Introduction to Reliability Engineering
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Lecture 21
Markov Analysis

Hello everyone. So, we are moving to next lecture and we will continue our discussion on Markov analysis. In the previous lecture, we discussed about two component system which are working together parallelly. So, we considered two cases in series system, if any one of the component fails, the system is considered to be failed, in parallel system where one of the failure is tolerated.

So, both the system whenever they fails then only the system failure occurs and we could see that when we know the state probabilities from there we can get the relative for both cases, series system case also, parallel system case also. Today, we will discuss first load sharing system. In load sharing system, as we discussed initially, we have two systems or two components.

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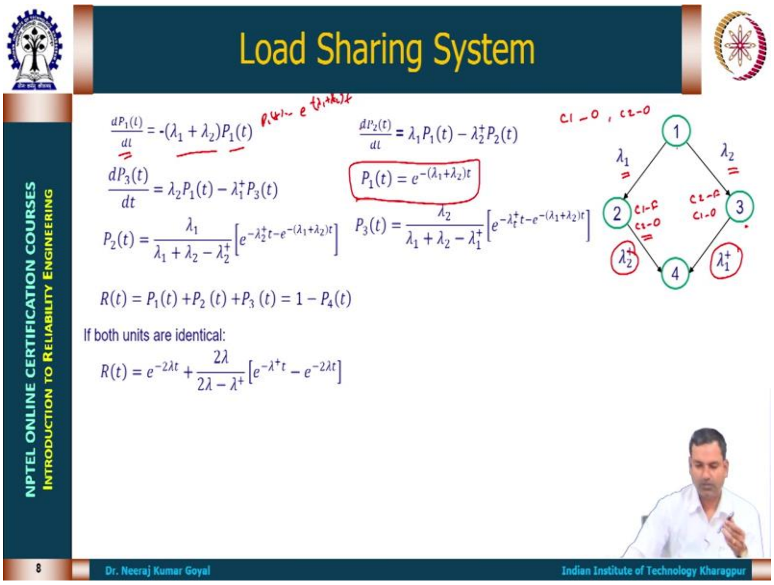

So, we have component 1 and component 2 like we discussed earlier, we have component 1 and component 2 both are operating here in state number 1. In state number 2 or state number 3, there are two possibilities here either component 1 can fail. So, if component 1 fails component 2 continuously operate here that is our system state 2. There is another possibility in place of component 1, component 2 fails. So, component 2 is failure, but component 1 is operating in this state.

In load sharing system, as we discussed briefly previously in previous class that whenever one system is failing, let us say we are talking about the generator system. So, if two generators are working together they are taking half half load, but if one of the generator fails then all the load is shifted to the another generator.

So, that generator now becomes is working in a higher load under higher load because now here generator 2 is working or component 2 is working. In higher load as we see the probability of failure will not be or the failure it will not be same as lambda 2. Failure it may become higher. It may remain same also but in general as we can see the concept of load sharing because of failure of one portion of the system, another system is now having the higher stress.

In higher states the chances of failures are high. So, conditional probability of failure per unit time is or that is failure rate is also high. Similarly, here component 1 is only working component 2 is failed, this is also under higher load. So, let us see the failure rate for this component is now the higher one which is lambda 2 plus. So, plus is denoting that the failure rate has increased to a new value which we are calling as lambda 1 plus. So, similarly here component 2 fill (()) (3:19) filler it has increased to a new value which is lambda 2 plus.

So, if we follow the same notation, then as we know that for state number 1dP 1 over dt will be negative of from P 1 only 2 rates around. So, lambda 1 minus of lambda 1 plus lambda 2 into P 1 t. This equation we have already solved. This is the same equation which we had in the two component case and the result was P 1 t was equal to e to the power minus lambda 1 t lambda 1 plus lambda 2 into t same thing which we have caught here. Now, here we have another equation. This equation for is for lambda 2.

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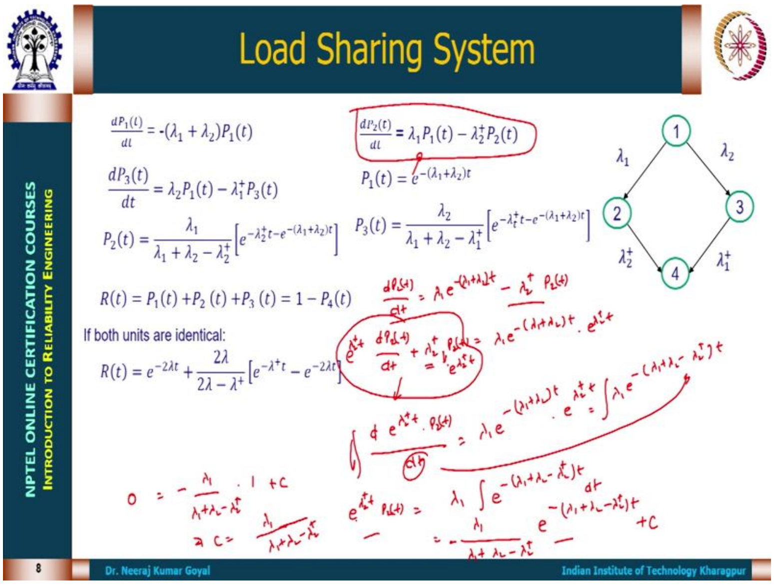

So, for lambda 2 equation when we look into now, let us see how do we solve this. Solution of this equation will be similar to how we solve the P 2 equation in previous case. So, here P 1 t we can pick up from here, this we can pick up and place it here and this will become dP 2 t over dt will be equal to lambda 1 e to the power minus lambda 1 plus lambda 2 into t minus lambda 2 plus into P 2 t.

$$\frac{dP_1(t)}{dt} = -\left(\lambda_1+\lambda_2\right)P_1(t)$$

$$\frac{dP_3(t)}{dt} = \lambda_2 P_1(t) - \lambda_1^+ P_3(t)$$

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - \lambda_2^+ P_2(t)$$

$$P_1(t) = e^{-(\lambda_1+\lambda_2)t}$$

$$P_2(t) = \frac{\lambda_1}{\lambda_1+\lambda_2-\lambda_2^+}\left[e^{-\lambda_2^+ t - e^{-(\lambda_1+\lambda_2)t}}\right] \qquad P_3(t) = \frac{\lambda_2}{\lambda_1+\lambda_2-\lambda_1^+}\left[e^{-\lambda_1^+ t - e^{-(\lambda_1+\lambda_2)t}}\right]$$

$$R(t) = P_1(t) + P_2(t) + P_3(t) = 1 - P_4(t)$$

$R(t) = e^{-2\lambda t} + \frac{2\lambda}{2\lambda-\lambda^+}\left[e^{-\lambda^+ t} - e^{-2\lambda t}\right]$, if both units are identical.

As we discussed earlier this lambda 2 P 2 t we can take left sides so, this will become dP 2 t over dt plus lambda 2 plus into P 2 t is equal to lambda 1 e to the power minus lambda 1 plus lambda 2 into t. As we discussed earlier this since this is lambda 2 plus we have to multiply here with the exponential of lambda 2 plus t all the terms here also we have e to the power lambda 2 plus t, here also we multiplied e to the power lambda 2 plus t, once we multiply this what happens this left hand term is becoming the differentiation of e to the bar lambda 2 plus t into P 2 t over dt. If you differentiate this that two term will be generated these are the same term is equal to lambda 1 e to the power minus lambda 1 plus lambda 2 into t into e to the power lambda 2 plus t.

This lambda 2 plus t will actually we can take inside here if you want so, this we can say that lambda 1 e to the power minus lambda 1 plus lambda 2 minus lambda 2 plus because this is plus sign so, this is minus sign this will become minus (()) (6:17) into t. Now, as we saw earlier we can integrate on we can take dT towards here and we can integrate on both sides. Once we do that, then this left side integration will give e to the power lambda 2 plus t into P 2 t will be equal to integration of the whatever terms are independent of t we can take them outside. So, lambda 1 is independent integration of e to the power minus lambda 1 plus lambda 2 minus lambda 2 plus into t dt.

This integration if you do we know this will become lambda 1 upon lambda 1 plus lambda 2 minus lambda 2 plus the same term which is in exponent as multiplication of t minus of that will be coming as the division term and again integration of minus of lambda 1 plus lambda 2 minus lambda 2 plus into t plus a constant c.

Now, this constant c as we see, when we put our terms that t equal to 0 that P 2 t will be 0, because at this initial time t equal to 0 system is in the state 1, so, P 2 is 0. So, here once you put 0 then this term will become e to the power 0 that will be 1 this term will become 0 because P 2 is 0. So, c will so, here this equation will become 0 equal to minus of lambda 1 upon lambda 1 plus lambda 2 minus lambda 2 plus into e to the power 0 will be 1 plus c.

So, here we can see c is equal to lambda 1 upon lambda 1 plus lambda 2 minus lambda 2 plus. Same thing when we place it here then from this equation what we will get here I will erase some portion here.

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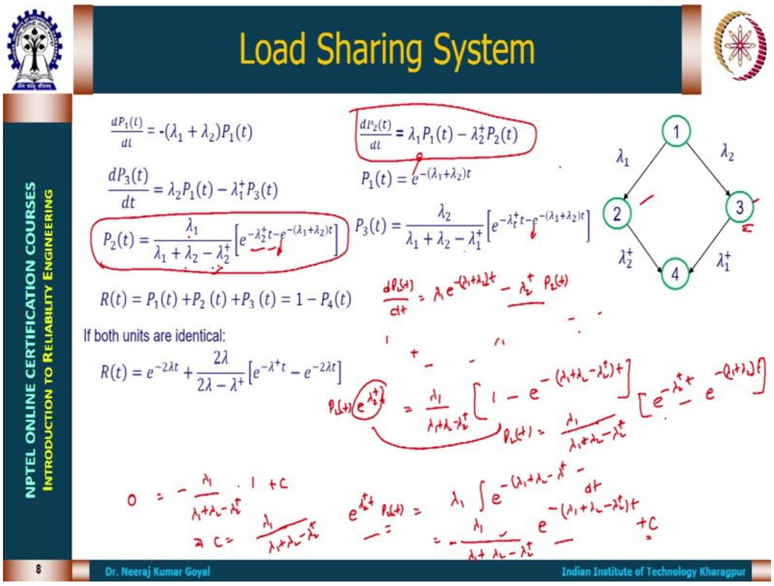

So, let us see if we again use a pen here. So, this equation which we had this equation is now P 2 t we want to calculate. So, to calculate P 2 t e to the power lambda 2 plus t we can divide. So, P 2 t will be equal to or P 2 t into e to the power lambda 2 plus t will be equal to lambda 1 upon lambda 1 plus lambda 2 minus lambda 2 plus that is for c. So, c though this value and this value is same, so, we can say this it multiply by 1 minus e to the power minus lambda 1 plus lambda 2 minus lambda 2 plus into t.

Now, here if you see e to the power lambda 2 plus t if you take on right hand side then P 2 t will be equal to lambda 1 upon lambda 1 plus lambda 2 minus lambda 2 minus multiply by e to the power minus lambda 2 plus t minus now, because this will go here and divide here. So, this lambda 2 will get cancelled, the remaining term is e to the power minus lambda 1 plus lambda 2 into t.

This gives me the final might be two value, if you look at here, this is my same P 2 is lambda 1 upon lambda 1 plus lambda 2 minus lambda 2 plus sorry this is lambda 2 plus multiply with e to the power minus lambda 2 plus t sorry this is below this is typing mistake/ This has to come below.

So, minus here minus exponential of minus lambda 1 plus lambda 2 into t. Similarly, then the if we try to solve for 3, similar equation will be there, only lambda 1 and lambda 2 will get interchange. So, this lambda 1 will become lambda 2. This will be lambda 2 plus lambda 1 minus lambda 1 plus and e to the power minus lambda 1 plus t minus the exponential of minus lambda 1 plus lambda 2 into t because of the similarity because this is this state is very similar to this state, only changes lambda 1 and lambda 2 are interchanged. One component is changed with the second component.

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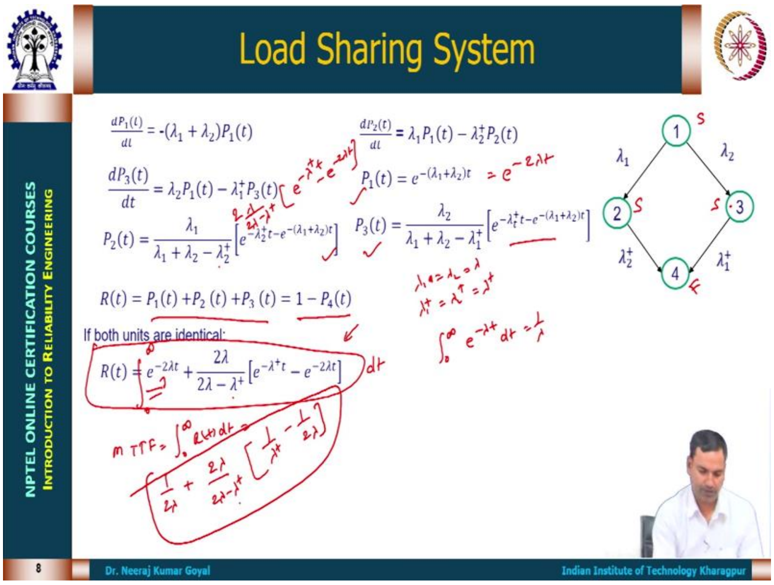

So, here we are able to get P 1 t, P 2 t and P 3 t. Now, what is the reliability? Now here if you look at this state this is a success state because both components are working. All nodes are served but in a shared mode, but this system if it is in state 2 then also it is a success because here also my complete load is being served. In state 3 also my state all the load is being served. State 4 is a failure state because here both components are failed and my requirements are no longer fulfilled. So, my reliability will be nothing but the system being in a state 1 or state 2 or state 3 that means reliability is nothing but this probability that system is in state 1 or state to or state 3, all three are mutually exclusive state so, reliabilities are summed up and this gives me the probability.

If we sum of these three then what will happen? So, here this consideration is given all when we if we consider that both the components are similar if both are identical unit that means, they are having same type, same design, same manufacturer, purchased on the same day. So, if both the needs are identical their failure behavior is also expected to be identical. So, in that case lambda 1 will be equal to lambda 2 and lambda 1 plus will be equal to lambda 2 plus, we can call this as lambda, we can call this lambda plus.

So, when we do this then this will be equal to e to the power minus 2 lambda t and this if we write this will be equal to lambda upon 2 lambda minus lambda plus lambda upon 2 lambda minus lambda plus into e to the power minus lambda plus t minus into the power minus 2 lambda t. The same thing is coming here whatever P 2 is here, same P 3 will also come, lambda upon 2 lambda minus lambda. So, this becomes twice of this. So, this will become 2 lambda upon 2 lambda minus lambda plus into e to the power minus lambda plus t into minus e to the power minus 2 lambda t. So, this becomes our answer here.

So, as we see here we are able to calculate reliability. If we want to calculate MTTF, then MTTF, we know is the integration from 0 to infinity of R t dt. And as we know, as we have seen earlier, if we do this then integration for this dt from 0 to infinity, when we apply this then integration of first term will give 1 upon 2 lambda as you know, integration of 0 to infinity e to the power minus lambda t dt is equal to 1 upon lambda.

So, same thing (()) (14:14) so, this become 1 upon 2 lambda plus constant term will remain outside that is 2 lambda upon 2 lambda minus lambda plus when we integrate this this will be given upon lambda plus when we integrate this, this will give 1 upon 2 lambda. So, this will be our MTTF. We can solve this further to get the final answer.

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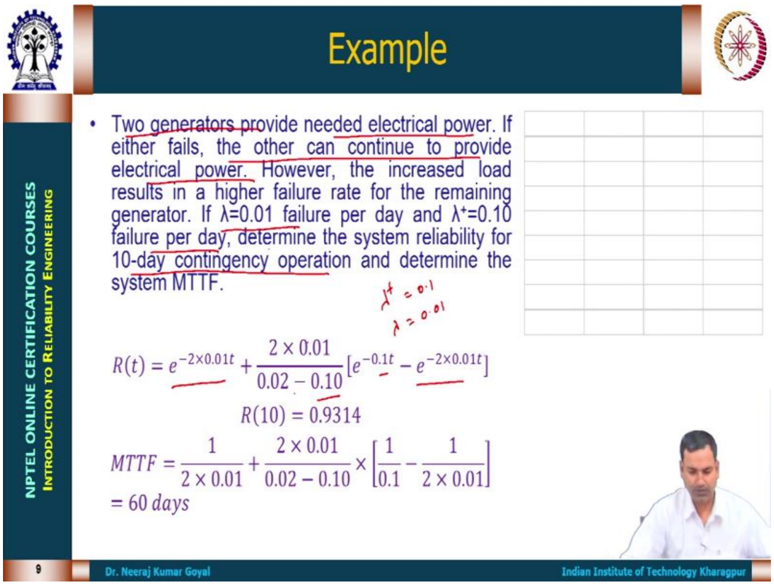

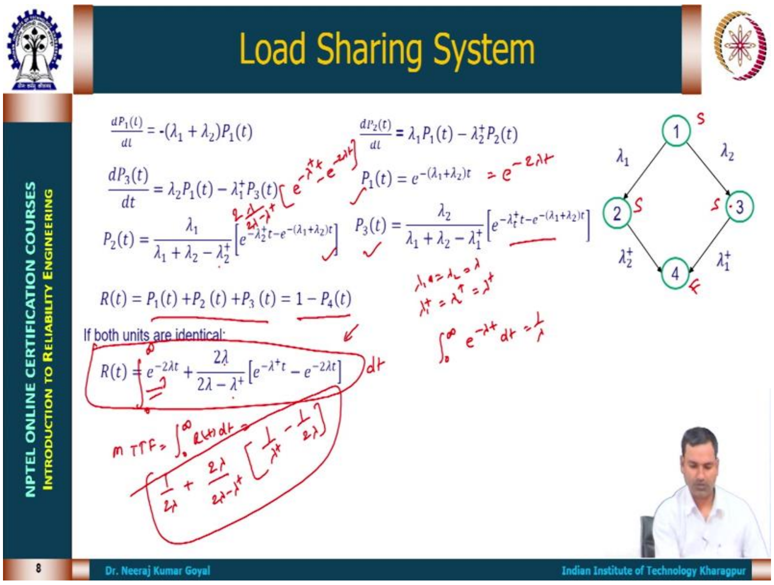

Let us take one example. We have two generators which provide the electrical power. If one fails, the other one can continue to provide electrical power. However, the increased load result in a higher failure rate for the remaining generator. So, let us say lambda is 0.01 failure per day. That means, when both are sharing the load then failure rate is 0.01 and lambda plus is 0.1 Right. So, that is failure rate is becoming 10 times when the load is not shared when one unit is taking the full load failure per day.

Now, what is the system reliability for 10 day contingency operation that means, there is no power and for 10 days these two generators has to supply the power. So, in that case what will be the reliability? If you want to know this reliability then we can apply the same formula what we have got here directly that is e to the power minus 2 lambda t plus 2 lambda upon 2 lambda minus lambda plus into this. So, e to the power minus 2 lambda t e to the power minus 2 into 0.01 into t plus 2 into lambda divided by 2 into lambda that is 0.02 minus lambda plus that is 0.1 multiply by e to the power minus as we see here e to the power minus lambda plus t so, lambda plus is 0.1 lambda plus.

(()) (16:09) over lambda plus is equal to 0.1 and lambda is equal to 0.01 minus e to the power minus 2 into lambda t between 2 into 0.01 into t.

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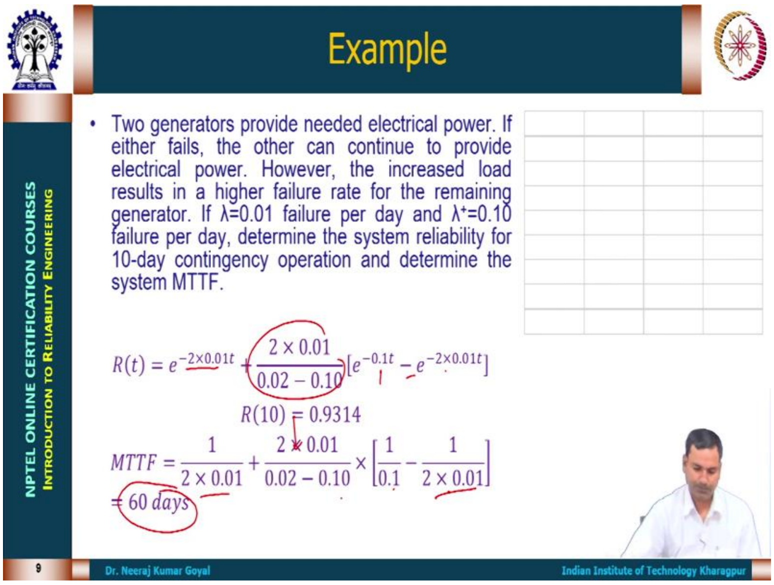

So, this if we try we can solve using the this I can show it this calculations if we can do it on Excel or we can do it on this Excel sheet also. So, here an MTTF as we saw earlier MTTF is nothing but 1 upon 2 lambda. So, this is 1 by 2 in to 0.01 plus the same term will come as it is here multiply by 1 upon 0.1 minus 1 upon 2 into 0.01. This if we solve because failure rate is in per day. So, when we solve this the answer MTTF will also come in days that is coming to be 60 days.

$$R(t) = e^{-2\times0.01t} + \frac{2\times0.01}{0.02-0.10}\left[e^{-0.1t} - e^{-2\times0.01t}\right]$$
$$R(10) = 0.9314$$
$$MTTF = \frac{1}{2\times0.01} + \frac{2\times0.01}{0.02-0.10}\times\left[\frac{1}{0.1} - \frac{1}{2\times0.01}\right]$$
$$= 60 \text{ days}$$

So, using this example as you can see that if there is a shared load then we can solve the problem together reliability.

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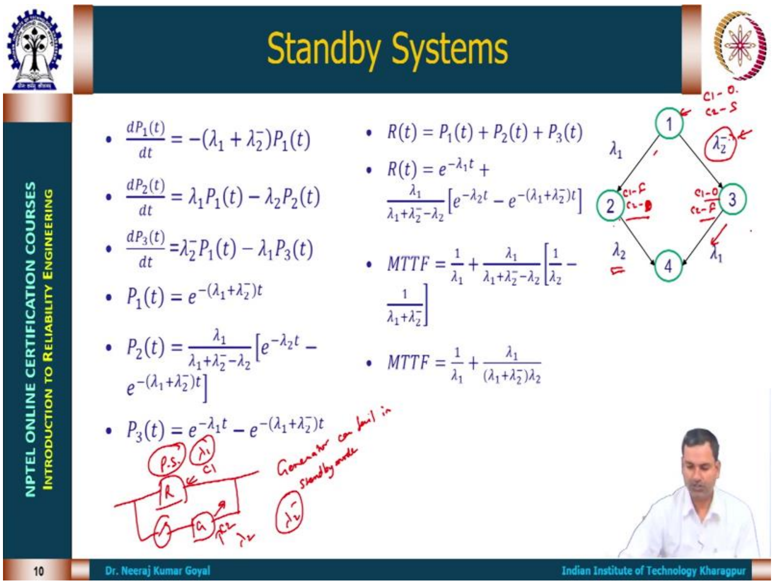

Now, if we look at standby systems. Standby systems means as we discussed earlier like of generator can be one standby system. In case of generative, what is happening we generally have the power supply so, power supply or electrical input is generally working. Now, so, we have the electrical supply coming from a state electricity board.

So, this power supply is working now, we have certain reliability for this or we have the failure rate for this let us say lambda 1. When this power supply fails then we have a generator here if power supply fails then this generator will be plugged in and the generator will supply the power. When the generator supplies the power then our sound system will continue to work.

So, as we see here that two possibilities we are considering here. Here we are considering that generator can fail in, generator or any system this is for all I am just taking an example here, but does not mean generator here. It can be any system where the main function is being served, but whenever main function fails then another component or the standby system this can be pump also like you may have been some primary pump is doing the pumping of the water in let us say power plant, nuclear power plant, but when it fails, then another pump will which is there that will be started to run then and that will supply the power supply the water.

So, what happens, whatever the system we are considering that second system will be plugged in and then that will start to do the function which was supposed to be done by the primary unit. Now, here we are considering that generator or the secondary unit can fail in standby mode and the failure but what will happen, generally as you consider the failure probability in standby mode is smaller than failure probability fully operational mode.

So, here what we consider that the second unit which is in secondary unit which is there in the standby. This is our component 2 and this is our component 1. Component 2 failure rate is lambda 2 if it is operational and component 2 failure rate is lambda 2 minus when it is not operational. Since, this is the difference which we generally see in terms of standby versus the parallel system.

In case of parallel system, both the units are working so, the whenever we started the failure rate was lambda 2 only, but in case of standby system, the system is in standby, it is not operating. Since it is not operating it is not experiencing this stress, since it is not experiencing the stress the chances of failures are less. So, the failure rate in standby mode many times is considered 0, but if it is not considered 0, then we can consider some stress some failure it is there in standby mode that is lambda 2 minus, minus we are denoting to say that this is the lesser failure rate compared to the failure rate it would have experienced in general operation mode.

Now, in system mode 1 what is happening component 1 is in function operational state and component 2 is in standby state. So, there are two possibilities here, either component 1 fails when it is operating or component 2 fails in standby mode itself, but component 1 keeps on working. So, here what will happen component 1 works and component 2 is failed. It is failed in standby mode. And other possibilities that component 1 fails, but component 2 is continues to operate in standby mode.

Now, what will happen here? From this state the moment component 1 is failed, what will happen, component 2 will be plugged in and component 2 will become now operational fail operational. So, component 2 which was standby will become operational here. So, now component 2 can fail with the priority lambda 2 failure rate lambda 2 and here all the components either component 2 is failed only component 1 is working that failure rate is lambda 1. So, this becomes our markup diagram for the same mark of diagram as we have solved earlier same thing will be used same equations we can develop and similarly, we can solve.

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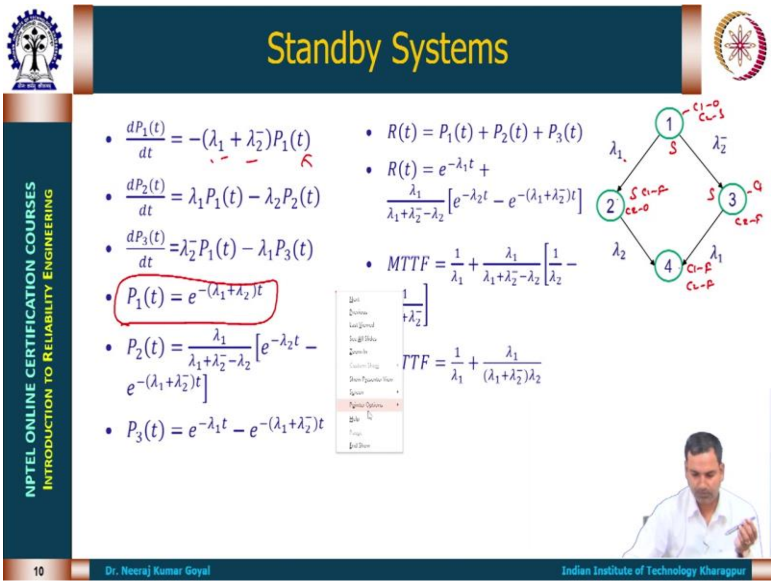

$$\frac{dP_1(t)}{dt} = -\left(\lambda_1 + \lambda_2^-\right)P_1(t)$$

$$\frac{dP_2(t)}{dt} = \lambda_1 P_1(t) - \lambda_2 P_2(t)$$

$$\frac{dP_3(t)}{dt} = \lambda_2^- P_1(t) - \lambda_1 P_3(t)$$

$$P_1(t) = e^{-\left(\lambda_1 + \lambda_2^-\right)t}$$

$$P_2(t) = \frac{\lambda_1}{\lambda_1 + \lambda_2^- - \lambda_2}\left[e^{-\lambda_2 t} - e^{-\left(\lambda_1 + \lambda_2^-\right)t}\right]$$

$$P_3(t) = e^{-\lambda_1 t} - e^{-\left(\lambda_1 + \lambda_2^-\right)t}$$

$$R(t) = P_1(t) + P_2(t) + P_3(t)$$

$$R(t) = e^{-\lambda_1 t} + \frac{\lambda_1}{\lambda_1 + \lambda_2^- - \lambda_2}\left[e^{-\lambda_2 t} - e^{-\left(\lambda_1 + \lambda_2^-\right)t}\right]$$

$$MTTF = \frac{1}{\lambda_1} + \frac{\lambda_1}{\lambda_1 + \lambda_2^- - \lambda_2}\left[\frac{1}{\lambda_2} - \frac{1}{\lambda_1 + \lambda_2^-}\right]$$

Now here success states are 1 and 2 and 3 in all three cases of a system is working because here component 1 is operating here also here component 2 is operating and here component 1 is operating. In this case component 1 is failed and component 2 is failed. Here component 2 is in standby mode. Here component 1 is failed and here component 2 is failed.

So, we have different states here and the different states will have the different probabilities. So, these state probabilities how do we determine as we discussed earlier for this state lambda 1 and lambda 2 minus are the two outgoing so, dP 1 over dt will be nothing but minus of from P 1 we have two outgoing lambda 1 and lambda 2 so, lambda 1 plus lambda 2 minus into P 1 t.

Now, if we solve this as we know earlier this becomes e to the power minus lambda 1 plus lambda 2 minus into t. P 1 t will be equal to e to the power minus lambda 1 plus lambda 2 minus into t. I am removing this markers so, that we can see properly.

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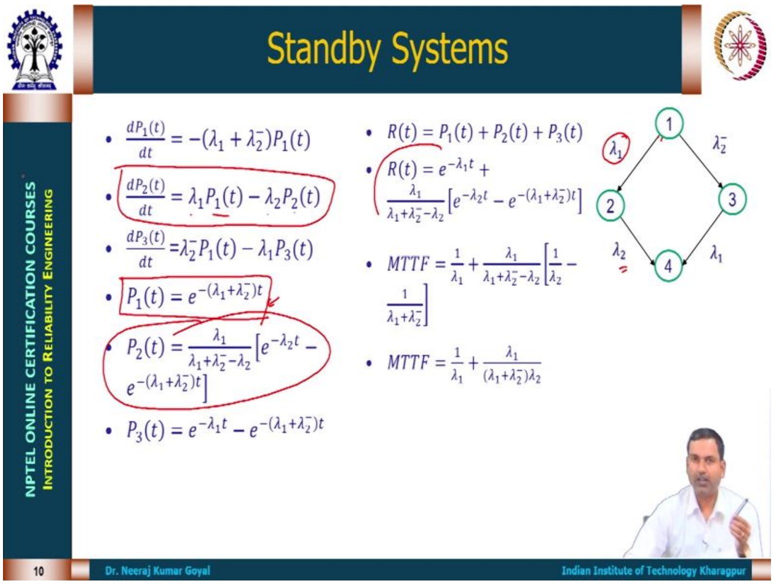

So, when we see it here so, solving this equation gives me directly this value. Now, we take the second equation. Second equation if you look at here or from state 1 incoming is lambda 1 So, lambda 1 into P 1 t and outgoing is only lambda 2 from state 2 so, lambda minus lambda 2 into P 2 t. This again as we solved earlier we can solve this equation also and then we solve this equation we get P 2 t is equal to this value. We can try this again so, that you will get a little bit more practice by seeing this.

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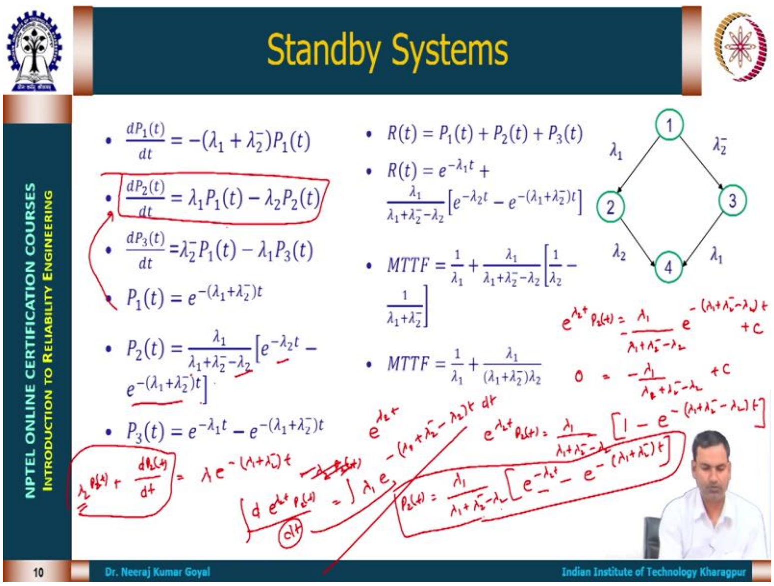

Let us try to solve this equation. Same process as we did earlier similar thing as we as you see that this diagram is also similar and the values the only changes in values. So, here when we solve this we can look into this. We replace the P 1 t which we have got in here. So, my dP 2 t over differentiation of P 2 t over dt will be equal to lambda 1 e to the power minus lambda 1 plus lambda 2 minus into t minus lambda 2 P 2 t. So, this P 2 I can take on left sides, so, this will become lambda 2 P 2 t. These are removed from here and I have taken from that side.

Now, this as we see here to resolve this what I have to do? Since lambda 2 is coming as multiplication of P 2 t I will multiply whole term by e to the power lambda 2 t. Once I multiply with e to the power lambda 2 t this left hand side will become e to the power lambda 2 t into P 2 t differentiation versus t will be equal to lambda 1 e to the power minus lambda 1 plus lambda 2 minus.

Now, here I will multiply this e to the power lambda 2 t. Since this is minus and this was positive so, inside when we take this will become minus, minus lambda 2 into t. This we have done same. So, I have just reduced one step. Same thing I have done in one less system. Now, again when we do the this dT if you take on here then if we integrate then our values will come. So, what will be one left hand side? Left hand side is e to the power lambda 2 t P 2 t will be equal to lambda 1. Now, whatever is the multiplication here negative of that same thing will come in denominator lambda 1 plus lambda 2 minus minus lambda 2 exponential of minus lambda 1 plus lambda 2 minus minus lambda 2 into t plus c.

Again when we put t equal to 0 then P 2 0 will be 0. So, 0 e to the power minus 0 will be 1, so, minus lambda 1 upon lambda 2 sorry lambda 1 plus lambda 2 minus minus lambda 2 plus c so, c will be equal to positive of this because when it goes to the left hand side will become positive. So, here we know c value, so, c value will be positive of this so, we get it clearly that e to the power lambda 2 t into P 2 t will be equal to lambda 1 upon lambda 1 plus lambda 2 minus minus lambda 2 into because of c this will become 1 minus this is minus term, so, minus term e to the power minus lambda 1 plus lambda 2 minus minus lambda 2 into t. We can solve this using the same way as we have solved earlier.

So, P 2 t will be equal to we can take divide this by e to the power minus or multiply both sides with e to the power minus lambda 2 t this will become 1. So, P 2 t will be equal to lambda 1 upon lambda 1 plus lambda 2 minus minus lambda 2 multiplied by e to the power minus lambda 2 t minus e to the power minus. Now lambda 2 minus minus plus and this minus when we multiply them this lambda 2 will be removed here, minus of lambda 1 plus lambda 2 minus into t. The same becomes our P 2 T which is here. If you see this p 2 t same lambda 1 upon lambda 1 plus lambda 2 minus minus lambda 2 multiply by e to the power minus lambda 2 t minus e to the power minus lambda 1 plus lambda 2 minus into t. So, same way as we have got earlier probabilities we are able to get these probabilities also.

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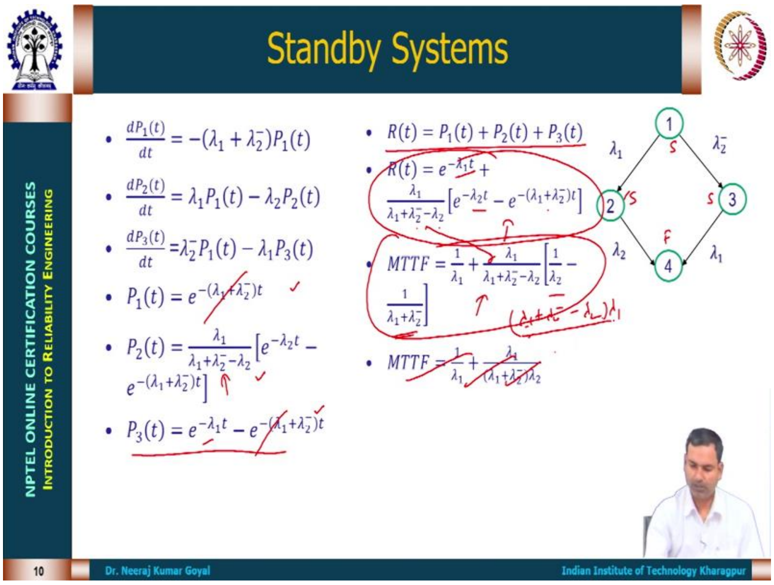

Similarly, PTT also we are able to get by taking by solving this. This is almost similar to what we have solved earlier in case of two component case. So, the only difference is rather than lambda 2 it is lambda 2 minus. So, once we solve this then P 2 t comes out to be e to the power minus lambda 1 t into minus e to the 1 minus lambda 1 plus lambda 2 minus into t. So, we have got all probabilities here P 1 t, P 2 t and P 3 t so, we can get the R t here. R t is because this is also success, this is also success, this is also success this system is some state is the failure state.

So my success state probabilities when I sum up I get the R t and my P 1 t is e to the power minus lambda 1 plus lambda 2 into t. And this if you see this will get cancelled, this will remain e to the power minus lambda 1 t, and this P 2 t whatever it is that will also come, lambda 1 upon lambda 1 plus lambda 2 minus minus lambda 2 and same. So, this becomes my reliability value. If I want to calculate MTTF, then the integral from 0 to infinity of this so, this becomes 1 upon this will give 1 upon lambda 1. This term will come as it is multiply by 1 upon lambda 2 minus of 1 upon lambda 1 plus lambda 2 minus. This gives me the MTTF.

If we solve this further by taking lambda 2 into lambda 1 plus lambda 2 minus then this will become lambda 1 if you take this inside common then this will become lambda 1 plus lambda 2 minus minus lambda 2 into lambda 1 divided by this value so, this lambda 2 so, we hold this this may this have to check. So, let us leave this. This will be the final value lambda 2 lambda 1 plus lambda 2 lamda 1 plus lambda 2 minus minus lambda 2 minus divided by into lambda 1 divided by lambda 1 plus lambda 2. So, we this solution may have been put in error.

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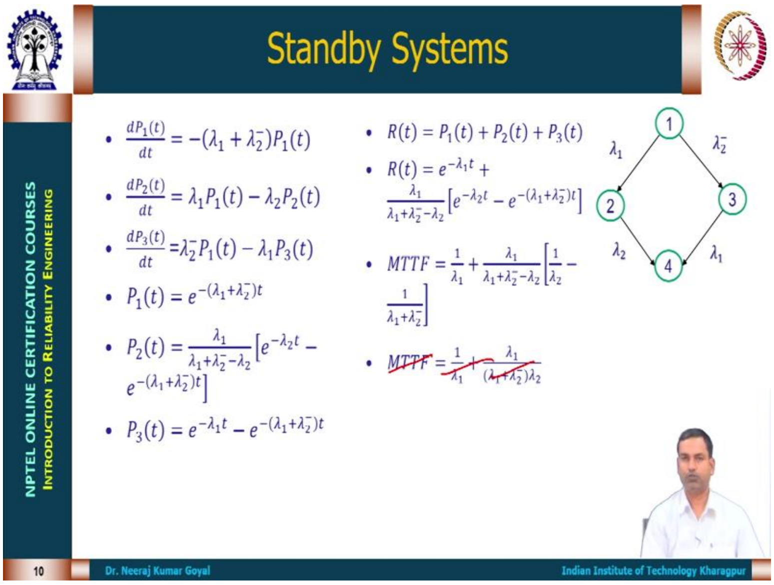

So, this is not applicable let us hold this here. System by system can be solved using this equation standby system considering that unit can fail in standby mode. But switch failure is not considered here that means he has a switch failure probability 0 or switch is going to be perfect. It is perfectly reliable. So, we will take an example of the same in discussing next class. We will stop it here today. Thank you.