Introduction to Reliability Engineering
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Lecture 02
Introduction to Reliability Engineering (Contd.)

Hello everyone. So, we are now up to the second lecture, which is a continuation of our previous lecture. So, today we will discuss the terms used in reliability engineering. So, here are some basic terms we are going to discuss today.

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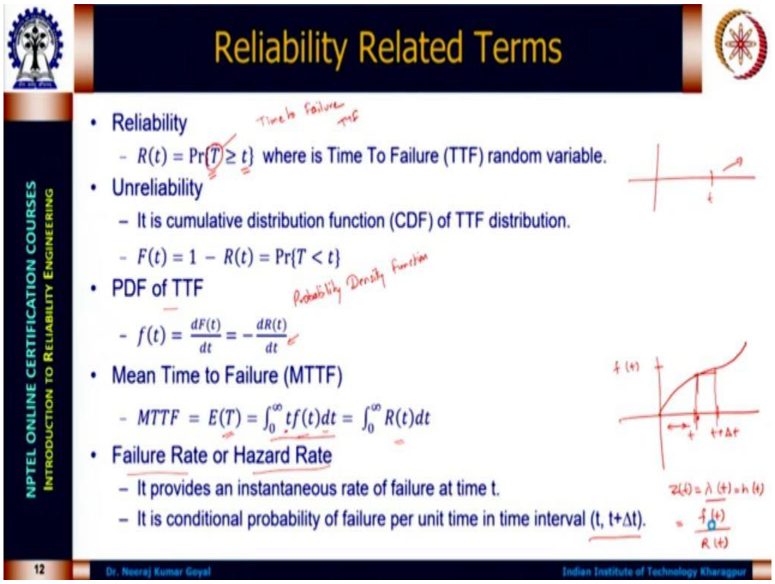

Let us first see what is reliability? As we discussed, reliability is the probability that the system performs a specific function for a given period of time under given stated conditions. Now, here what is that given period of time? Let us say it is over t; we can also call this time(t) the mission time. So, mission time back for the railway, as we discussed, mission time is like 1 hour trip time. So, the whole trip time can be a mission time which is completed without failure. How do we see that this is considered or it is without failure? We considered the random variable here.

As we discussed, this random variable is time to failure TTF. Now, this TTF time to failure has to be beyond time(t). What does it mean? What if I am going to use the system for certain time t; then, if my TTF is lying somewhere here, then I am fine. My system is reliable because I do not have any failure in this duration. So, what is the probability that my failure does not happen in this region? That means failure happens outside this region. That means my time to failure is larger than the time(t) that is my reliability. Moreover, what will be my unreliability that failure happens within this period? So, unreliability will be the cumulative TTF, or we can say it is (1-R); that is the probability that my failure is before time t, or within time t.

So, failure happens before completing the mission or the trip. Moreover, if that happens the system is considered to be unreliable. So, unreliability is the probability that failure will happen before completing the operation. So, here we use reliability and unreliability; these are prevalent terms. Then, we also use PDF. PDF is the probability density function. We will discuss a little more detail in the following classes and introduce basic probability. We can say the probability density function is the slope of the unreliability curve, or slope of CDF.

Alternatively, we can say the negative slope of the reliability curve, or we can say that is the negative differentiation of reliability. So, it is the differentiation of unreliability or CDF or negative differentiation of reliability. So, if we say that F(t) or R(t) unreliability and reliability. However, the probability density function may not be the probability; because it is the per unit time. So, it will have the time inverse dimensions, while reliability and unreliability will be dimensionless quantities. Then, comes the mean time to failure. The mean time to failure as we know, is the expectation or mean.

How do we calculate expectation of any random variable t? That is, since it is a continuous function, time is a continuous function we take the integration from all ranges. That is generally taken from -∞ to ∞, but time is a positive quantity. So, we do not have anything from -∞ to 0. So, it starts from 0 and is integrated up to ∞; and the random variable because it is E(t). So,

$$MTTF = E(T) = \int_0^{\infty} tf(t)dt = \int_0^{\infty} R(t)dt$$

There is an important term which we will be discussing in this overall course. Furthermore, this is the term which is very highly used in the reliability theory. That is the failure rate or we sometimes call it as the instantaneous failure rate versus the hazard rate. So, instantaneous failure rate or hazard rate what is it? Instantaneous failure rate or hazard rate is an instantaneous rate of failure at time t. It is the conditional probability of failure per unit time in time interval t to (t +∂t). That means that if we have a time interval t and (t +∂t) what is the, if we take f(t) here?

So, whatever is the value of f(t) in this short period? This f(t) value is my probability of failure per unit time. However, here, there is a conditional probability. What is that conditional probability? Conditional probability is that the item should not have failed before time t; that means it should be reliable here. So, this becomes f(t)/R(t). So, Z(t) is also represented as λ(t). It is also represented sometimes as h(t) as the hazard rate. We will be mainly using λ(t). This equals the probability density function, which is failure probability per unit time.

Failure probability per unit time is conditional. What is the conditional? That it should have been working; means out of the working unit what the probability that it failed. So, the working unit probability is R(t). Out of the working unit what is the probability that it failed is hazard rate; we can get this in a stepwise manner before going for a description of hazard rate. Let us first see what conditional reliability is.

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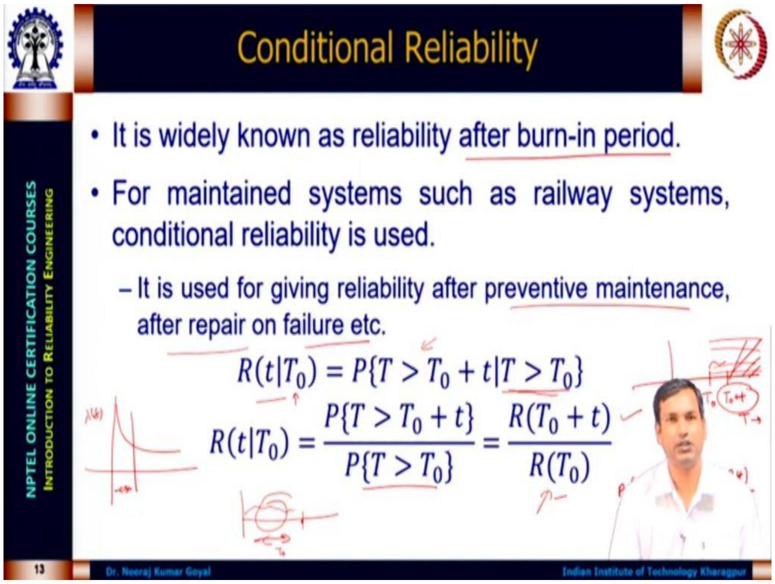

So, conditional reliability is often used for reliability after the burn-in period for systems like railways, aircraft etc. It also gives reliability after preventive maintenance or repair on failure etc. Repair on failure means once it fails, it is repaired. Preventive maintenance means the component or the system has not failed yet. However, we know that it may fail, so to reduce the chances of failure, we are doing some maintenance there so that it does not fail. So, here as we see this is $R(t|T_0)$. So, T_0 is the time for which it has been operated. If you look at here, this is work for T_0 and this is $(T_0 + t)$. So, what is the reliability during this period?

That probability that it works during is T_0 to $(T_0 + t)$, given it has worked up to time T_0. So, this is the probability that the time to failure is beyond $(T_0 + t)$, given time to failure is greater than T_0;

that means it has not failed till T_0 that is my condition here. Moreover, in this condition what is the probability that it works from T_0 to (T_0+ t); that is failure time is beyond (T_0+ t). How can we get this? This probability conditional probabilities, probability we know conditional probability of probability of (x|y) is equal to the P (x∩y)/ P(y).

Similarly we can get P(T > T_0+ t). So, the intersection of the two if you see is the same as probability that P(T>T_0+ t). So, the intersection term is R(T > T_0+ t)., probability that P(T > T_0+ t).; that is my R(T_0+ t). Similarly, the condition here is that P(T>T_0) is R(T_0). So, this is how we can get conditional reliability.

$$R(t \mid T_0) = P\{T > T_0 + t \mid T > T_0\}$$
$$R(t \mid T_0) = \frac{P\{T > T_0 + t\}}{P\{T > T_0\}} = \frac{R(T_0 + t)}{R(T_0)}$$

This conditional reliability concept is often used for burn-in; what is the burn-in period? Initially, because of the manufacturing etc., there are certain times the number of failures is very high. So, failure rate is high.

In that case what happens to remove that big population or the faulty population, what we do? We do burn-in. That means we keep the device under elevated temperature and observe that if it is keeps on working or not. So, stresses are applied, and then we see that whether it is failed or not like, like a motorcycle or car, what can be done initially before giving it to the customer, they will run it for eight hours. They will run it for one or two days, or one or two months, and then they will give it to the customer. That happens because of the initial running whatever weaknesses are there, whatever problems are there that can be detected and corrected, before giving it to the customer.

Same way, if you talk about the trains, like trains, before being used for public uses. What they will do? Like a nuclear power plant, they will launch the train in the dry run. So, even for one or two years, they will run that train without any passenger to see whether there is a failure. Because initially, chances of failures are high, there can be a problem due to the manufacturing or installation; those problems are difficult to remove. But, initially, if we have some failure-free period, then we can ensure that the device will run normally after that period of time; or it will have good reliability.

So, this becomes our conditional reliability; during this period (T_0), the customer has not observed any failures. So, during this period, the customer is not using the product as it is used within the organization. Furthermore, whatever failure happens, here it is corrected; the customer does not come to know. So, the customer sees the reliability after time $T_{0,}$ which is (T_0+t); this is conditional reliability. Because already T_0 time we have spent, and we have ensured that component is not in failed condition here. So, this becomes the conditional reliability here. Similarly, for preventive maintenance, the same philosophy works here.

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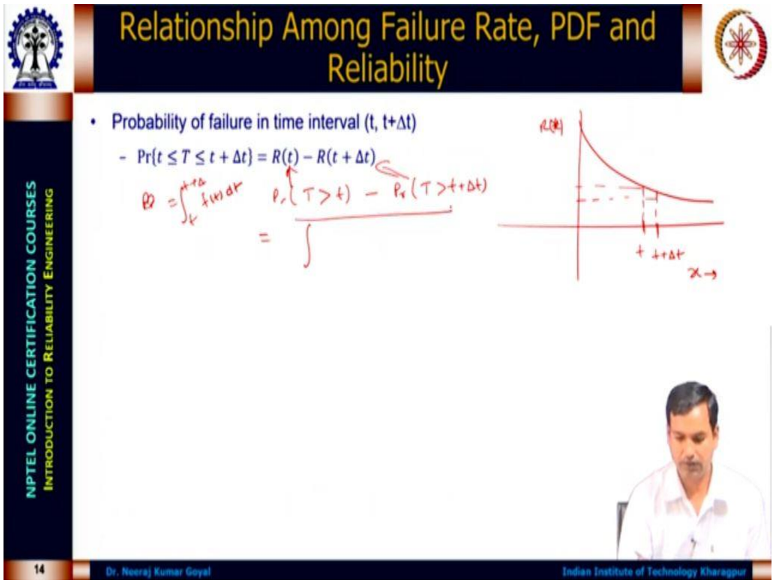

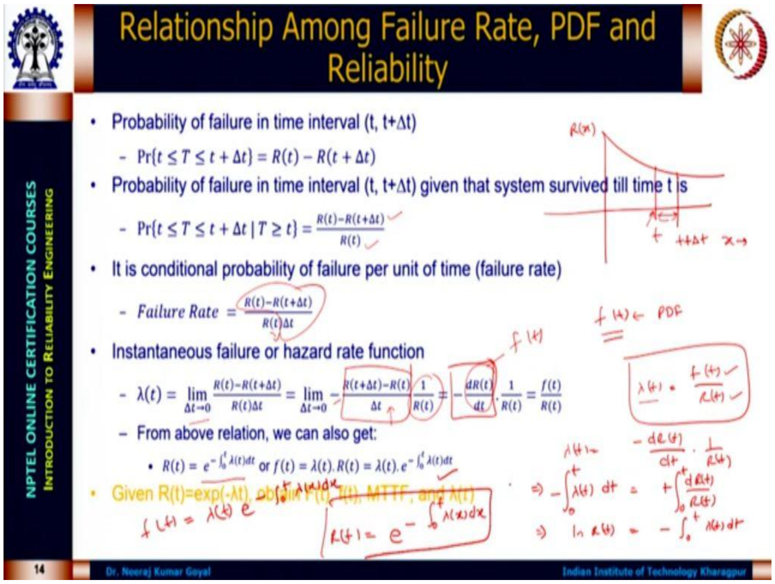

Let us see how the failure rate relates to the other thing. So, though we have shown there that failure rate is f(t)/R(t); but how does it come? Let us see what the concept of failure rate here is. Let us assume that we have time (t +∂t). Now, during this time interval, what is the probability of failure? Now R(t) is a decreasing curve. We will discuss all this. This is or we can say our R(x); let us see this is x. So, as we see here, if we want to know from this is the probability of success; so what are the chances of failure here?

The chances of failure here mean that it is failing during small t to (t +∂t); that means we can write the same thing that this is a probability that my time to failure is beyond small time t, because it has not failed here. But, it has failed before (t +∂t). So, the probability that time is greater than (t +∂t). But time has exceeded (t +∂t). So, the same thing we can add this to R(t), and this is R(t +∂t). So, same thing or we can say this is probability that in another way we can see this is my probability. So, I can calculate that in the limit from 0 to t (t +∂t)ft dt. This gives me the failure probability in time interval (t +∂t).

We can write again the same way that is called the T is greater than t and T is greater than (t +∂t). Once we get this, this gives us the conditional probability of failure; let us again see this. So, now, I want to know my probability of failure in the same interval; but, the condition is it is survived by here. We are keeping this because we want to ensure that the product is in working condition. So, out of working condition, out of the working product, what is the proportion of failure per unit time? That is of concern; that is what we call the failure rate.

While f(t) which we discussed earlier is PDF which is concerned with the failure probability per unit time during this interval, there is no condition. It is out of the whole population. Out of the whole population, what is the priority that it will fail in this duration? However, for failure rate, what is the probability it will fail in this duration given that it is working here; that means only the population surviving up to here. How can we get this means conditional probability? So, that the probability of failure during this period divided by the probability that it is survived up to time that is R(t). So, this becomes conditional probability of failure per unit time which is failure rate. So, failure rate becomes the same value as what we have written here.

So, the same thing we are calling as instantaneous hazard rate function. So, now we can say that limit t delta t tending to 0 will give the instantaneous failure rate or hazard rate i.e R(t)- R(t +∂t)/R(t +∂t). So, I can take 1/R(t)outside here, this portion I can take outside; remaining portion is our R(t) minus R; negative if I take outside, this becomes R(t)- R(t +∂t)/R(t +∂t). So, if I apply the limit, this portion will become dR(t)/dt. So, we get the following

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{R(t)\Delta t} = \lim_{\Delta t \to 0} - \frac{R(t + \Delta t) - R(t)}{\Delta t} \frac{1}{R(t)} = - \frac{dR(t)}{dt} \cdot \frac{1}{R(t)} = \frac{f(t)}{R(t)}$$

So, this is how we get it. In a more straightforward way if you see, this is the PDF applicable given that conditional PDF given that there is no failure up to time t; that becomes the same thing. So, we can now, if we try to use this formula, we will also be able to get R(t); because we know this $\lambda(t)$= *f(t)/R(t).* This is uniquely defined that means if we have one value of $\lambda(t)$or one function $\lambda(t)$, then f(t) will be uniquely defined by that. We cannot have two different ft when given a $\lambda(t)$function. For one function of $\lambda(t)$, we will have unique f(t) ,unique R(t); we cannot have two different values of them. So, they are uniquely defined by each other.

So, here we can calculate reverse for, if we have any one of the functions, we can get all other functions; if we are having $\lambda(t)$,then we can calculate R(t). This R(t) comes out to be e to the power minus 0 to t, lambda t dt. How does it come out? We can see that this $\lambda(t)$= -dR(t)/dt, and that is 1/R(t). If we say that dt if we take it here, then $\lambda(t)$dt will be equal to -dR(t)/dt, if we take integration on both side from 0 to t. So, this will become negative sign we can take here, this will become plus; and this will become lnR(t). lnR(t) is equal to minus integration from 0 to t lambda t R(t), $\lambda(t)$dt.

So, from here my R(t) comes out to be e to the power minus; because ln if I take it here, this will become exponential, zero to t lambda x dx. Same thing we have written here. I can say x so that we are able to differentiate between mission time and the integrating variable. Similarly, like ft is equal to $\lambda(t)$R(t), from f(t) $\lambda(t)$into R(t); so, R(t) is my already here. So, if we want to calculate f(t), then f(t) in terms of $\lambda(t)$ if I say, then this will become $\lambda(t)$into R(t). R(t) e to the power minus 0 to t, lambda x dx, this one same. So, similarly, whatever we have if we have R(t), we can get $\lambda(t)$, we can get f(t) all the functions can be interchangeably evaluated. So, simple mathematics we can do it.

$$R(t) = e^{-\int_0^t \lambda(t)dt} \text{ or } f(t) = \lambda(t) \cdot R(t) = \lambda(t) \cdot e^{-\int_0^t \lambda(t)dt}$$

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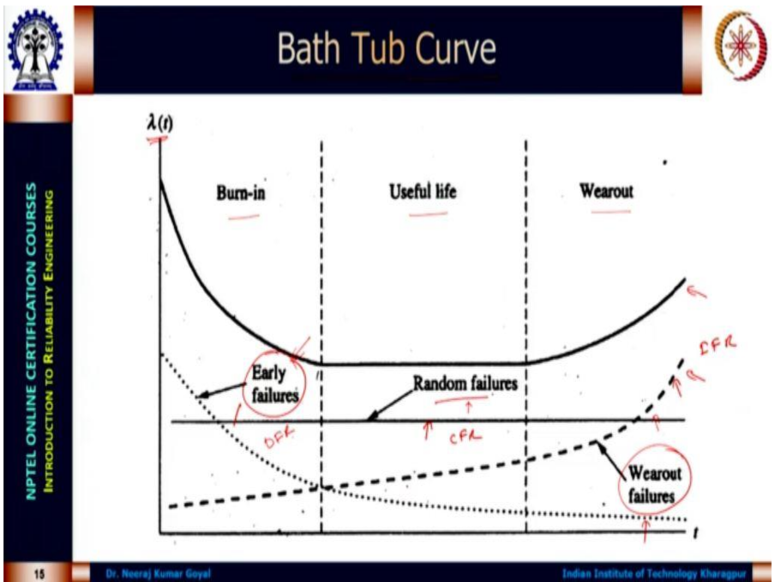

There is one important curve which is very famous curve; which is called bath tub curve. This curve defines how life goes on. This life is similarly experienced by humans, for animals or even the products. If we look at it, we have three phases here. We have burn-in, we have useful life, and we have the wearout. How is it coming up? There are actually three kinds of failures involved here. There is wear out failures, there is a early failures, and there is a random failures. Now, let us see what is this early failures. This bath, generally there are three source general source of the failures; one is the early failures.

These early failures are happening because of the mistakes we haves made during the design, or the mistakes which have been done during the manufacturing; or there has been weakness in the parts. Or there has been installation problems or transit problems, transportation problems. So, because of this what happens that in a very short time of product uses, the product will fail. But, initially only the failure rate is high; that means per unit time failure probability is high. But later on, those products that have survived continue to survive longer. So, my failure rate, this is a curve for failure. So failure rate keeps decreasing, also called decreasing failure rate DFR.

Then, another source is the random failure. Random failure happens due to the various reasons it can happen with any device anytime. So, it is not generally having a particular reason or particular; any component can fail for any reason, accidents can happen. So, accidental failures in human life can be called as the random failures. Because, early failures like for human life if we compare, early failures are the ones which are happening because, let us say infant mortality; so the child at the time of birth is most vulnerable. So, at that time, we have high death rates. So children at the time of pregnancy or at the time of birth, or during initial feed up to three to five years, there is a high chances of deaths.

Why that is there? Because the system or the human being or the baby requires care. Why do they require care? Because they are vulnerable to the environment, vulnerable to human uses and various accidents. So, this early failure probability or early failure rate has to be insured; so they require care. So, it is insured that one; but once they reach age five and more, the chances of failures due to these kind of baby-related issues becomes less. But, there is that there is a random failure. Random failures can be due to any reason; it can be due to the development of certain diseases, or the development of certain accidents, or something suddenly happens with someone.

But, does not happen with most of them, it happens with some of them. Similarly, something happens or goes wrong when using the system. So, they have these kind of random failures. Then, we have the wear out failures. These wear out failures are the age related failures like when we grow old, then our body (weaks) weakens, and we are susceptible to the diseases. And after certain life, let us say 70 years or 80 years, we wear out is so high that our body gives up. So, initially the failure rate is low, there is no wear out; but later on, on the life we see that due to wear out, chances of deaths are rising. Same thing happens with the human being and same thing happens with the components or the systems. The systems also wear out like our tires, they will they will wear; so, and we have to replace. Similarly, most of the components and parts of our systems cannot survive long; they cannot continuously work for infinite period of the time.

So, at a certain period of time, after that time, because make a change in chemical properties, mechanical properties, change in system will lead to these failures. So, everything ages and once it ages, because aging wears out what happens. If we add these three curves, this curve, early failure curve, then random and wear out failures. If we add them, then we will see that see the shape. In the initial period, the dominant failure mode is the early failure, DFR. During the useful life, which is the like for our case, it is the useful life maybe around 5 years to somewhere around 50 years; we do not see much chances of failures. The chances of failures are same. Some person can die without having any not because of age; it can die at any moment because of those. There are random failures; so it is a constant failure rate.

Then, we have the increasing failure rate that is our wear out failures. Here we have the failures because of the aging, because of weakness, which has been there for the system of the human being; and this is increasing failure rate, IFR. So, this kind of curve or failure rate pattern is generally observed with most of the systems. So, if we understand that, we can devise our expectations better and manage our systems better.

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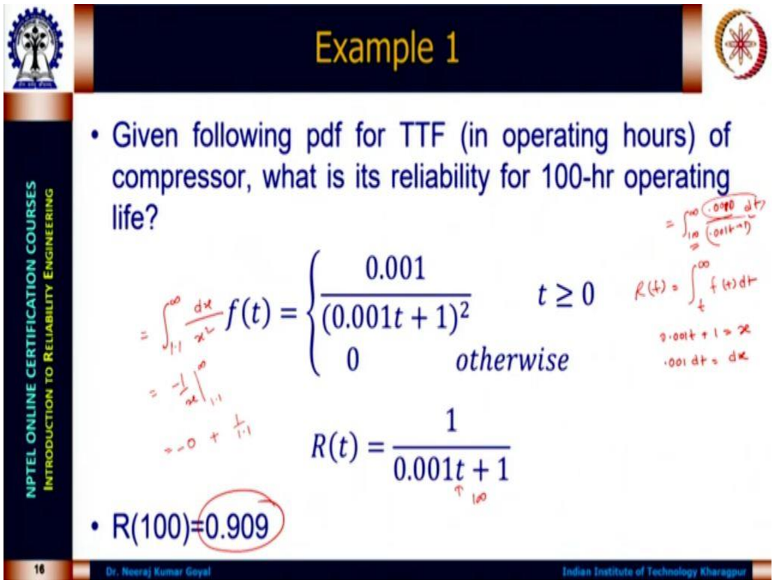

Let us see one example: if we have TTF distribution given here, if f(t) is given, then what will be the reliability? As we discussed, reliability equals integration from t to infinity f(t)dt. So, here if you want to calculate reliability for 100 hours of operating life, we can calculate it as from 100 to infinity, f(t) is this; because this is defined for t greater than 0, we are talking about t greater than 100. So, we can use the same 0.0001, sorry 0.001, divided by 0.001 t, plus 1 whole square dt. If we solve this, then like we can assume that 0.001 t plus 1 is equal to x; so 0.001 dt will be equal to dx. So, this will be replaced by dx and what we will have here let us solve it here.

So, if I apply t equal to 100 here, so 100 into 0.001 will become 0.1 plus 1; so that is 1.1. That is and for infinity that will become remain infinity; and this will become dx upon that is again my x square. So, what I will get is minus 1 upon x that is 1.1 to infinity. So, that infinity this will be 0 and minus minus will become plus; and that will become 1 upon 1.1, this will be my Rt. Same if you see here, if I apply t equal to 100, I will get the same value. I have solved it in terms of values, we can first solve in terms of t and then apply the t; or we can solve directly also. So, my value if I solve this 1 upon 1.1 that is turning out to be 0.909.

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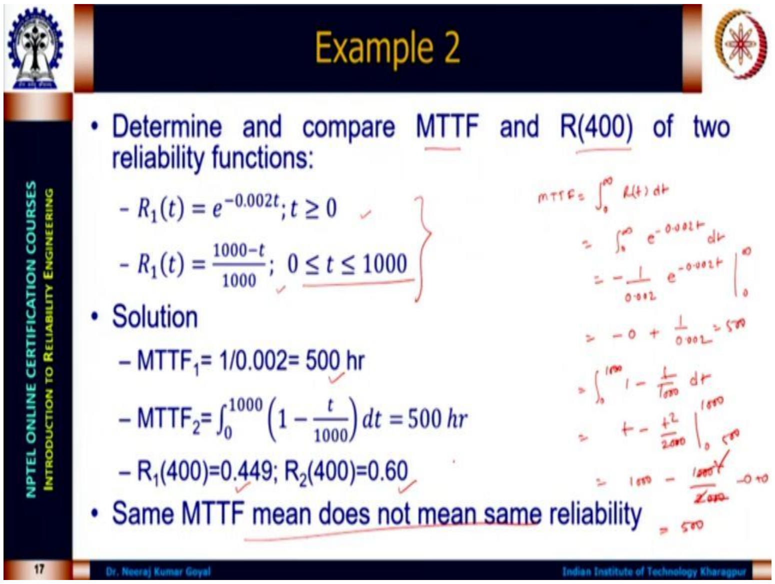

Similarly, if I do another example that computing MTTF and reliability. Now, let us say there are two reliability functions given reliability e to the power minus 0.002t, and another reliability function is 1000 minus t upon 1000. This function is defined for t values of t from 0 to 1000; this is defined for all positive values of t. If we want to calculate MTTF for this, now MTTF is integration from 0 to infinity, R(t) dt. If I integrate this, what will I get? I will get integration from 0 to infinity, e to the power minus 0.002t dt. I will get 1 upon 0.002 minus, e to the power minus 0.002t, from 0 to infinity. If I put infinity here, this will become 0; e to the power minus infinity will be 0.

So, minus 0 and then I put 0; this will become 1 and this will become 1 upon 0.002. This is 500 hour. Similarly, if I solve this, then this is integration from. Now, here this is defined for 0 to 1000 only; so I will calculate from 0 to 1000. And if I divide by 1000 that is 1 minus t divided by 1000 dt; so that will become t minus t square by 2000, from 0 to 1000. So, that will become if I do this, this will be 1000 minus 1000 square, divided by 2000 minus 0. If I apply t equal to 0, both will become 0. Now, here if we see from 1000, it will be gone. I will have 1000 divided by 2 that will be 500; 1000 minus 500 will give me 500. So, as we see in this case, both are giving me 500 hours.

But, if I calculate the reliability for 400 hours, I simply apply the values here. So, this will become e to the power minus 0.002 into 400; this gives me reliability as 0.449. If I apply the t equal to 400 here, that will become 1000 minus 400; that means 600 divided by 1000 that will be 0.6. So as we see here, two functions are having same MTTF; but their reliability behavior may be different. So, we should not say that if two systems are having same MTTF, then they are having the same reliability also.

So, we should keep in mind that reliability may be different, even though system is having the same MTTF.

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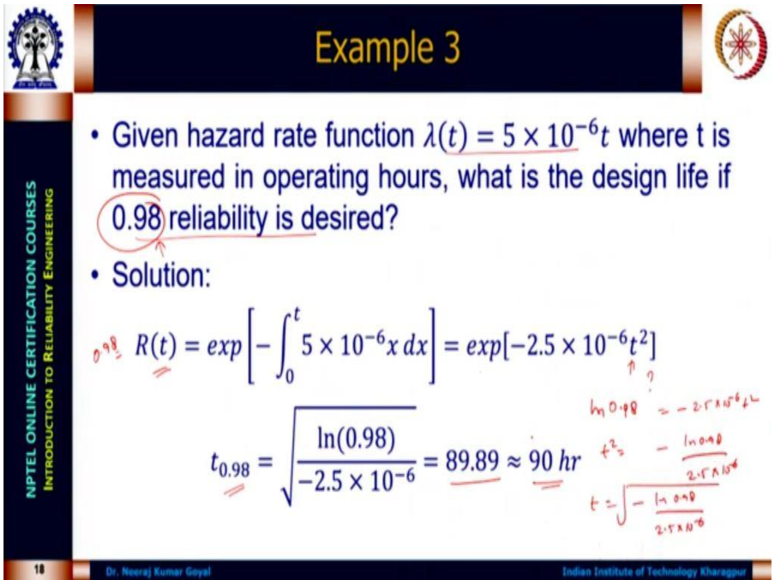

Similarly, if we have the lambda t available to us, we can calculate the design life. So, what is the design life? That means we want to know the life at which our reliability is 0.98; our reliability is not falling below this. So here, I can write down the R(t) function that is exponential minus 0 to t ft dt. If I solve this, my value will become exponential. This x square by 2 this will become; so this will become 2.5 into 10 to the power minus 6, t square. Once I apply the values here, what is my design life? I want to know what is the value of t for which this reliability will be 0.98? So, I will put 0, this will be 0.98. And I want to know the final value of t; so, here I can take log of this.

So, ln of 0.98, then this will be right side minus 2.5 into 10 to the power minus 6, t square. Then, t square is equal to minus ln of 0.98, divided by 2.5 into 10 to the power minus 6. Then, t will be equal to square root of minus ln 0.98, divided by 2.5 into 10 to power minus 6. Since, ln value is calculated for value less than 1; this will be negative value. So, negative negative will become plus; if we solve, we get the value 89.89; that equals 90 hours. So, using these functions, we can evaluate reliabilities as we see. So, we will stop it here today; we will discontinue our discussion in the next lecture. Thank you.