states one is system is in good state there it is working fine chances have from here there are two possibilities it can get degrade or it can fail also. So, if it fails the failure rate for failure from system state 1 2 3 that means, it does not go to degrade, but directly failed in that case that failure rate is lambda 1 and there is a rate lambda 2 by which the system can degrade.

So, generally what happens and in in a system or component of failure can be happened with due to the random reason or sudden reasons also, degradation mostly is understood mostly this is more of systematic way of failure where slowly slowly either bear like for tire this will become bear or slowly slowly it will bear or if we are dealing with metals, they may get rusted and slowly slowly their strength will decrease. Similarly, other parts they may have with time they may be their properties may be changing of the material and because of that over the time, the system strength will not be same as it was when it was new.

So, there will be degraded state and from degraded state we have another failure rate by which it will be reaching to the failure state generally this failure rate lambda 3 will be much larger than our failure rate lambda 1 because in under degraded state chances of failures becomes high the probability of failure becomes high.

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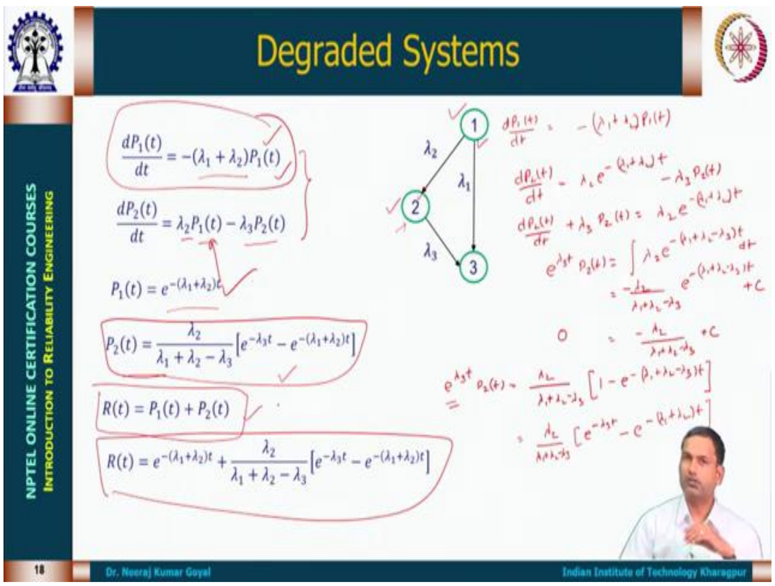

Now, to solve this system we will use like what we have solved already. So, here we will try to evaluate this now, as we know what will be DP 1 over dt that is outgoing only outgoing states we have so, minus sin lambda 1 plus lambda 2 into P1 t same thing and for state number 2 we have incoming that is lambda 2 from P 1 lambda 2 from P 1 and outgoing is from itself that is lambda 3 minus lambda 3 into P 2.

$$\frac{dP_1(t)}{dt} = -\left(\lambda_1 + \lambda_2\right) P_1(t)$$

$$\frac{dP_2(t)}{dt} = \lambda_2 P_1(t) - \lambda_3 P_2(t)$$

$$P_1(t) = e^{-(\lambda_1+\lambda_2)t}$$

$$P_2(t) = \frac{\lambda_2}{\lambda_1 + \lambda_2 - \lambda_3}\left[e^{-\lambda_3 t} - e^{-(\lambda_1+\lambda_2)t}\right]$$

$$R(t) = P_1(t) + P_2(t)$$

$$R(t) = e^{-(\lambda_1+\lambda_2)t} + \frac{\lambda_2}{\lambda_1 + \lambda_2 - \lambda_3}\left[e^{-\lambda_3 t} - e^{-(\lambda_1+\lambda_2)t}\right]$$

Now, if we solve these then from this we already know as we have solved many times earlier, this equation solution gives e to the power minus lambda 1 plus lambda 2 into t. So, p 1 t is equal to e to the power minus lambda 1 plus lambda 2 into t. If you put the same thing here and we fix all this then the 2 comes out to be this value this also we have solved many times or you can try again and you can solve this.

Or if you want we can also solve this very easily that DP 2 P 2 t differentiation versus t is equal to lambda 2 e to the power minus lambda 1 plus lambda 2 into t minus lambda 3 into P 2 t as we can take this left side this will become DP 2 t over dt plus lambda 3 this is p 3 P to t lambda 3 P 2 t will be equal to lambda 2 P to the 1 minus lambda 1 plus lambda 2 into t.

As we have discussed earlier if we multiply E to the power lambda 3t on both sides then this will give E to the power lambda 3 T P 2 t will be equal to integration of this term multiply with e to the power lambda 3t lambda 2 e to the power minus lambda 1 plus lambda 2 once you multiply with lambda 3 this will become inside it will become minus lambda 3 into t dt.

So, this if we solve this will be equal to lambda 2 divided by lambda 1 plus lambda 2 minus lambda 3 minus sin e to the power minus lambda 1 plus lambda 2 minus lambda 3 t plus C and we know at t equal to 0 this will be 0 this will be minus lambda 2 upon lambda 1 plus lambda 2 minus lambda 3 plus C.

So, C will be equal to lambda 2 upon lambda 1 plus lambda 2 minus lambda 3 and this equation will be to E to the power lambda 3t E 2 t will be equal to lambda 2 upon lambda 1 plus lambda 2 minus lambda 3 into 1 minus e to the power minus lambda 1 plus lambda 2 minus lambda 3 into t once you divide by this on right hand side this will become lambda 2 upon lambda 1 plus lambda 2 minus lambda 3 into e to the power minus lambda 3 t minus e to the power minus lambda 1 plus lambda 2 into t the same thing is here what we have done.

And what is reliability here now, it depends on how we define reliability in most of the cases in whether system is in degraded state or good state in both the cases the system is working though it may not in degraded state it may be having little lesser capacity, but still it is functioning so, if you consider degraded state as a reliable state or as a working state in that case reliability will be equal to probability of system state in p 1 or system state in P 2.

So, if we sum up the 2 probabilities we get the reliability if we consider only state 1 as the reliable state state 2 and 3 are considered to be fairly state then the reliability will be only this value. But generally we will consider P 1 2 plus p 1 t plus P 2 t as the reliability.

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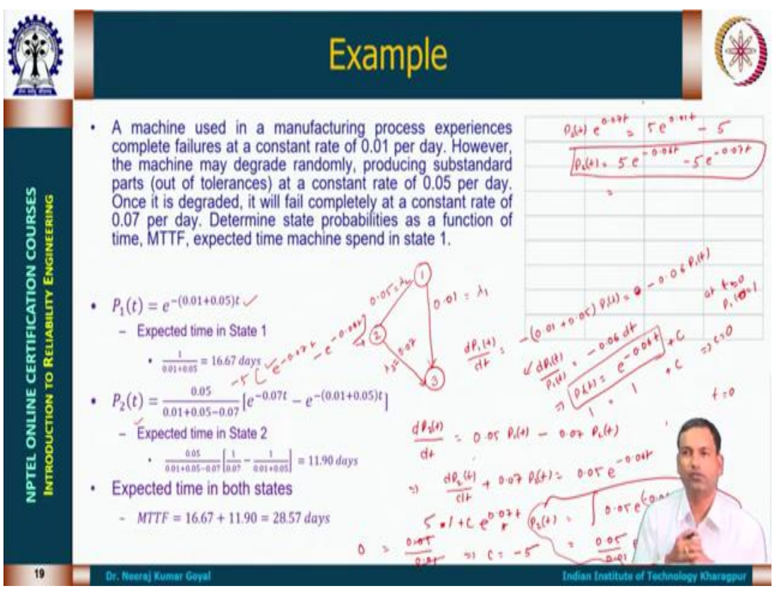

Now, let us see this is this an example. So, let us say we have a machine which is used in a manufacturing process and experiencing complete failures as constant at 0.01. So, we have the system state 1 complete failure that is our 0.01 in previous figure that has been given us lambda 1 and this is state number 3, this is lambda 1, then, however, the machine will degrade randomly producing substandard parts at a constant rate of 0.05 per day.

$$P_1(t) = e^{-(0.01+0.05)t}$$

– Expected time in State1

$$\cdot \frac{1}{0.01+0.05} = 16.67 \text{ days}$$

$$P_2(t) = \frac{0.05}{0.01+0.05-0.07}\left[e^{-0.07t} - e^{-(0.01+0.05)t}\right]$$

– Expected time in State2

$$\cdot \frac{0.05}{0.01+0.05-0.07}\left[\frac{1}{0.07} - \frac{1}{0.01+0.05}\right] = 11.90 \text{ days}$$

Expected time in both states

- MTTF $= 16.67 + 11.90 = 28.57$ days

So, 0.05 is the rate by which it may get degraded once it is degraded, it will fail completely at a constant rate of 0.07 per day 0.07 per day. So, this is my lambda 2 and this is my lambda 3. If I want, I can use the formula which we developed earlier or what like all the equations, we have developed by taking them by taking some representation here like lambda.

So, by putting the values in the formula we can got it, but we can also solve this directly. If you see that, if we solve this directly, we know that DP 1 t over dt will be equal to minus of 0.01 this 1 and minus of 0.05 into P1 t. So, here this is equal to minus of 0.06 P1 t. So, if we solve this then DP 1 t over P1 t, if we say will be equal to minus 0.06 dt. So, if we solve this then P1 t will be equal to e to the power minus 0.06 t plus c at t equal to 0 p 1 t is equal to p 1 0 is equal to 1.

So, this will become 1 l2 to the power 0 is also 1. So, C will become equal to 0. So, this becomes my P 1 t similarly, I can use the equation for this state now, for this state so, this we have already got this.

So, once we solve this we will be able to get p1 t similarly P 2 t if we want we can solve again from the figure directly. So, we can say DP 2 because sometimes solve if let us say you are not able to remember these equations. So, if you are not able to remember these equations, then solving preparing the Markov diagram and then solving it for lambda well lambda representations or the symbolic representation would be a little bit tougher compared to if we take the numeric values solving for numeric values maybe sometimes simpler, like if you write dP dT over dt that is positive 0.05 or p 1 T and negative 0.07 P 2 t.

Now, here if we see if we can resolve this, so, DP 2 t over dt plus 0.07 e to t is equal to 0.05 or P1 t we have already calculated here if we want we can put it this value directly here if we have calculated for a certain time t we can use that directly, but we can use this so, that is e to the power minus 0.06 t.

Now, here this as we discussed this become e to the point 0.07 t into P 2 T which is equal to integration of 0.05 e to the power minus 0.06 plus 0.07 into t this 0.07 will get multiplied here dt this if we solve this will be equal to 0.05 so, 0.06 and 0.07 so, this will be E to the power 0.01 T.

So, this will be 0.01 e to the power 0.01 t plus c and we know at t equal to 0 so, when you put equal to 0 left hand side will be 0 and right hand side will be 0.05 divided by 0.01 which is nothing but 5 it and E to the point 0 will be 1 plus C. So, this implies c will be equal to minus 5. So, this p 2 T which I have got here I can solve it here.

So, my P 2 t will be equal to P 2 t into e to the power 0.07 t will be equal to 5 e to the power 0.01 t minus 5 now if I divide this by exponential term that my P 2 t will be equal to 5 minus 5. So, I can take 5 common here or I can take then I will just do this little differently so here, P 2 t will be equal to we will divide by here so this will become 5 e to the power minus 0.01 minus 0.07 will be 0.06 T minus 5 e to the power minus 0.07 t.

So, if I take a common I can take or I can simply put this value as the P2T and same value is given here, if you solve this, you will get the same value. So, this gives me a P 2 t and like this if I solve 0.05 0.06 minus point, so minus 5 and this will become e to the power minus 0.0 7 t minus e to the power minus 0.06t, which is same as this.

So, here as we see that, we can use the formula directly if we forget the formula, then we can use this numerical or numerically also we can follow the same process of solving and we can find out the values p 1 2 p 1 t and p 2 t. I am removing this because all the formulas are already mentioned here. So, I will erase this.

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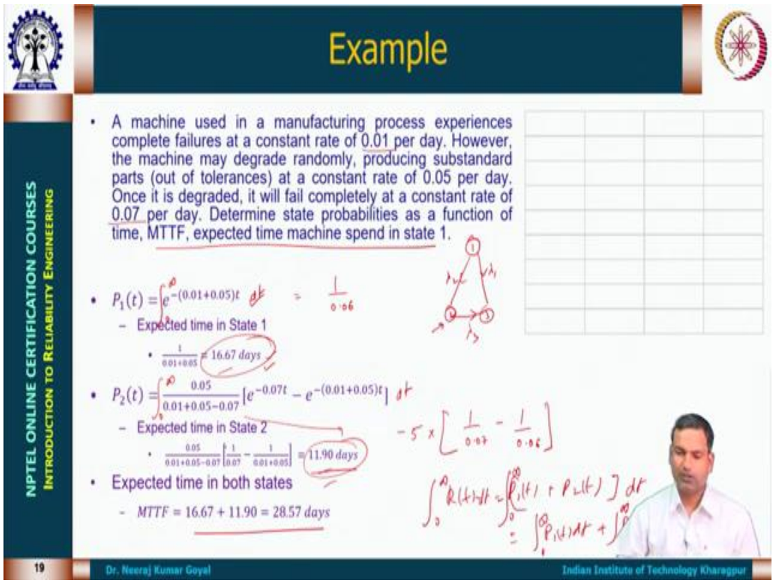

Now, let us see here, we want to know the time spent in state 1, what we have we have state 1, state 2, state 3. So, how much is the average time it will be (stan) (stan) spending in state 1 and state 2 good state and degraded state. So, to know that we use the same what we have discussed earlier. So, if I am interested in only state 1, so that means state 1 probability is this value. So, if I integrate this probability with respect to time from 0 to infinity, I will get the state 1 average time or expected time spent in state 1.

So, I will integrate this from 0 to infinity dt. And once I do that, we know that integration of this will be 1 upon lambda and what is lambda here that is 1 upon 0.06. So, 1 upon 0.06 that can also be around 16.67.

Similarly, if I want to know how much average time I will be spending system will be spending in state 2, then, I will take the P 2 probability and I will integrate this from 0 to infinity dt and we know that when we integrate this then as we have seen earlier, this is minus 5 multiply by then we integrate this this will give 1 upon 0.07 and when we integrate this this will give minus of 1 upon 0.06.

This when we solve we will be able to get the expected times spent in state 2, which is around 12 days, which you can clearly see that in degraded state the probability of failure is faster, because lambda 2 is high, sorry this lambda 3, this is lambda 2, lambda 1, lambda 2 lambda 3. So, because lambda 3 is significantly higher than lambda 1, my lambda 1 is 0.01. And my lambda 2 is lambda 3 is 0.07.

So, failure rate is high that is why what will happen, it will spend less time in this state, it will be (dep) the chances of departure from this state will be higher. So, the probability of being in this state will be lower. And also means it will quickly try to departure to the state 3 compare to departure from the stage 1 because the departure rate lambda 3 is higher than the departure rate lambda 1.

So, because of that average time spent in stage 2 is only it is quite less than those, whatever time is being spent in state 1. And how much is the MTTF as we discussed earlier, for MTTF, we need need to consider the reliability. So, reliability here will be p 1 t plus P 2 T and if we take the MTTF, then that will be this. So, RT is equal to P 1 T plus P 2 t.

So, if I integrate RT from 0 to infinity dt that will be summation of this. So whatever I have got the values I already have the values for p 1 t dt and I have already have values for p 2 t dt integration from 0 to infinity.

So, these values I already have 16.67 and 11.9. Once we apply this then my MTTF comes out to be 28.57 days. So, this is the expected time which I am expanding in both the state together. So, in in one way is giving me the MTTF.

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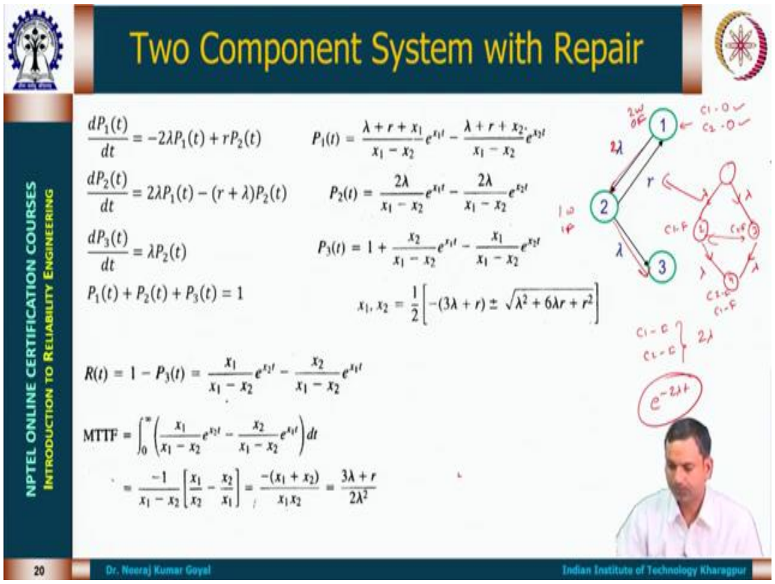

So, here as you as you have seen, we can use this for calculating the reliability and MTTF for the degraded system. So, 3 state system also can be explored in same way, but since 3 state system we have already discussed in terms of RBD. So, we are not discussing the same thing again but the same can be evaluated here also.

Now, let us consider another case in our earlier evaluations, we have considered that the system can only fail it cannot be repaired, because there was no repair path available, but, if we consider the system that it can get repaired then RBD cannot be used for that purpose because system is state repair will depend on the failure state once it fails then only it can go go for the repair. So, that is why we have to use this Markov diagram as a Markov process to solve the problem.

So, here like we can represent the same thing as a state diagram here so, this is state is my where both the components are working. So, component 1 is working operating and component 2 is also operating. So, there is 1 problem here that is this should be 2 lambda so, this I will correct later.

So, because what happens here the failure rate is 2 lambda because there are 2 components working since 2 components are working we know that any one of them can fail. So, either component 1 can fail or component 2 can fail.

So, the failure rate becomes double that becomes 2 lambda because this becomes e to the power minus 2 lambda t the reliability of staying in the same state is e to the power minus 2 lambda t. So, the failure rate becomes 2 lambda.

So, same if I had made this diagram into this fashion 2 and if I made these 3 here lambda here lambda here this is for component 1 component 1 failure and this is for component 2 failure then again I can make this as a 4 that is again lambda again lambda this is for component 2 failure and component 1 both are failed here.

So, either I can make like this or rather than doing this I can because both are similar component both are lambda so, rather than departure it is 2 lambda here lambda for component 1 lambda for component 2 and this state 2 and state 3 are the similar state in both the state 1 component is in failed state and 1 component is in working state.

So, this diagram I can represent through this the 2 lambda going here and lambda going here because here 1 component working and a 1 failed and here 2 working 0 failed. Now, here I am removing this for clarity.

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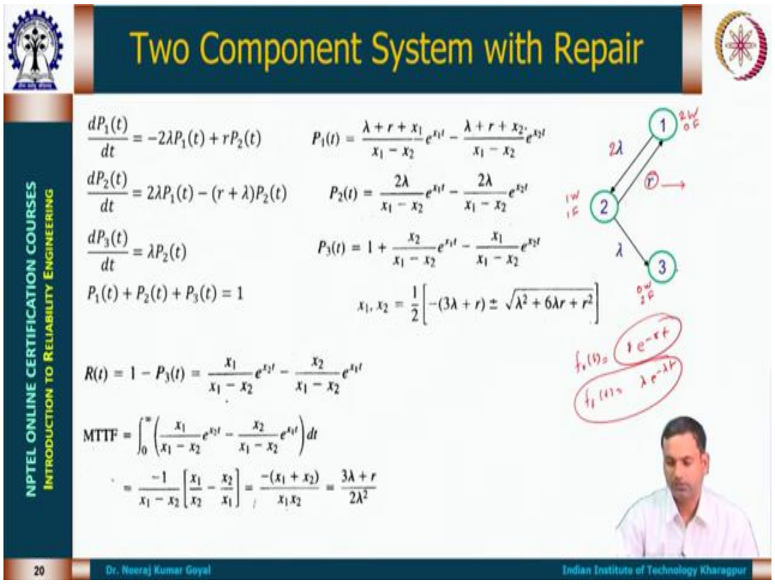

So, this is 2 lambda remember and here we can say that 2 components working and 0 component in failed state this is 1 component in working 1 component in failed state. And here both component in failed state 0 working.

So, here if I want to write here I am considering the repair also what I am considering that my repair is also following the exponential distribution and the (repar) rate of repairs r similar to lambda lambda is the failure rate r is the repair rate so, I can see that repair distribution is r e to the power minus r into t this is f repair distribution pdf of repair. Similarly, my failure distribution is lambda failure is lambda e to the power minus lambda t.

So, similar to that, if my repair is also following exponential distribution, because for Markovian we try to use distribution which is following the exponential distribution so this r becomes the rate of transition from state 2 to state 1. So, this gives me the repair possibility.

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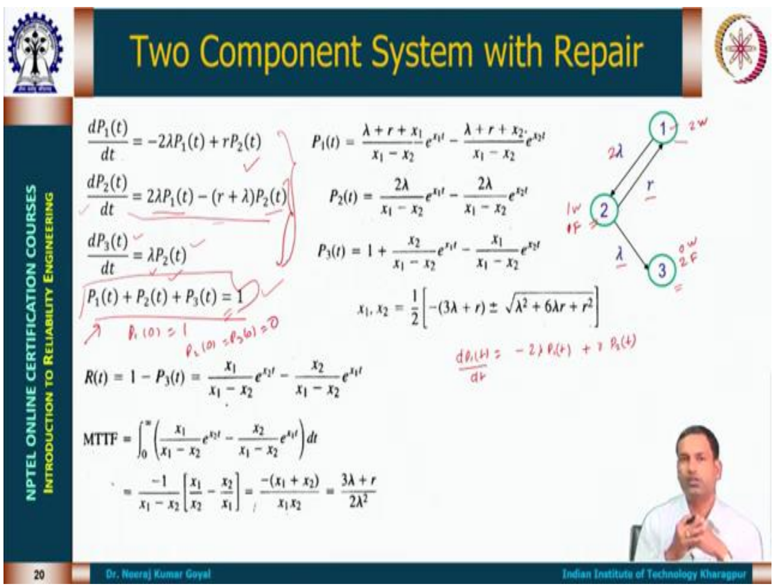

$$\frac{dP_1(t)}{dt} = -2\lambda P_1(t) + rP_2(t) \qquad P_1(t) = \frac{\lambda + r + x_1}{x_1 - x_2} e^{x_1 t} - \frac{\lambda + r + x_2}{x_1 - x_2} e^{x_2 t}$$

$$\frac{dP_2(t)}{dt} = 2\lambda P_1(t) - (r+\lambda)P_2(t) \qquad P_2(t) = \frac{2\lambda}{x_1 - x_2} e^{x_1 t} - \frac{2\lambda}{x_1 - x_2} e^{x_2 t}$$

$$\frac{dP_3(t)}{dt} = \lambda P_2(t) \qquad P_3(t) = 1 + \frac{x_2}{x_1 - x_2} e^{x_1 t} - \frac{x_1}{x_1 - x_2} e^{x_2 t}$$

$$P_1(t) + P_2(t) + P_3(t) = 1 \qquad x_1, x_2 = \frac{1}{2}\left[-(3\lambda + r) \pm \sqrt{\lambda^2 + 6\lambda r + r^2}\right].$$

$$R(t) = 1 - P_3(t) = \frac{x_1}{x_1 - x_2} e^{x_2 t} - \frac{x_2}{x_1 - x_2} e^{x_1 t}$$

$$\text{MTTF} = \int_0^\infty \left(\frac{x_1}{x_1 - x_2} e^{x_2 t} - \frac{x_2}{x_1 - x_2} e^{x_1 t} \right) dt$$

$$= \frac{-1}{x_1 - x_2} \left[\frac{x_1}{x_2} - \frac{x_2}{x_1} \right] = \frac{-(x_1 + x_2)}{x_1 x_2} = \frac{3\lambda + r}{2\lambda^2}$$

Now, what happens here? This brings us to the scenario what is the scenario here that we initially have 2 components working, here, 1 of them failed, we reached to this state. Now, what will happen in this state 1 component is failed that goes into the repair and the component which is working that continues to work.

So, because of that what happens from this state to this state, there are 2 possibilities. Now, 1 is working 1 is in failed condition, if the component gets repaired and nothing happens to this what will happen this will reach to the this state where both are working. But, before repair happens, it is possible that the component which is working can also get failed. So, in that case what will happen both will be failed 0 will remain in fail working condition. So, from this 2 state we have 2 transition possible either failed component get repaired, if it is repaired, we are reaching to the state 1 where both components are working then the system will assume that both components are working and other states is that the working component fails that means, repair does not happen, but before that the working component gets failed.

So, in this case what will happen we will have the both components failure. So, now here what is happening because of the repair, my reliability is supposed to improve the reliability which I had earlier because earlier what was happening the failed component was thrown away from the system the failed component was not was no longer considered to be part of the system, but here the failed component can be repaired and because of that the system can again get strengthened to working conditions that is the 2 components working.

Now, to solve this, we can evaluate this and get this for P 1 DP 1 T over dt will be equal to outgoing is 2 lambda. So, minus 2 lambda from state p 1 p 1 T and what is incoming incoming from state 2 so, plus and how much is that R R into P 2 t this becomes our first equation similarly, for state 2 incoming is 2 lambda here so, 2 lambda p 1 T and outgoing is lambda 2 that state 3 r to the state 1 from the state 2, so, minus P 2 T into lambda plus r that becomes about dP T over dt.

Similarly, P3T over or DP3 change in P3T only incoming is there so, that will be the positive lambda into PTT here when we are solving this this equations, we have to consider 1 more condition that is p 1 T plus P 2 t plus P 3 t is equal to 1 this is the absolute absolute probability which is true always whether time is 0 at time t equal to 0 p 1 0 is equal to 1, but, P 2 0 is equal to p 3 0 is equal to 0 that means, initially the system is in state 1 that is why it p 1 0 is 1 and P 2 0 and P 3 0 is 0 anytime, whenever we consider the system has to be in one of those state either 1 2 or 3.

So, this equation has to be satisfied all the time. So, when we are solving these equations, generally we drop one of the equations and we consider this equation as the additional equation. So, here to solve this actually what is happening now, if I take these equations, then none of the equation is having the single parameter like in earlier cases when we solve whenever we solve for P 1 t all the equation was in terms of P 1 T there was no P 2 there was no p 3.

So, the equation could be solved easily and once we got the value of p 1 we put that value in the equation for P 2 and then once we put the equation for p value for p 1, it becomes the equation for P 2 only. And we could solve that easily.

So, single para because this was single probability values, it was easy to solve, but now, because in every equation we have more than one terms like here we have p 1 and P 2 both in the second equation we have p 2 p 1 both in third equation we have p 3 and P 2. So, none of the equation can be solved directly. So, we have to use as system of equations here. So, we have to consider that this is set of differential equations, which we need to solve together.

To do that, first we will try to get these equations modified little bit here actually we have DP3t which is the shortest equation. And if we see p 2 t that is in p 1 and P 2, this is p 3 and P 2 now, if I use this equation, I can convert this equation into if I replace P 1 value from here as 1 minus P2T and P3T, then this equation will become only p 2 and p 3. So, we have this equation in P 2 and p 3 this will also be in p 2 and p 3.

So, that means, my number of parameters which I need to solve will will become 2. So, that will be easier case to solve compared to if I take mix all three equations and then solve it. So, here we will try to solve these two equations by taking as a system of equations and this to solve this equation because both are differential equation, we have to use the Laplace transform. So, that we will discuss in next class and we will continue our discussion on this system how to solve this using the Laplace transform. So, we will stop it here. Thank you.