Introduction to Reliability Engineering
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Lecture 19
System Reliability Modelling (Contd.)

Hello everyone, we will continue our discussion on system reliability modelling. We started discussion with the series, then parallel, then k out of m, parallel series, series parallel. We also discussed last time that if we have the complex system which cannot be reduced directly to series parallel network, then how can we use various methods like disjoint theorem or we can use the cut set method or path set method to solve them. We will continue our discussion to some more system.

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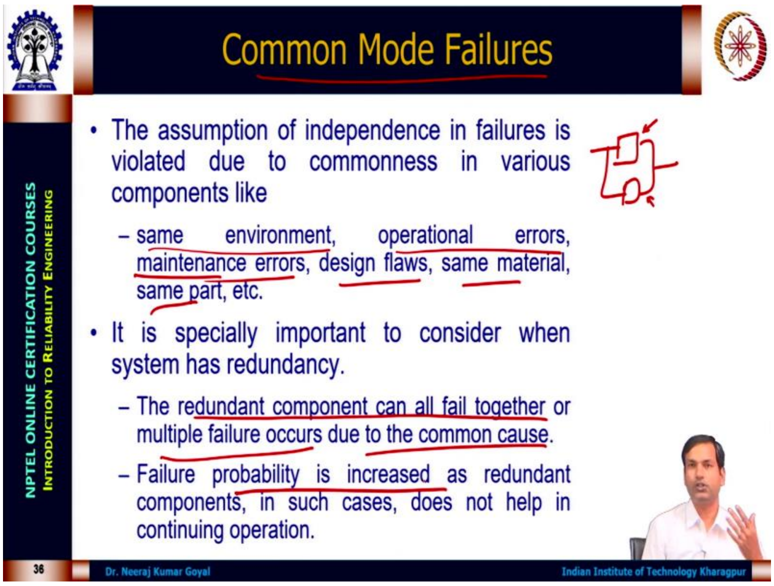

Here, we are discussing about the common mode failures. Sometimes what happens that when we consider that see components are in series of parallel, the inherent assumption as we discussed initially in RBD that components are independent. That means, one failure, one component failure does not affect the failure of another component. But sometimes this redundancy which we are building in the system may not exactly be holding good. The reason being the common mode failures. There are some failure modes which can cause the failure of such components together.

So, this commonness can be due to the same environment. Let us say something goes wrong like some electric pulse comes; that electric pulse can cause failure of if you have let us say if you are talking about a room where we have the multiple light bulbs. So, but that same electrical pulse can cause the failure of all electrical light bulbs. So, we will have the dark. Though there are let us say 4 bulbs, but all 4 may fail together because they have the similar electrical circuit and they are made of the same material, same design. So, they are susceptible to the same pulse.

So, because of the same design, same environment, same people who are operating, maintenance errors, like many times what happens, someone who is maintaining the system or operating the system, if they know, they have learned something which is wrong, which is not the right way of doing it, but they will do the same thing for all the components which we have put in redundancy. So, in that case what will happen, when that particular situation arises, in that situation the way we have maintained or the way we have operated is incorrect and that will be exposed. In that case what will happen? All elements will fail together.

So, human elements in terms of maintenance or operational, design flaws like if multiple components which we have taken for the same material same design, so, if there is a some weak component, then that is weak in all. So, whenever stress is coming high, that means, if my design is like that that it cannot sustain 240 volt, then if suddenly 240 volt come, then all bulbs will fail or all designs will fail. So, because of these similarities what happens, they also have the common causes and these common cause result in a failure of multiple systems together.

While during the design of the system we assumed that these systems are in redundancy that means, if one fails my another system will work and my function will continue, but because of common cause, all redundancies are becoming useless, all redundancies are becoming non-effective. So, the redundant components can fail together or multiple failure can occur due to the common causes. So, either it may not be all sometimes, it may be like a major portion of the components fail together due to the common cause. So, with this what happens?

The failure we were assuming the independence because of which our reliability was becoming higher. Because of parallel system our reliability becomes higher, but in this case because of common cause, the reliability achieved will not be so higher and our reliability will actually reduce. So, our failure probability is increased because of the multiple failures happening together due to the common cause and this becomes a major reason for system failure many times. So, how do we take care of this in evaluating the reliability?

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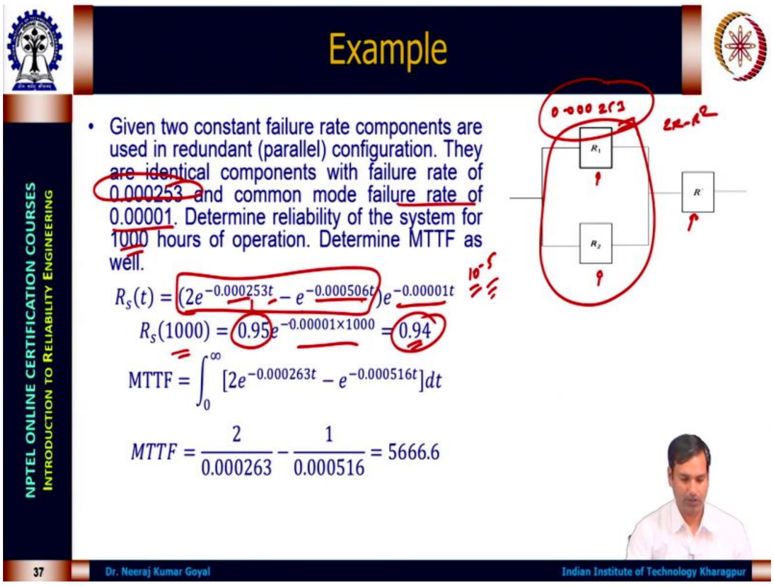

So, let us say if we have two components, R1 and R2. Generally, these components are of similar type. So, we have two components which can fail independently due to the reasons and they can also fail due to the common reason. So, if we say that two components are there, they are in redundant and they use identical components with failure rate of 0.000253. So, that is the failure rate for this, that is the failure rate for same, for R2. So, but common mode failure rate is also there, that is 10 to the power minus 5. Though, this failure rate is high and this failure rate is small, but as we see, here we have the redundancy.

$$R_S(t) = \left(2e^{-0.000253t} - e^{-0.000506t}\right)e^{-0.00001t}$$
$$R_S(1000) = 0.95e^{-0.00001\times1000} = 0.94$$
$$\text{MTTF} = \int_0^\infty \left[2e^{-0.000263t} - e^{-0.000516t}\right]dt$$
$$\text{MTTF} = \frac{2}{0.000263} - \frac{1}{0.000516} = 5666.6$$

So, due to the redundancy the reliability for this redundant system will be 2R minus R square. This we have evaluated many times. So, 2 into R means e to the power minus lambda t, this is lambda into t and R square means multiplied by 2 into t, e to the power minus this multiplied by 2 will give 0.000506. 506 t. This becomes the reliability for this and how much is the reliability for this? Reliability for this is e to the power minus point 10 to the power minus 5 into t.

Now, let us say t is 1000 hours, if we put t equal to 1000 hours, Rs turn out to be 0.94 we apply this. So, this reliability is 0.95 and because of this reliability is turning out to be 0.94. So, similarly if we see here that because of the common cause, the common cause will appear as the series. Though components are in parallel, the common cause because the same cause and system can fail either due to the common cause or system can fail due to the independent causes both components fail.

So, this reliability is calculated using this. So, common cause if we are considering, then common cause reliability has to appear in series of these parallel components and similarly, we can solve this. If we want, we can solve this using excel though I have not, you can take it from here.

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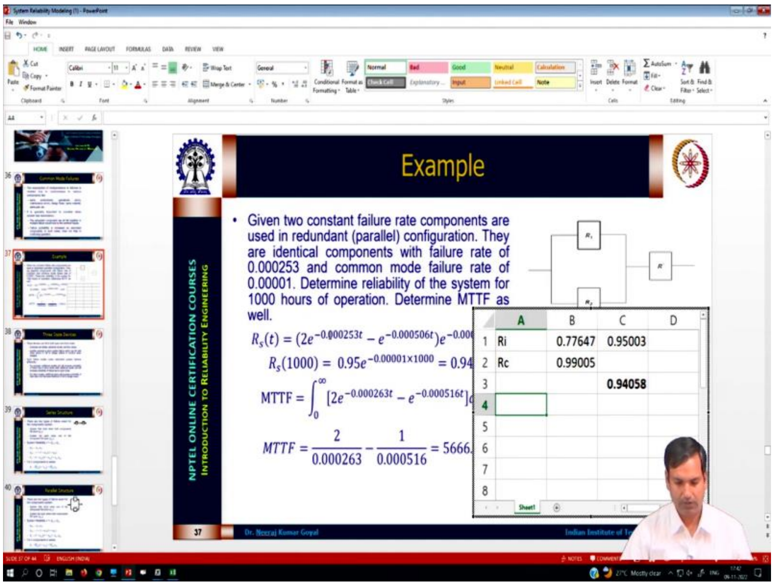

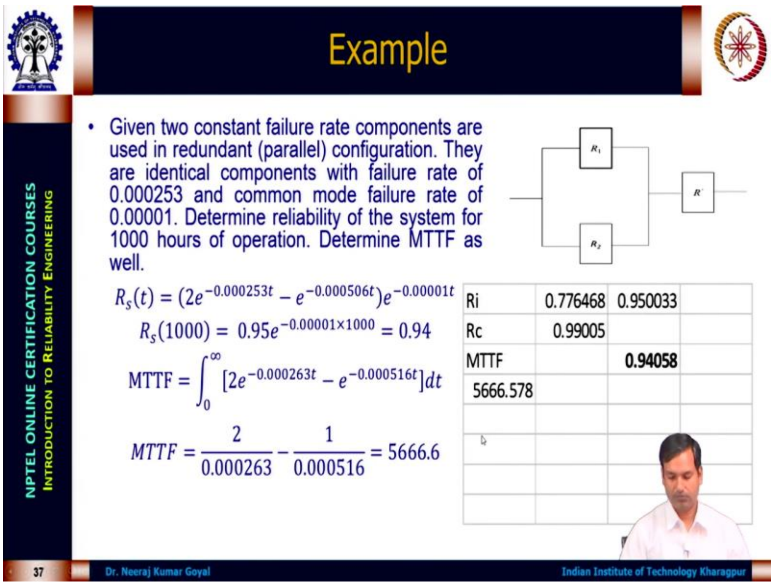

So, let us say if we take this here. Let us try to see how do we solve this. So, for one component independent, I will say Ri, independent failure reliability. So, that is equal to exponential minus 0.000253 into 1000 and common cause reliability is equal to exponential minus 1 e minus 5 into 1000. Now, if I am taking parallel combination, that will be equal to 2 into R minus R square and RC is this. So, my system reliability will be equal to this multiplied by series of this.

I get this and for MTTF calculation as we see, this lambda here if we see this will this is 2, 2 e to the power minus this multiplied by this. So, this will become addition to here. So, this will become 0.00021 will be added here. So, 263 and similarly 1 will be added here. So, this will become 516. And once we integrate this, then we know integration from 0 to infinity of e to the power minus lambda t dt is 1 upon lambda. So, that will become 2 divided by this value.

So, what I get is this is equal to 2 divided by my lambda is 0.000263 and this is negative sign already minus, but there is no multiplication. So, 1 divided by 0.000516. I have to take a another bracket here, this is fine. So, my MTTF turns out to be 5667 or we can say 5666.6. So, if we are going to consider common cause we can consider the common cause in reliability calculations like this.

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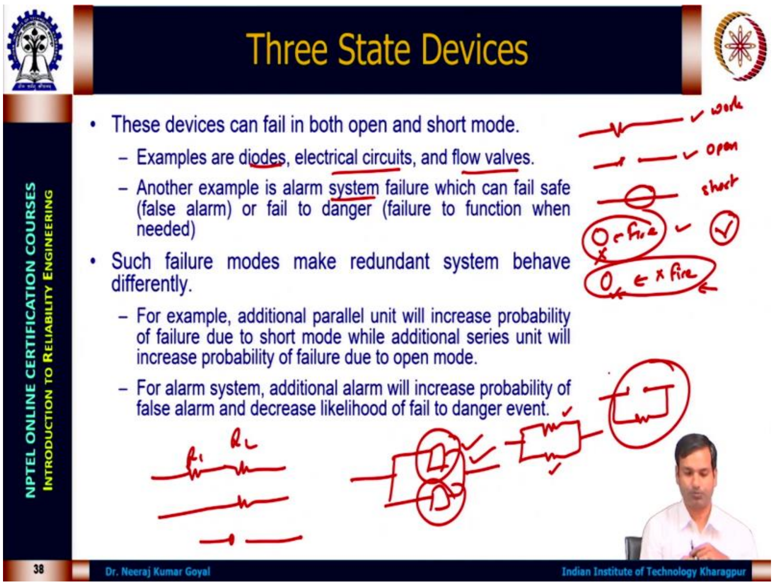

Many times, we are concerned with 3 state devices. Like we have resistors, capacitors, valves, they can fail in multiple failure modes. Like resistor if we have resistor, so, a resistance can fail in 3 major, there can be more failure modes like, but 3 major failure modes is like resistor, 2 major failure modes is like resistor becomes open, resistance becomes infinite; another failure mode is resistance becomes short. So, either resistance works or resistance fail open or fail short, this can be two major state.

There can be other states where resistance is becoming smaller or higher, but the major state variations are this. So, here what will happen? When resistance fails in open or fails in short, it will have the different impact on the system and the redundancy which we have put into the system will have a different way of reacting. So, like diodes are there, electrical circuits, flow valves, we have alarm system. Like for alarm system also let us say alarm system can fail in 2 failure modes - one is fail to alarm, like when let us say if we talk about the fire alarm.

So, we have the fire here, but alarm does not ring. In that case also failure is there because it is supposed to ring in case of fire. That is one type of failure mode. Another failure mode is alarm works, but there is no fire. So, even though fire is not there, alarm rings - false alarm and this is alarm failure. So, in this case what will happen? Now let us assume that I have more alarms. So, let us say rather than 1, I put the 2 alarms. If I put 2 alarms, then what will happen? If there is a fire then detection probability by one of the alarm will be higher because now we have 2 alarm. If one fails to detect, another one will be able to detect.

So, my failure probability of detecting fire will be reduced or my reliability to detect fire will increase, but my probability of failure that is that without fire it is ringing will become high because there are 2 alarms. So, now, either alarm 1 can also ring, another alarm 2 can also ring even when there is no fire. So, my chances that fire without fire alarm will ring will also increase. So, my redundancy is helping me to reduce one probability, but it is increasing another probability.

Similar thing happens with the open and short as we will shortly discuss that if we have 2 components in parallel, 2 resistance in parallel then let us see if it fails short then even if this fails short or this fail short, anyone fail short my probability will, my system circuit will become short. So, in that case what is happening that my chances of short is increasing because whether this fail short or this fail short, in both the cases my system will fail short, but against opening chances my reliability is increasing, failure chances are increasing because even if this resistance fails open, my another resistance will continue to serve the purpose.

Similarly, if they are in series, then I am protected against the short because if one fails short, another will be provide the resistive path, but in this case if anyone fails open either R1 or R2, in both the cases my system will fail. So, open failure in the series, if resistance opens, then my failure probability is increased, probability of getting that system will fail in open mode will be higher, but probability that system will fail in short mode will become smaller.

For parallel configuration of resistance, my short mode failure probability is increased, but my open mode failure probability has decreased. So, depending on the configuration because when devices are having multiple states; the same configuration may not be able to give us the higher reliability. We have to see in which aspect our reliability is increasing and in which aspect we have to pay the penalty. So, here as I think this we discussed, so let us discuss the series structure.

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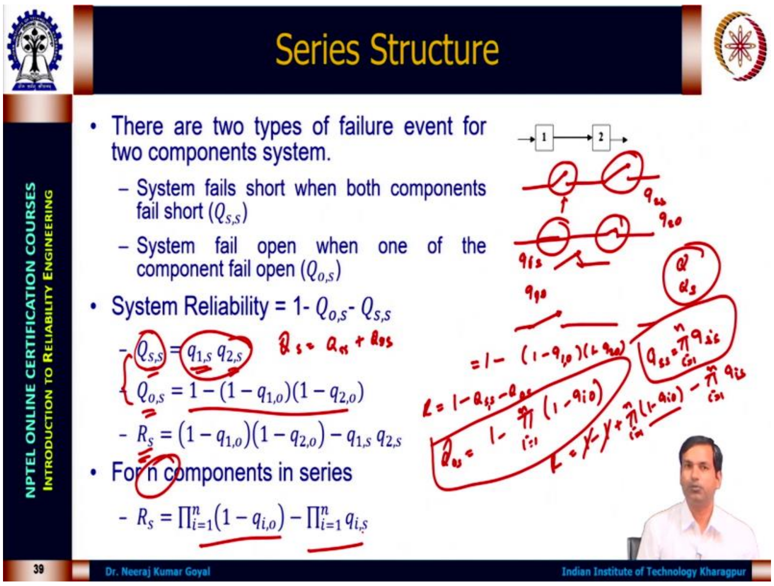

So, we have 2 components in series or as we discussed we have the 2 resistance in series. So, now, we have 3 possible cases for the elements also and for the system also. So, for element let us say element can have 3 state; either it is working or it fails. Let us say this is R1, this is R2 or better understanding we can also take switches or we can also take valves. If you are not having electronic background may be let us take the switches, that will be easier to understand. Let us talk about switches.

We have 2 switches in series. Now, switches can fail in 2 failure modes, it can stuck close or it can stuck open. That means, we can say it is failed short means it is shorted right? It is shorted and I cannot open, it is stuck. So, fail short, that probability is Q for first switch let us say Q1s and for second switch that is Q2s. Similarly, it may remain open that is I want to close it, but I am not able to close it my circuit does not complete, that is Q open circuit. So, let us say Q1O and similarly Q2O.

My system can fail in 2 failure modes and one is the success probability. So, my system can fail in open mode that is Qo and similarly my system can, or this component together can cause the system to be failing in short mode. Now, let us discuss here. Then my system will be shorted. That means, I am not able to open the circuit. This will only happen when both the switches fails in short mode. That means, when both becomes short and I am not able to open any one of them I will not be able to open the circuit or I will not be able to switch off.

So, here my system will fail in short mode when both the components fail in short mode. That means, my failure probability in short mode of the system is failure probability of component 1 in short mode and failure probability of component 2 in short mode, this becomes my failure probability in short mode. What will be the failure probability in open mode? For open mode if any one of is the component is stuck, open, then another component does not matter.

Because if one component is stuck open I will not be able to close the circuit. For closing the circuit both should close. So, any one of them is able to cause the failure. So, that means, the failure probability will be 1 minus 1 minus, parallel configuration here 1 minus Q1O and 1 minus Q2O, either of the of them failing in open mode will result in failure mode open for the circuit. So, QoS becomes failure of failure in open mode of system will be 1 minus of this. So, my reliability will be how much?

My reliability will be, as we know reliability is 1 minus Qs is minus QoS. That means, probability that my failure probability will be summation of this, Qs will be equal to Qs,s plus QoS, either system fails in open mode or system fails in short mode, that will be my system failure probability. And system failure will be 1 minus Qs, that is 1 minus Qs,s minus QoS. So, this if let us say I am having n components in series. So, Q open circuit will be equal to 1 minus multiplication I equal to 1 to n, 1 minus QiO; any one of them fails in open mode, it will result in open mode failure.

Similarly, Qs,s short mode failure probability will be any, all of them fails in short mode, that means, QiS, i equal to 1 to n, if I multiply with them, I will get the Qs,s right. If I subtract them from 1, so, then reliability will be equal to 1 minus 1 plus 1 minus 1, then this will become minus minus plus pi of i equal to 1 to n, 1 minus QiO minus this minus pi i equal to 1 to n QiS, this if we solve 1 and 1 will get cancelled and this will become multiplication of 1 minus QiO i equal to 1 to n minus multiplication of QiS same thing is written here.

So, this gives me the series structure when I am following and I if I am taking two failure modes one success mode, that means, three state devices, then this will be giving me the reliability of series system.

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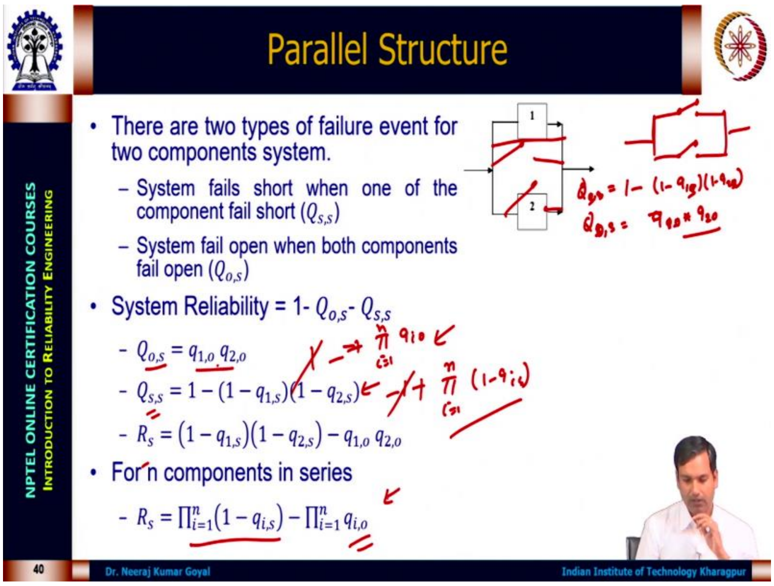

Similarly, if I have parallel system, for parallel system this concept will be just reversed. So, for parallel system let us say this 1 and 2. So, parallel system will fail if any one of them fails short. If this is shorted what will happen? My system will be short. If any one of them is shorted, I am not able to open, then my system will short. So, that means, parallel, that means, 1 minus 1 minus Q1O and 1 minus Q2O, this will be my QoS. But for open mode failure if this becomes open, then it is not sufficient because, then my system will work through through 2.

So, both has to open, then only my system will fail in open mode. So, this is s right, this is s. So, You, this is s s, short mode failure will be equal to sorry, open mode failure will be of system will be when all components fail in open mode. So, that will be QoS into 1 Q1O and Q2O. So, here open mode failure is multiplication of all open mode failure probabilities and short mode failure is 1 minus 1 minus Q1s minus 1 minus Q2s. As we discussed earlier for n component this will become 1 minus pi i equal to 1 to n 1 minus QiS and this will become QiO i multiplication i equal to 1 to n.

And when I subtract them from 1, so, then this will become 1 minus this, minus 1 plus 1 plus this. So, 1 and 1 will get cancelled and this will become multiplication of this minus QiO. So, multiplication of 1 minus QiS i equal to 1 to n minus i equal to 1 to n QiO. So, here because of parallel structure when we use this will be giving us the system reliability.

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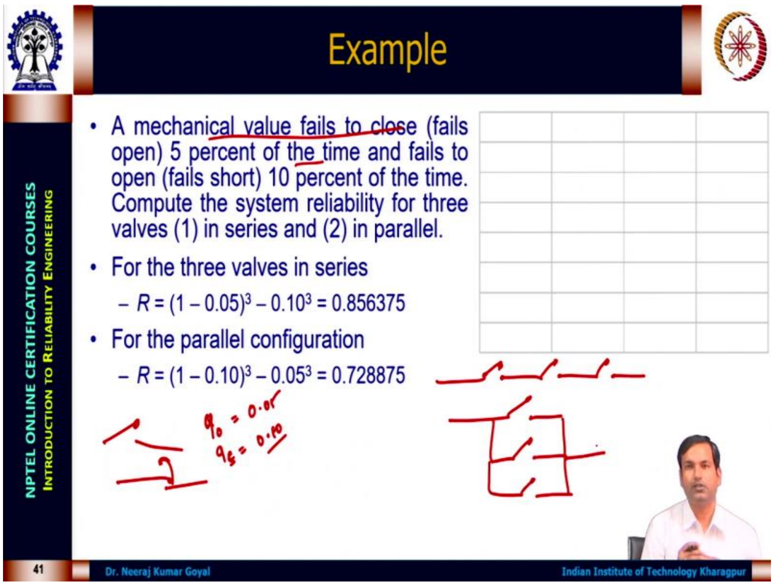

Let us take one example that let us say there is a mechanical valve which fails to close; fails to close means fails open, that it remains open, I am not able to close it 5 percent of the time. That means, failure probability of valve is in open mode is equal to 0.05 and fails open and fails close or fails short, that means, this is closed and I am not able to open it, that is 10 percent, point 10. Now, I want to know the reliability for 3 valves. So, I have 3 valves. First case is, I put them in series, another case is I put them in parallel. What will be my system reliability in both the cases?

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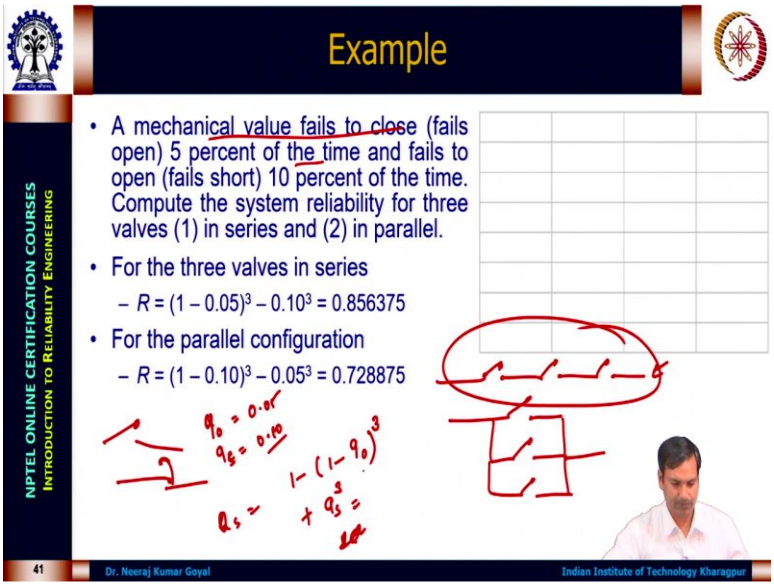

Now, this reliability as we see, we have already developed the formula, we can use the same formula here. When we use the same formula then my reliability will come to this value. So, as we have discussed or we can do this here again. What will be my failure probability in let us first consider the series this. So, for series my system will fail in two cases. System will be open if any one of them in open. Any one of them in open means 1 minus 1 minus we have the 3 components and Qo, any one of them is in open raised to the power 3.

And what is the failure probability in short mode? Short mode means all 3 has to fail in short mode, then only my system will fail in short mode. That means, Q short raise to the power 3. How much is this? And if I take reliability, this is my Qs. So, Rs will be equal to 1 minus of Qs. Now, let us calculate this.

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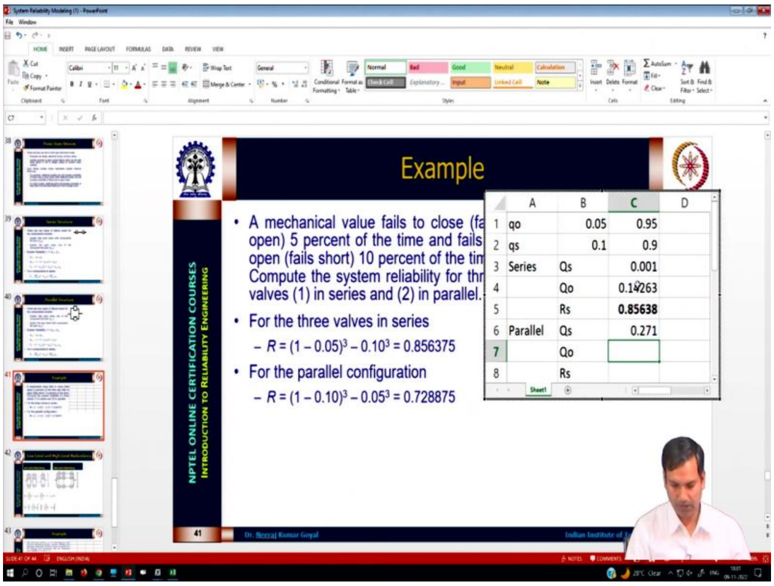

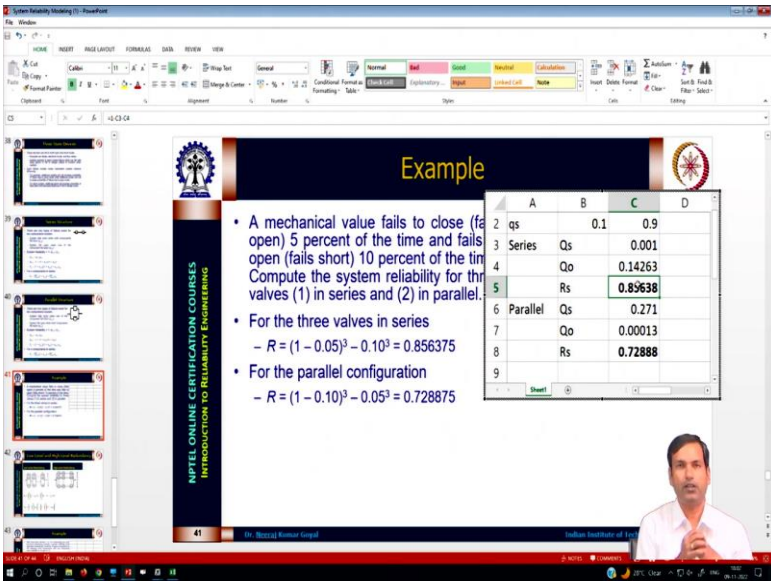

Let us try to do this in my Qo, Qo is 0.05 and Qs is equal to 0.1 for series system. For series when I am discussing or let us say I am interested in two thing, Qs in short mode. So, when in series it will fail in short mode, then only when all components are failing in short mode. That means, Qs raised to the power 3, Qo means any one of them fails in short mode. So, that is equal to, I can take 1 minus of this here itself, this will help me to solve problem much easily. So, that is equal to 1 minus each one right fails in short mode raised to the power 3. So, how much will be my reliability Rs? Rs will be equal to 1 minus this minus this. This comes out to be my reliability in series.

Similarly, let us say parallel. For parallel again if we do same way, in parallel system as we discussed earlier, for parallel system because elements are in parallel, any one shorts, the system will short. So, Qs means anyone. That means, 1 minus 1 minus Qs raised to the power 3. Open mode failure means all has to open because it is in parallel. So, system will become open only when everyone is open. So, that means, open mode failure probability raise to the power 3.

System reliability will be equal to 1 minus neither it fails in short mode, nor it fails in open mode. As you can see, we are able to calculate the reliability for series configuration as well as the parallel configuration.

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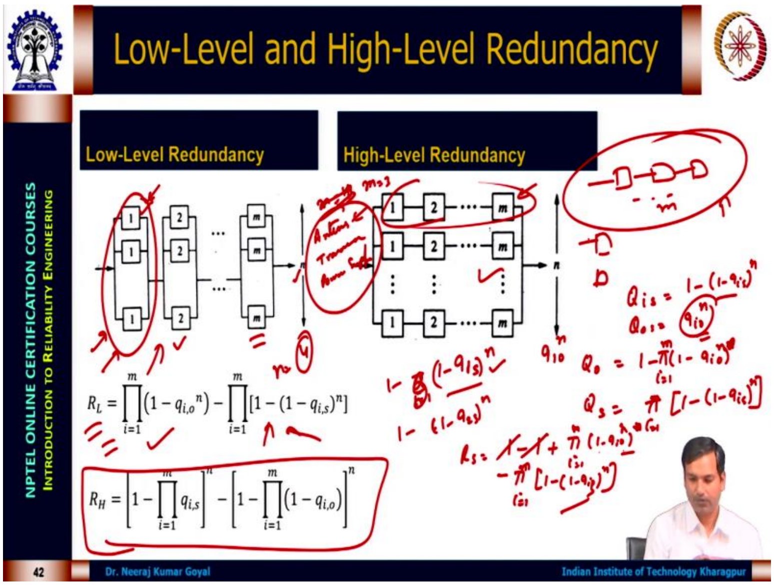

Similar concept is applicable when we discuss with the low level redundancy and high level redundancy. For low level redundancy means, see, if the system requires M element like M can be let us say if M is 4 where we were discussed earlier that we need let us say antenna, we need a transceiver, we may need a power supply right. Let us say M equal to 3. So, what can happen? In low level redundancies, let us say N N elements.

So, we have N equal to 3. We need these three systems to work. For one system if we have one set of these three, my system will work. Now, let us say for these three I have let us say different-different, I have 4-4 elements of this, N equal to 4. I have 4-4 of if each. Then, I can arrange in two ways, I can have the low level redundancy, that means, all antennas are in parallel. Then all transceivers are in parallel, then all power supplies in parallel. For high level redundancy that means, one set of antenna transmitter transceiver and power supply is in parallel with similar four sets.

This is high level redundancy. For low level redundancy when we are evaluating, we evaluate the reliability in this case. So, same as we have evaluated earlier, the low level redundancy reliability can be evaluated. Now, this low level redundancy when we see, then these are in parallel. So, first let us look at the parallel of this N element which are same, element all antennas are same let us say. So, any one of them works, the system will work, but we have the three state system here.

So, if any one of them fails short, so, for QiS, if any one of them fail short, the system will fail short. So, how much will be the short failure probability? That will be 1 minus QiS or for one system I will be equal to 1, any one fails short, multiply I equal to 1 to the how many such devices are there N devices are there. So, I can simply put power N 1 minus S. So, the probability that this section will fail short is 1 minus 1 minus QiS raise to the power N Q1S.

Similarly, 1 minus 1 minus Q2S raise to the power N will be the failure probability of this in short mode. So, now, this has become series of parallel. So, each element I am able to get the two probabilities, this is my short failure probability for each. Similarly, I can get the open mode failure probability for this. As we know for this system to fail in open mode all systems need to fail in open mode. So, Q1O raise to the power N will give me the failure probability for this block in open mode.

So, now, I am having the M such blocks ok and for each block I know the failure probability for short mode. So, let us say QiS is equal to 1 minus 1 minus QiS raise to the power N and QoS is equal to QiO raise to the power N. Now, I want to know the failure probability of this whole system. So, for whole system there are M such blocks. So, which are different. So, they may have the different, i is different. So, here now this block will fail in open mode if any one of them fails in open mode.

That means, Qo will be equal to, each one can fail in open mode, failure probability is this. So, 1 minus 1 minus QiO raised to the power N, this multiply I equal to 1 to N. Similarly, for short mode failure, all has to fail in short mode, then only system will fail in short mode. So, for system level Qs will be equal to multiplication i equal to 1 to N of this short mode failure, what is the short mode failure? 1 minus 1 minus QiS raise to the power N. And how much will be reliability of system? Reliability of system will be 1 minus Qo.

So, 1 minus 1 plus pi I equal to 1 to M 1 minus QiO raise to the power N minus this pi I equal to 1 to N 1 to M 1 minus 1 minus QiO raise to the power N. 1 and 1 will get cancelled and if you see this is my same formula, multiplication of 1 minus QiO raise to the power N I equal to 1 to N, this was QiO raise to the power N. So, not this and another is 1 minus 1 minus QiO QiS raise to the power N and multiplied i equal to 1 to N, same formula these two terms comes here and this becomes our load.

Similar to this, we get the high level probability. Same thing happens, same way we evaluate and we will be… So, you should try and get this value that whether you are able to get this high level probability is the same way. We have to use the same principle here. First, let us consider each series. So, this series system will fail if any of the component fails in open mode then system will fail in open mode. If this series will fail in short mode if all the components 1 to M fails in short mode right. Then again this will become parallel of each and we will apply two step second step again. And this gives us this formula.

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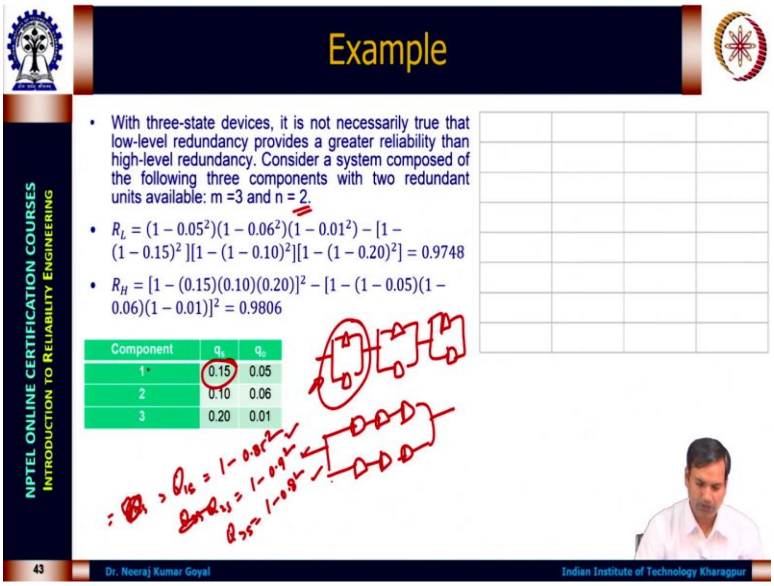

If we take an example here that if we have three elements here, 1 2 3 and N is equal to 2, that means, we have low level configuration and high level configuration. So, we have 2-2 elements here for low configuration and for high level configuration we have two parallel rows, 1 2 3, 1 2 3 and I want to calculate the reliability for this. Now, for this, we know that for short mode failure probability if I want to calculate, then short mode failure probability in this case, if any one of them fails in short mode, this will fail in short mode.

So, I can say Q1S will be equal to, either of the two fails in short mode that means, 1 minus 0.85 square. Then similarly Q2S will be Q2S, Q2S will be equal to this component 2 fails in short mode. That means, 1 minus 0.9 square and Q3S will be equal to 1 minus 0.8 square and my system will fails in short mode when all fails in short mode. So, let us do this in excel quickly.

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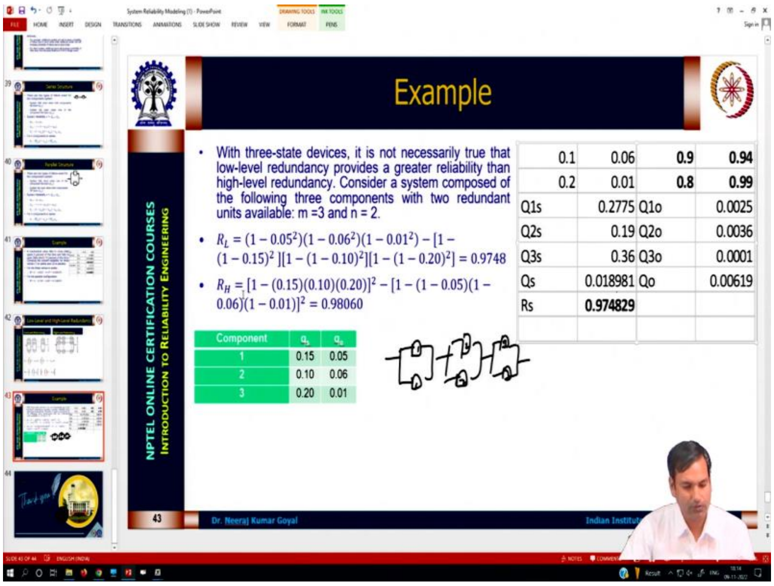

I will use the pen here. So, as we discussed here I will solve for series again and you can do it for, for low we will do and high you try to do. So, 1, component 2 and component 3. 1 1 2 2 3 3. Now, first let us evaluate the short mode failure probability. So, for short mode failure probability I will write down this value, 0.15, 0.05 or 0.1, 0.06, 0.2, 0.01. I will take 1 minus of this also and same value I will calculate here. So, because these values I will need, so, that is why I have taken it here.

$$R_L = \left(1-0.05^2\right)\left(1-0.06^2\right)\left(1-0.01^2\right) - [1-(1-0.15)^2\Big]\Big[1-(1-0.10)^2\Big]\Big[1-(1-0.20)^2\Big] = 0.9748$$

$$R_H = [1-(0.15)(0.10)(0.20)]^2 - [1-(1-0.05)(1-0.06)(1-0.01)]^2 = 0.98060$$

Now, I want to know the individual block like Q1 short, Q1 short means any one of them is short. That means, this is equal to 1 minus short failure probability is this, short success probability. Then similarly, I will get the Q2s. Q2s is equal to 1 minus short mode failure square. Q3s is equal to 1 minus, third component does not fail in short mode, square. Now, how much will be Qs here? Qs will be equal to if all of them fails in short mode, then only component will be failing in short mode.

That will be equal to this multiplied by this multiplied by this. So, this becomes my short mode failure probability. Similarly, Q1O come… this what we have here? This fails in open mode. Why it will fail in open mode? If both of them fails in open mode, then only system will fail in open mode. This first component system will fail. That means, my open mode failure probability is square. Q2O, I will just simply take this and Q3O. But when I take Qo when this fail in open mode?

This will fail in open mode if any one of them fails in open mode, either component 1 set, component set of component 2, set of component 3. That means, this is equal to 1 minus 1 minus of this multiplied by 1 minus of this multiplied by 1 minus of this and how much will be my reliability? Reliability will be equal to 1 minus of Qs minus of Qo, 0.9748. Same way you try to solve for high configuration.

So, we will stop our discussion here and this as we see, we have been able to take various configuration into the picture and we were able to solve them using the basic probability loss and using those loss, we were able to get the system reliability when we know the component of subsystem reliabilities.

So, with this we will stop for system reliability modeling. Next time, we will be discussing about the state-based system. When we consider that system is changing from one state to another state in that case how we can evaluate the system reliability using the Markov models. So, thank you. Thank you very much.