Introduction to Reliability Engineering
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Lecture 22
Markov Analysis (Contd.)

Hello everyone. So, we have been discussing about Markov Analysis and we discussed in previous two lectures, two different cases three actually different cases. In first lecture, we discussed two components system which are working together and then they can be series system can be series or parallel depending on the configuration, then we discussed about shared load system where if an equipment fails, then load goes to another system, in that case the failure rate of another system will be higher.

Then we also discuss the standby system. In standby system, we considered the situation where the system which is in standby can fail when it is in standby, but with a lesser failure rate, it will not have the stress so, the failure rate will be lesser when it is there in the standby mode, then the failure rate which is there in the general operating mode for the second unit. We derive the equations and we also saw that how we can solve those equations to get the values. Today we will try to do one small example for where we left yesterday that in last class that is first standby system.

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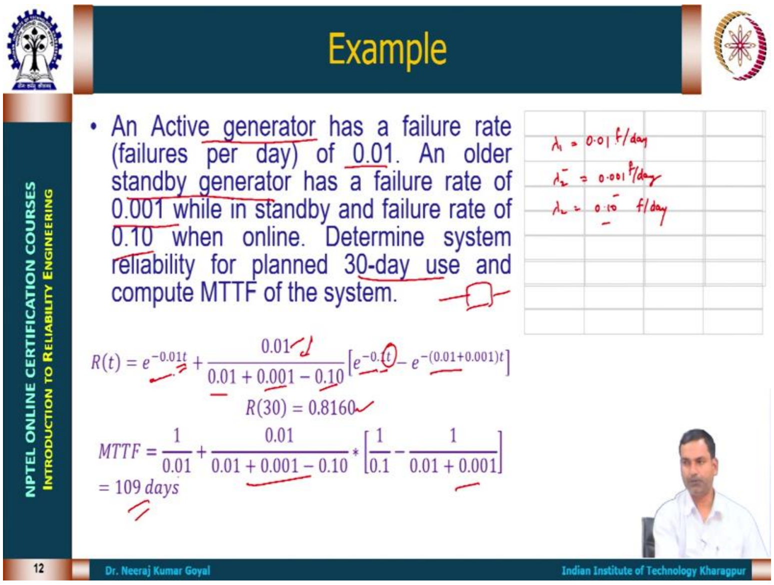
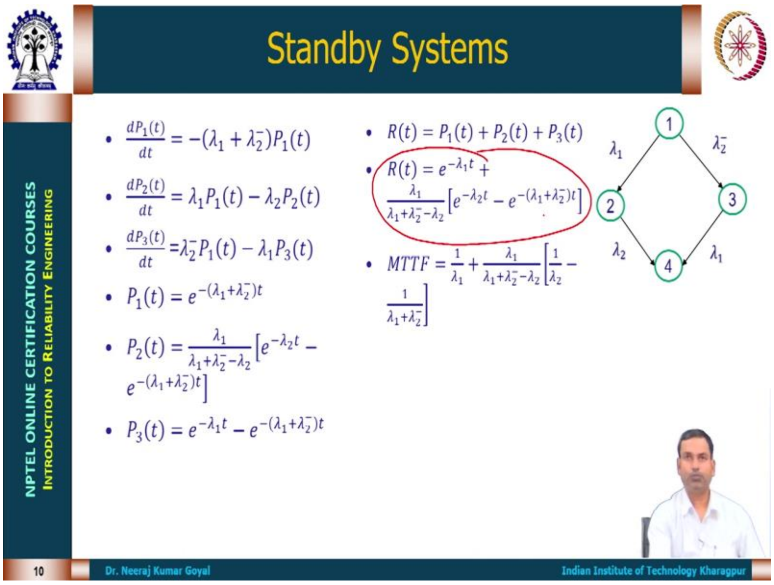


$$R(t) = e^{-0.01t} + \frac{0.01}{0.01 + 0.001 - 0.10}\left[e^{-0.1t} - e^{-(0.01+0.001)t}\right]$$

$$\text{MTTF} = \frac{1}{0.01} + \frac{0.01}{0.01 + 0.001 - 0.10} * \left[\frac{1}{0.1} - \frac{1}{0.01 + 0.001}\right]$$

$$= 109 \text{ days}$$



So, let us see the example that similar to what we considered earlier. We have active generator that means our timer unit, so, we have a generator which is working and but it has a failure rate of 0.01 failures per day. But, here we have an older generator that means, this is a new generator and this generator is new made and it is having a better reliability, the better reliability smaller failure rate, but there is another generator which we are keeping in standby.

This generator is having a failure rate of 0.001 when a standby that means it is not working just we are keeping it there, but then also may fail when we try to run this generator we may find that it is not working. Another thing is when we operate this old generator then failure rate is 0.4 which is significantly higher than the failure rate of the active generator. Now, we want to know what is the reliability for this system for a 30 day use?

As we know here, if you look at the our previous formula which we developed my lambda 1 is here 0.01. Failure rate of generator in standby mode that is lambda 2 minus that is my 0.001 per day failures per day we can say failures per day. Then lambda 2 in operating is 0.10 failures per day. Now, as we have solved earlier we can use this equation and we can get this value.

So, if we look at the previous presentation from the previous presentation, my value for reliability was coming this e to the power minus lambda 1 t plus lambda 1 upon lambda 1 plus lambda 2 minus minus lambda 2 e to the power minus lambda 2 t minus e to the power minus lambda 1 plus lambda 2 minus into t. Same equation we can use here that is e to the power minus lambda 1 t plus lambda 1 upon lambda 1 plus lambda 2 minus minus lambda 2 multiplied by e to the power minus lambda 2 t minus e to the power minus lambda 1 plus lambda 2 minus into t.

Same values we have put here for lambda 1 lambda 2 minus and lambda 2 and when we solve this we get this now, by putting t equal to 30 we get the reliability for 30 days which comes out to be 0.8160. Same thing we can solve and we can get the MTTF also for MTTF this is one upon 0.01 plus the same value will come as it is multiply by 1 upon 0.01 minus 1 upon 0.01 plus 0.01. This value comes out to be 109 days.

So, the formula which we evaluated earlier, we can use the formula to get this probability directly and we are able to know the reliability as a function of time. So, we can get the reliability for 60 days also, we can get reliability for 100 days also, whatever the value of t we put here we will get the reliability here so, we are having the relative function as a function of time t.

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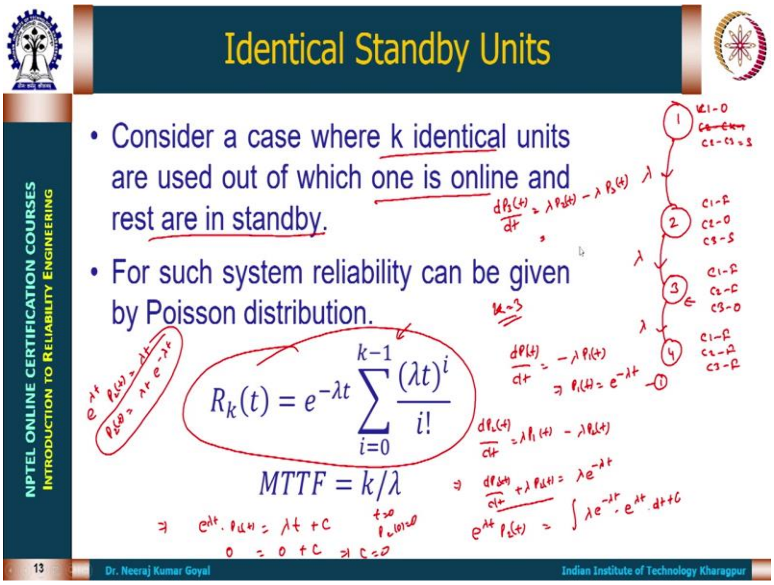

Now, let us see if our units were identical. And here we are considering that there are k identical units which are used and one of them is online and rest are in standby. So, for these kinds of systems the reliably can be evaluated and can be given using this formula. How this formula comes? Let us try to evaluate this.

$$R_k(t) = e^{-\lambda t} \sum_{i=0}^{k-1} \frac{(\lambda t)^i}{i!}$$

$$MTTF = k / \lambda$$

Let us see that we have initially we have k units working so, we have unit 1 or component 1 operating and component 2 to component k minus 1. So, let us take one value of k for an example and that will help us to understand. So, let us say k is equal to 3 here that means, I have three units for three units here. So, one unit is an operation and C 2, C 3 are in standby.

Now, from here we are assuming here that system cannot fail in standby mode. System can only fail when this is operating. So, the failure rate will be lambda here. All components are having same failure rate. Only one unit is operating. Since one unit is operating the failure rate of one unit is lambda.

Now, from here we reach to the system state 2. In system state 2 component 1 is failed, component 2 is operating and component 3 is standby. Now from here again because only one component is working and failure it is lambda it will reach to the system 3 state number 3. In the state number 3 component 1 is already failed component 2 is also failed and component 3 becomes operational.

Now, from here it will reach to system number is systems check number 4, where component with the failure rate lambda component 1 is failed, component 2 is failed and component 3 is also failed. So, this becomes our failure state. I am showing it for three the same can be expanded and can be done for the multiple states.

Now, for three as we see here, first state if we write then dP 1 t over dt is equal to only one outgoing is there that is minus lambda into P 1 t. If we solve this we know P 1 t will be equal to e to the power minus lambda into t this we already know. So, we are writing because we have solved so, many equations.

So, we already know. Now from P 2 t dP 2 over dt will be equal to incoming is from P 1. So, P 1 t into lambda and from outgoing that is from P 2 minus lambda P 2 t this if we solve this become dP 2 t over dt plus lambda P 2 t will be equal to lambda P 1 t. P 1 t is e to the power minus lambda t. If we solve this then this will become e to the power lambda t into P 2 t will be equal to, I am doing two step escaping here.

So, this actually is already taught in differential equations solutions if you remember you can directly do that. So, this term will be equal to e to the power lambda t P 2 t that will be integration of lambda e to the power minus lambda t into e to the power lambda t dt plus c. Now, if we integrate this, then what we get? We get e to the power lambda t into P 2 t which will be equal to lambda and this will become 1. So, lambda into t plus c if I put t equal to 0 then P 2 0 will be equal to 0. So, c will if I put that this will be 0, this will be 0 and c so, this implies c will be equal to 0.

So, if I put c equal to 0, then my equation now becomes e to the power lambda t into P 2 t is equal to lambda t. So, my P 2 t will be equal to lambda t e to the power minus lambda t. Similarly, if I solve for P 3 t. For P 3 t, if I write then dP 3 t over dt will be equal to plus from two, so, that is lambda into P 2 t and going out that is lambda from P 3 t. So, lambda P 3 t. If you see, this will be equal to or same way it will become like we solved earlier, same way it will become I will show this maybe I can show I need to erase this little bit so, that I can use some space here.

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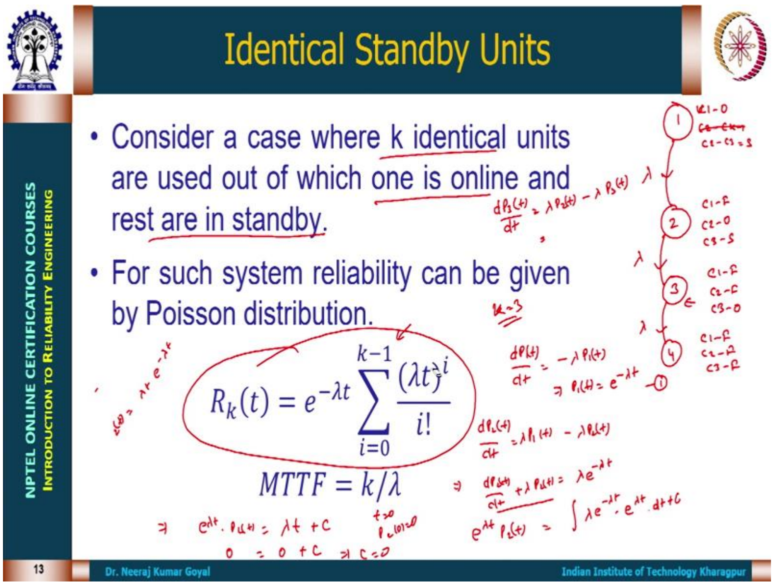

If you see here, lambda 1 if I put i equal to 0 here, then lambda t raised to the power 0, so, this will become 1. So, you will get e to the power minus lambda t that is the same term which I have got for P 1 t e to the power minus lambda 2 t lambda t, when I put i equal to 1, then this will become lambda t raised to the power of 1 divided by factorial 1. Factorial 1 is 1, so, this will become lambda t e to the power minus lambda t that same term for P 1 t.

Similarly, when I solve for P 2 t, P 3 t, my term will become i equal to 2, that will become lambda t squared divided by factorial 2. So, that means lambda t squared divided by two multiplied by e to the power minus lambda 2 t, lambda t. So, same thing we should get.

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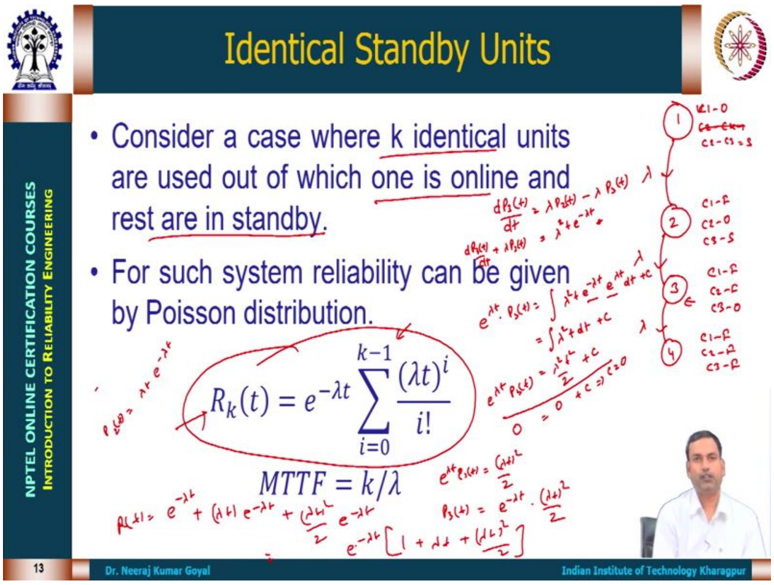

So, here I am trying to let us say if I solve the same thing, I have got some space here to solve this. So, as I see that I am removing this all. I am not removing the diagram because we need to use that again. So, let us see this now, I will put P 2 t value from here to here. So, this will become lambda so, lambda into lambda t, so, this will become lambda square t e to the power minus lambda t minus and this term when I take left hand side this will become dP 3 t over dt plus lambda P 3 t will be equal to this.

Now, again as we discussed earlier or e to the power lambda t into P 3 t will be equal to integration lambda squared t e to the power minus lambda t then again e to the lambda t will get multiplied dt plus c. Now, if you look at here this will become integration of lambda squared e to the power minus lambda t to the power lambda t will become 1 lambda squared t dt plus c. This lambda square t square by 2. If you integrate this t will become t square by 2 plus c that is equal to e to the power lambda t P 3 t.

Now, if I put t equal to 0 then P 3 0 will be 0, so, this will become 0, this is 0 plus c so, that implies c is equal to 0. So, from this equation I can get e to the power lambda t P 3 t will be equal to lambda t whole square divided by 2 and from this I can get P 3 t will be equal to e to the power minus lambda t into lambda t whole square divided by 2.

As we see here depending on the number of states same thing will come next thing will come is lambda t cube divided by for t cube when it comes t cube by 3, so, 2 into 3 that will be factorial 3. So, same way when I keep on solving depending on the number of items I have this equation will keep on evolving and this is the result which I will get.

So, my reliability is nothing but P 1 t plus P 2 t plus P 3 t and I have got all the values. So, my reliability will become e to the power minus lambda t plus lambda t into e to the power minus lambda t plus lambda t squared divided by 2 into e to the power minus lambda t. This I can write it as e to the power minus lambda t into 1 plus lambda t plus lambda t square by 2, which is same as this equation. Once we solve this further, you will get this generalized equation.

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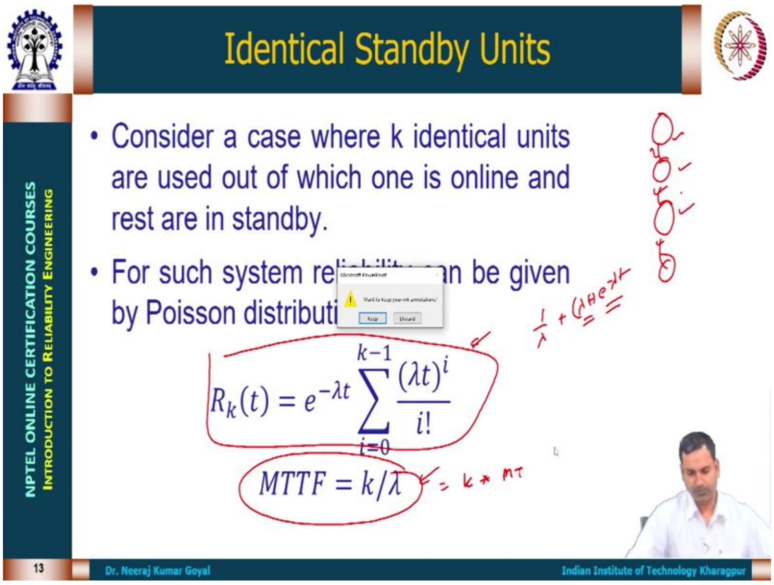

For this identical units, when we solve we are able to get this R k t. Similarly, when we integrate this function right then this integration will give 1 upon lambda for each term. So, when i equal to 0 then also we are getting 1 upon lambda. Then we have lambda t e to the power minus lambda t when we integrate this then again we get 1 upon lambda because this term on part when we solve this term is making no difference.

Then again whenever we solve further, so, for every term we are getting one upon lambda so, MTTF comes out to be k upon lambda, which is useful here also we can understand because if you look at our system, like we have k components here, or we had the k components there.

Like in this case four were there, when we consider, four states, first component and component 2 then component 3, so three components were there, they were working one by one. So, how much will be MTTF?

Because they cannot fail when this is in standby. So, on an average total time will become three times of that MTTF which you will get for the one component because they could not fail in standby mode. So, first component will have on an average time MTTF. Second component will also have an average time of MTTF. Third will also have on an average MTTF.

So, if we have k such unit, then average time it will be taking to failure will be MTTF will be equal to summation of average MTTF of individual. So, we can say this is k into MTTF of each system.

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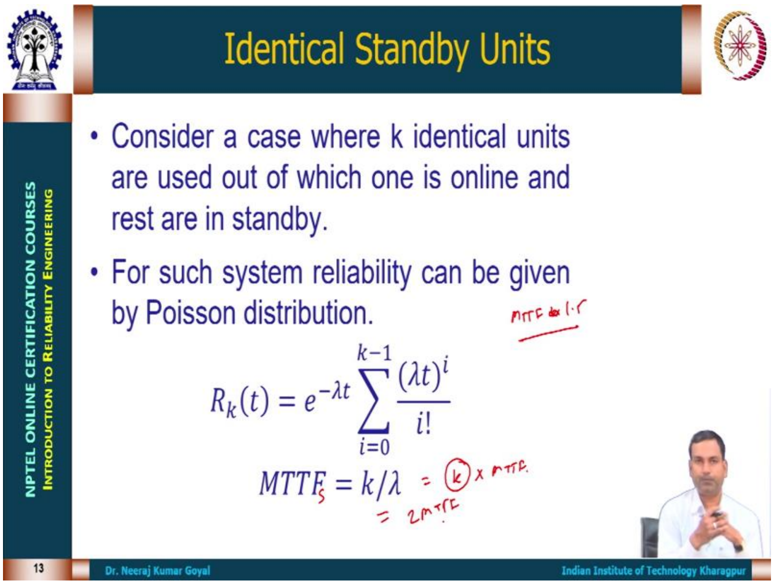

So, this is equal to k that is number of units multiply by MTTF. This is my MTTF of system. This is MTTF of component, which is unlike the what we considered in two component failure. For two components failure what was happening? The probability was MTTF into 1.5, 1 plus 1 by 2 which was much less. This was 1.5 for 2 unit but for two 2 this will become 2 MTTF. So, for standby system, because the standby system is not operating when the first system is operating, we get more time to failure or it works for a longer period of time.

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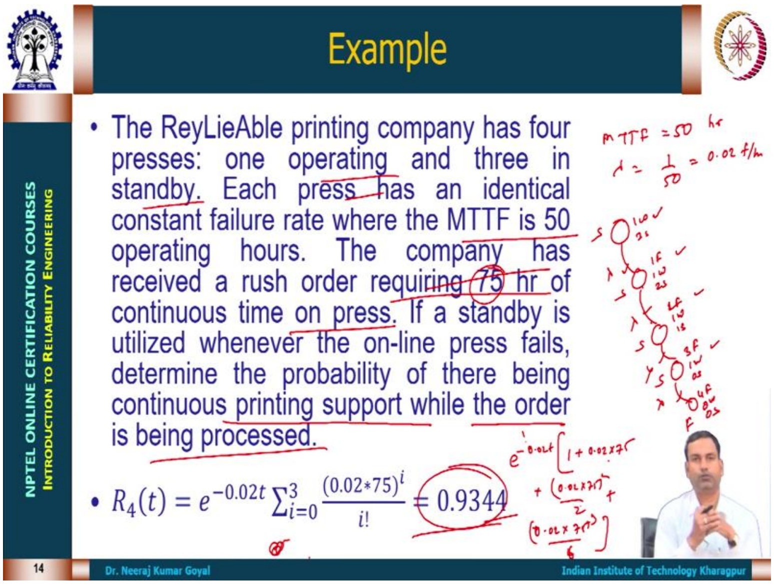

Let us take one example that are ReyLieAble printing company has four presses, it has four press. One operates and three are in standby. Each press has the identical constant failure rate which is MTTF is 50. So, individual MTTF is 50. So, lambda is equal to 1 by 50 operating hours so, that will be 0.02 failure per hour. Now, here if you see the company has received this rush order requiring 75 hours of continuous operation that means they will start working one by one if it fails, when one press fails, then they will plug in another failure and they will continue till last press is working.

So, what is the probability that the company will be utilized on the probability that? It will be able to provide the continuous printing support when the order is being processed. That means it does not consume all the that means it in 75 hours all units should not fail. So, what is the probability that at least one of them is working. That means either system as we see here, we have four states here, three in a standby.

So, one working, three standby. Then we have one failed, one working, two in standby. Then we have two failed, one working, one standby. Then we have three failed, one working, zero standby. Then from here we can have four failures and 0 workings, 0 standby. So, here as we see here, we have 1, 2, 3, 4, 5 state.

So, as we know our formula the because all are lambda here, all failure rates are lambda, and as we solved earlier the reliability will become the probability that system is in this state, this state, this state, this is failure state because here my order will not complete before order completion, I will have the failure, but in all these states if the system is in any one of these state, then my order will be completed.

So, this will become e to the power minus lambda t. So, lambda is 0.02 t into 1 plus lambda t. So, lambda t is 0.02 into t, t here is my time of interest is 75 hours, 75. Similarly, 0.02 into 75 whole square divided by 2 plus for third component 0.02 into 75 multiply whole cube divided by factorial 3. So, that will become 6. I have four states 1, 2, 3, 4, this will be my reliability. And what will be unreliability? Unreliability will be 1 minus this or I can simply say unreliability is e to the power minus last state probability can be calculated that it will be 1 minus of this.

$$R_4(t) = e^{-0.02t} \sum_{i=0}^{3} \frac{(0.02 * 75)^i}{j1} = 0.9344$$

So, once we know this, we use this sum when we evaluate this and sum it up the property comes out to be 0.9344. So, with this example, you can see that if we have the standby units, and we do not consider standby failure during the standby mode, then this can be a reliability value which we can get.

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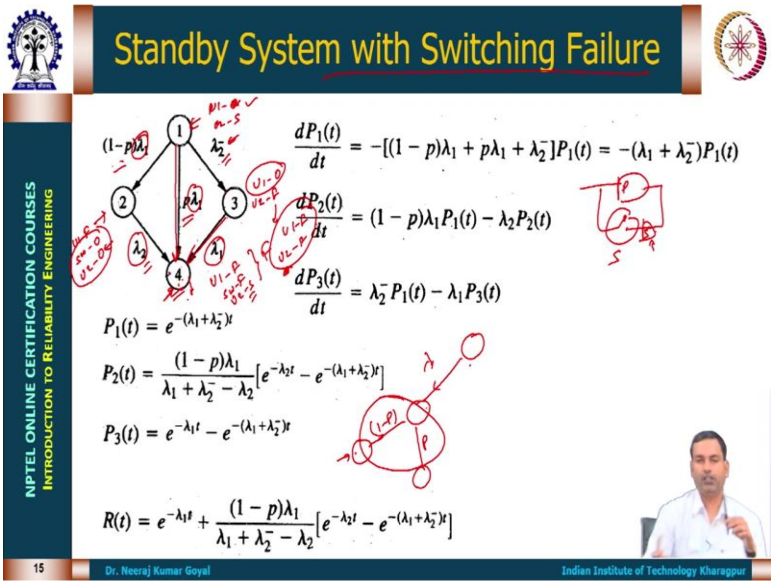

Now, if we consider another case, then we consider a standby system with switching failure. Switching failure means, as we discussed earlier, our primary unit is here then we have a secondary unit which is not operating. But, whenever primary unit fails then either by automatic or by manual switching the secondary unit will be put in operation. So, but this switch can also fail. If these switches failed, what will happen? I will not be able to plug in the standby unit and my system will be in failed condition.

$$\frac{dP_1(t)}{dt} = -\left[(1-p)\lambda_1 + p\lambda_1 + \lambda_2^-\right]P_1(t) = -\left(\lambda_1 + \lambda_2^-\right)P_1(t)$$

$$\frac{dP_2(t)}{dt} = (1-p)\lambda_1 P_1(t) - \lambda_2 P_2(t)$$

$$\frac{dP_3(t)}{dt} = \lambda_2^- P_1(t) - \lambda_1 P_3(t)$$

$$P_1(t) = e^{-\left(\lambda_1+\lambda_2^-\right)t}$$

$$P_2(t) = \frac{(1-p)\lambda_1}{\lambda_1 + \lambda_2^- - \lambda_2}\left[e^{-\lambda_2 t} - e^{-\left(\lambda_1+\lambda_2^-\right)t}\right]$$

$$P_3(t) = e^{-\lambda_1 t} - e^{-\left(\lambda_1+\lambda_2^-\right)t}$$

$$R(t) = e^{-\lambda_1 t} + \frac{(1-p)\lambda_1}{\lambda_1 + \lambda_2^- - \lambda_2}\left[e^{-\lambda_2 t} - e^{-\left(\lambda_1+\lambda_2^-\right)t}\right]$$

If it is automatic switch then also if it is manual switch then also so, my objective of reliability will not be completed here. There are various cases considered for this which is like hot standby, warm standby, et cetera. So, we are not going into that we are taking a simpler concept here that first unit is only working, second unit is in standby and there is no and but this can fail in standby and there can be also a switch failure. So, now, if you look at this state, state number, here my first unit is primary unit or the unit 1 is working or operating I can say operating and unit 2 is in standby mode.

Now, here there are three possibilities here. First possibility is that the equipment primary equipment fails in prime whenever primary equipment fails the failure rate is lambda 1, but this lambda 1 has again two possibilities like from here we have lambda 1. But here we have two possibilities here. One is it is when this failing that switches also switches working and other cases which is failed if P is the probability of switch failure the same state but this is not related with time. The switch failure probability is not related considered to be related with time because switch is not continuously operating.

When we consider that primary unit is failed at that time, the switch can be found in two condition either it is working or it is failed. So, this probability of failure and probability of working gets multiplied with the lambda 1 and this gives me the transition to two different states. One is state this state is that unit one is failed switches operating operated. Since switch is operated, what will happen? Unit 2 will become operational.

Now, this unit if we see this is a failed state because here what is happening? Unit 1 is failed for this state if we see then switches also failed. Since switches failed now, because switches failed then you need to remains in standby. We cannot put it in operation mode. Since we are not able to put it in operation mode this is a failure state. So, this state directly leads to the system failure. This state in this case we have the unit 2 as the operating. Since unit 2 is operating so, the failure rate of unit 2 lambda 2 is coming into the picture here.

In this case, what is the second case that is that my standby unit can also fail with the failure rate lambda 2 minus in the standby mode. In that case, what will happen? Unit 1 is operating here and unit 2 is failed here, which is the standby failure.

Now, here because unit 2 is already failed, so, no switching nothing even switch works or does not work in both cases, the unity cannot be put into the operation so, switch becomes irrelevant here. So, here since unit 1 is operating that will keep on operating until it fails. So, the failure rate of unit 1 only is coming into the picture and that gives me the state number 4. So, state number 4, I am having two different possibility. One in one we have the unit one failure, unit 2 is in standby, but switch has failed. Another case is unit 1 is failed, from here unit 2 is also failed already and here because of that my system is state is here that is failed.

So, my failure state is reached in two ways either unit 1 is need to be failed in both the cases, second cases that unit switches failed, but you need to comes on keeps on working. Another case is unit 1 fails, unit 2 fails in standby mode, another case is unit 1 fails, switch works but unit 2 fails in operation because of that is lambda 2. So, three cases three possible paths are coming from three different directions and but all are leading to the system failure and this is my fourth state is my system failure state.

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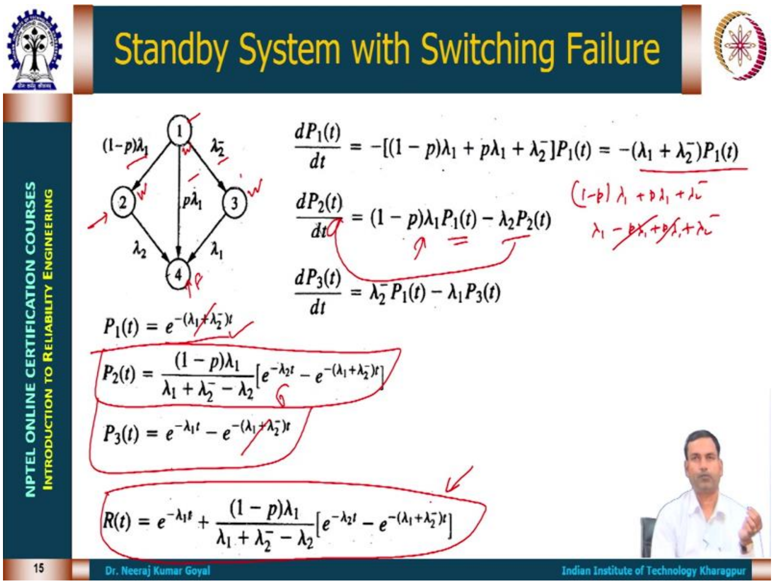

This problem again, as we have solved other problems same way this can be solved. So, P 1 this is stage 1 if you look at it here, then as we see outgoing probability is this, outgoing rates are these these these. So, when I summed up this, this will become 1 minus P into lambda 1 plus P lambda 1 plus lambda 2 minus. Now, here P lambda 1 P lambda 1, this will become lambda 1 minus P lambda 1 plus P lambda 1 plus lambda 2 minus. This will get cancelled. So, what I will remain is lambda 1 plus lambda 2 minus into P 1 t.

This if we solve, my P 1 t will come e to the power minus lambda 1 plus lambda 2 minus into t which is similar to what we have for the earlier case is standby with failure problem, that system can fail in the standby mode. But P 2 t which is the probability of this state where the switch is working in that case, this if we solve from here similar process we have to follow. So, this I will leave as an exercise to you.

So, try to evaluate this that by putting P 1 value here P 1 t what we have got this e to the 1 minus lambda 1 plus lambda 2 minus this lambda 2 P you take it here, then you apply the same formula that you multiply with e to the power lambda 2 t in both side. Then you integrate on both side, you will get the answer for P 2 t which will be equal to this.

Similarly, P 3 t you can get this and once you get P 1 2 P 2 t and P 3 t from there you can get the R t. R t is nothing but in this is the only failure state. This is also working state, this is also working state, this is also working state. So, when we sum up these three probabilities I get the R t. So, R t comes out to be this value, because this value this will get cancelled and e to the power minus lambda 1 t plus this value e to the power (()) (30:04) plus this value when we take this case in our reliability.

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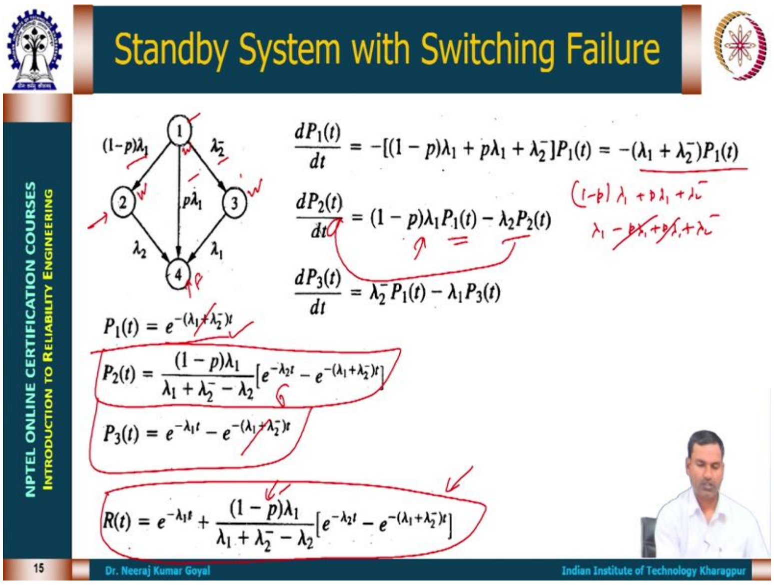

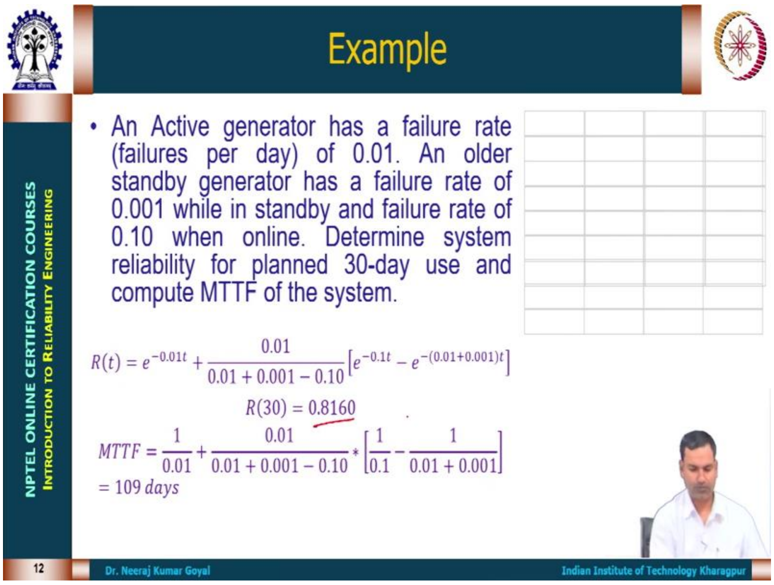

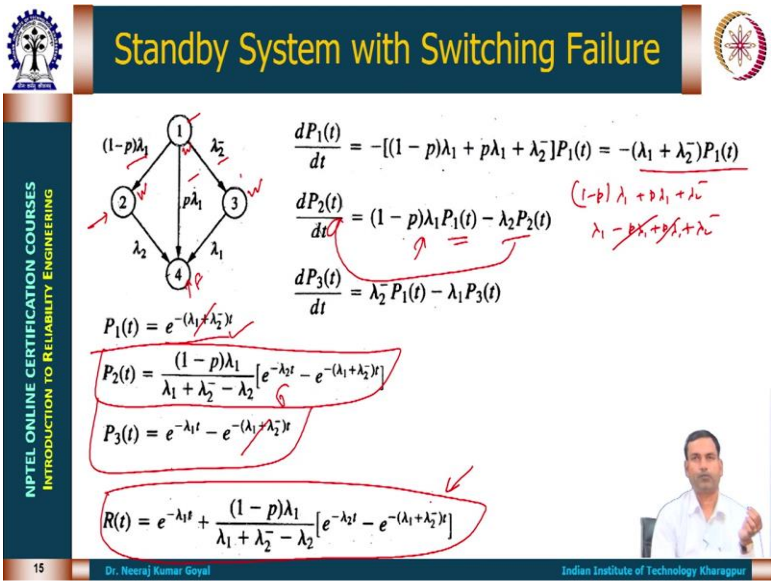

Like we solved earlier example, we can solve similar example for this by simple formula application, we can apply the formula, but we have got in previous slide and then here the same problem is here that we have the generator which is having the failure rate of 0.01 so, lambda 1 is equal to 0.01.

Older standby generator, so, lambda 2 minus is equal to 0.001 and failure rate of this then online is lambda 2 that is 0.10. This problem is very similar to what we saw earlier, but earlier when we solve we did not take the switching failure probability here the switching failure probability P is equal to 0.1 10 percent. 10 percent means 0.1, 10 by 100 is 0.1.

Same thing then we solve now, this new formula we use that is same e to the power minus lambda 1 t 1 minus P lambda 1 divided by. If you see the only thing which is changed is 1 minus P is coming here. Earlier there was no P 1 minus P coming here. This was lambda 1 plus upon lambda 1 plus lambda 2 minus minus lambda 2 that is the equation is same. Then we apply this my probability comes out reliability comes out to be 30. As you see here, some addition will be less because of 1 minus P because 0.9 whatever value was here that will get multiplied by 0.9. So, reliability will fall that is 0.8085. If we see the previous example two component then load sharing then standby.

So, here when we solve the example just 0.8160 that has become 0.8085. And same way, we can calculate the MTTF also that will also reduce because of this point and multiplication. So, we are able to solve standby system in two cases when we consider switches perfect that means switch failure probability if we do not take and we consider that system can fail in standby mode or during the operational mode. Here we have considered that switch can fail can also fail. So, switch failure probability also we have considered and using this we are able to get the reliability.

We have also considered then when units are identical, then how can we get the system reliability, by assuming that there is no switch failure possibility and there is no failure in standby mode. So, all these cases provide support generalized cases which you can use in practical reliability valuation for various systems where standby is involved.

So, we will stop it here and we will continue our discussion for further system configurations. Thank you.

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