Introduction to Reliability Engineering
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Lecture 15
System Reliability Modelling

Hello everyone. Now, we are moving to lecture number 15 this is on system reliability modeling. In earlier lectures we discussed how component reliabilities can be represented by the various distributions like exponential distribution, Weibull distribution, logarithmic distribution, log normal distribution, normal distributions etcetera. We also discussed various terms and definitions used in the reliability theory and how they are related to these distributions.

If you know the distributions, how we are able to know the reliability parameters or various concerns which we wanted to know the probability values. Today we will be discussing about system reliability modeling, generally what happens that systems are generally few and used in few numbers only. So, in that case and many times when we are designing the system, we do not have the past history of the system.

So, in that case we try to use the historical data from the component reliability values, this component reliability values if we know then we can evaluate the system reliability. So, at the time of design stage or further when we are launching the product, we are able to know how much is the system going to be reliable.

Similarly, when we see the deficiency in reliability for various products, we can use various approaches like redundancies so that we are able to achieve better reliability even though component reliability is not so good or subsystem reliability are not so good or system reliability are not good, not so good. So, today we will be discussing that how various components or systems when they act together in a certain manner and contribute to the reliability how they can be model to evaluate the reliability of the system.

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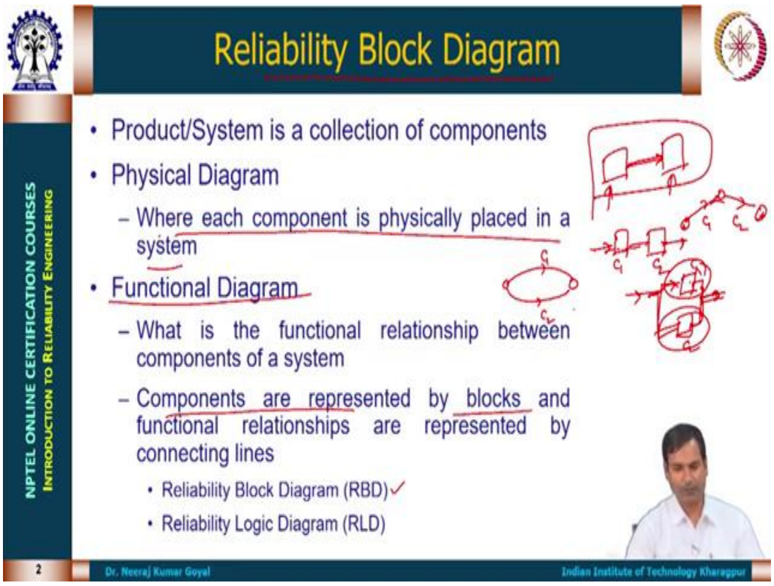

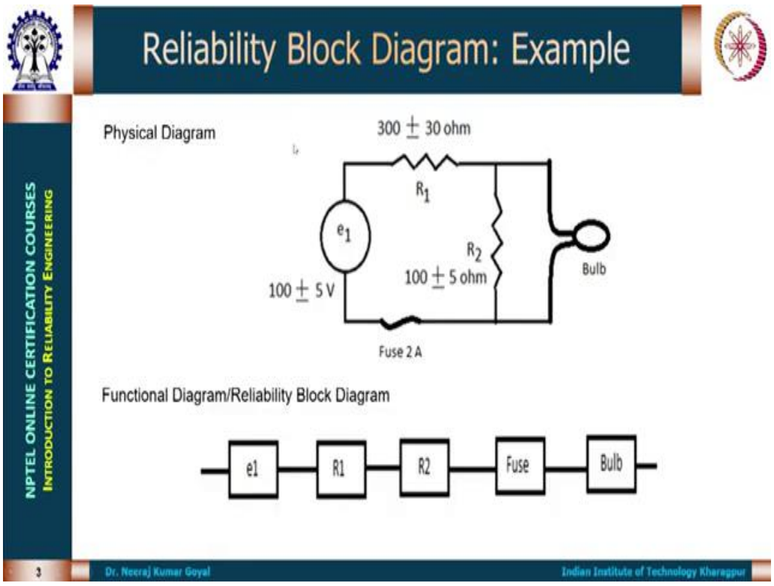

For this purpose, we will be using reliability block diagram in case reliability block diagram. So, the reliability block diagram we use for representing the system like we generally use flow diagram functional flow diagram we use of other kinds of system the representation the system reliability can be represented by the relative block diagram. So, as the name suggests it is a blocks of reliability. So, like we will be using these kinds of blocks where each block will represent a component or subsystem reliability values.

Here generally physical diagrams are we already know like layouts and system layouts like piping layouts, other layouts, process layouts but here we are using a logic this is more of a logic diagram which we use in the reliability block diagram we also have functional block diagrams for the system where we try to tell or explain the how the system is functioning with respect to various components how they are contributing to the system functions.

So, in case of reliability we try to represent the component by the blocks like this is one component block this can be another component block and functional relationships are represented by the connecting lines. So, these connecting lines which we are using here these connecting lines represent whether systems are required to function together or whether they are compensating each other.

Generally, when we use a system in series like this, in that case it means that we need to have both the components let us say C1, C2 both the components need to work for the system to work. As we can see we can make out that for the system to work it should start and it should end here. So, to have a start and ending here all those functional blocks should work if any functional block is not working then our system function will not complete.

Similarly, we use the parallel configurations here these parallel configurations represent that my system can function in two ways either component one is working then also I will be able to start from here and I will be able to reach to my target. Similarly, if component two is working then also I will be able to achieve my functionality. So, when we have the redundancy then we have the parallel combinations with redundancy means it is extra.

I could do with C1 only my work was complete. I could do the function C1 only but C2 I have kept extra for reliability purpose. So, that when C1 is not working C2 can continue to work and my reliability or my system continues to function. So, my system does not fail though components C1 has failed. Similarly, if C2 fails then also C1 can keep some working. So, here one component failure it is able to tolerate and even in case of one failures the system continues to work.

Whenever we use blocks we use we call it reliability block diagram generally line diagram if you use like same thing I can represent through like this. So, C1, C2 and this is the starting node and this is the terminal or ending node. So, for my system to be successful these both C1, C2 need to work. Similarly, for parallel I can represent like this, like this. So, here the by default I do not have to make the blocks if I am using the presentation here over the line edges over the edges, I can get the same so both can be solved in the same way both means almost same thing. Just it is a little bit different in representation only.

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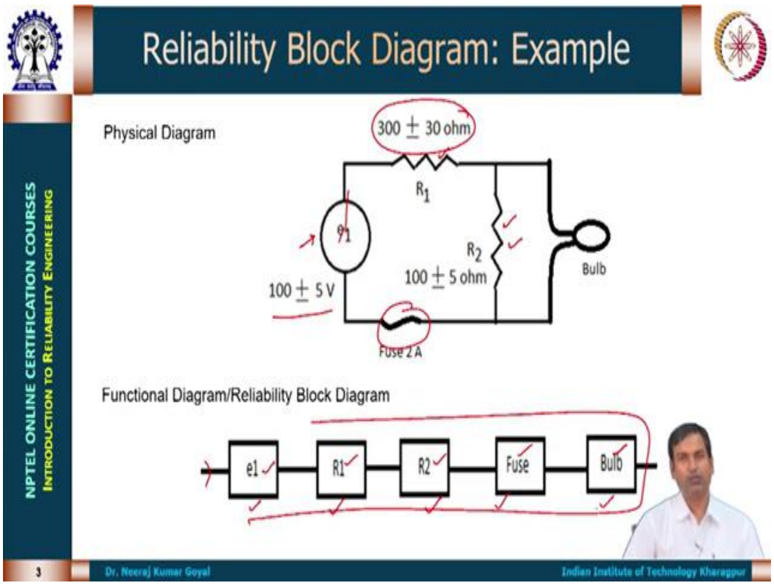

Let us say we want to understand what is the reliability block diagram let us see if we have a simple voltage divider circuit like this where we have a supply voltage we have two resistances, one is this resistance another is this, R1 is here R2 is here and we also have a fuse we have the bulb connected across the R2. So, whatever voltage is here this will divide and whatever voltage comes as output that will be used for the bulb.

Now, here if you look at the reliability block diagram for this system to work properly I need all the elements of the system to work if let us say my supply fails then my system will not work. So, supply has to work for my system to work then this resistance should not fail resistance should be working within the specification 300 plus minus 30 ohm then only my system work if it is violated my system will not work properly.

Similarly, then my R2 resistance has to also work if this does not work then also my system will not work the voltage division will not take place. The fuse also need to work fuse should also not blow up unnecessarily or when it is required it blows up then also there is a problem. So, here the fuse reliability it also comes into the picture.

Similarly, the if the bulb itself is not working then also the circuit will fail. So, in this case for this circuit to work all my system components supply, R1, R2, fuse, bulb need to work. So, everything I need to work this comes into the series, so it is a series system where all these components need to work. So, for my system to be reliable this has to be reliable, this has to be reliable this, this, this everyone has to be reliable. So, then only I will get the system reliability.

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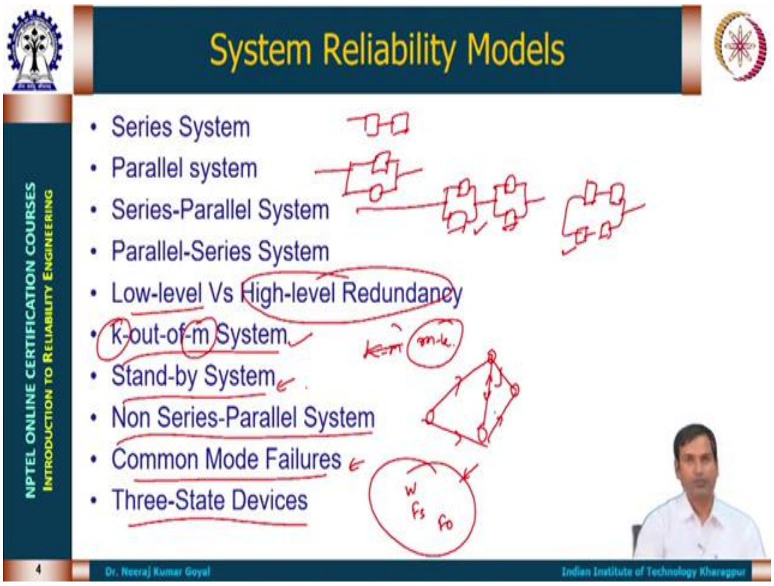

Now, let us see how do we evaluate such systems general combinations which you will be discussing is the series system, series system will look like this where components are in series that means all components need to work for the system to work. Parallel system is whatever components or series of components are in parallel any one of them works the system will work. Series parallel system we may have, series combinations of parallel or parallel combinations of series like this or we may have like this also, have we may have other.

So, this is series of parallel and this is parallel of series, parallel series, series parallel we have low level versus high level redundancy where low level redundancy means that we are applying the redundancy or we are giving more components at the lower level that means at the component level and high level redundancy means we are giving redundancy at the higher level that means we are using duplicate either system.

So, high level redundancy means at the system level and low level redundancy means at the component level. k out of m system is that for a system which is consists of m sub systems a component out of them if k systems are working, then system will function that means it can tolerate k minus m number, m minus k number of failures.

But if failures are more than this then system will fail. Standby system like our generators or UPS etcetera these are the system when they come into operation they remain standby they are not used in general, but when the primary function fails, then these bring are brought into the system and then they start functioning. Non-series parallel systems these are mostly used in various networks which cannot be directly series of parallel where multiple ways of connections are possible like if we have a connection like this.

So, in that case it is neither series neither parallel because of this element. So, this kind of systems need to be analyzed little differently than the series parallel system. Common mode failures are there. Common mode failures represent that there are some failures which can happen because of common reasons. So, those systems are in redundant fashion but because of the common reason all the systems can fail together.

Three state devices generally like we discussed resistance, so resistance can have three states one is that it is working, another is it is failed in short mode, another can be it is failed in open mode. So, this is having three state there can be more states but these three are the major or potential states for the various devices. So, various devices can fail in multiple ways. So, they may have the multiple states.

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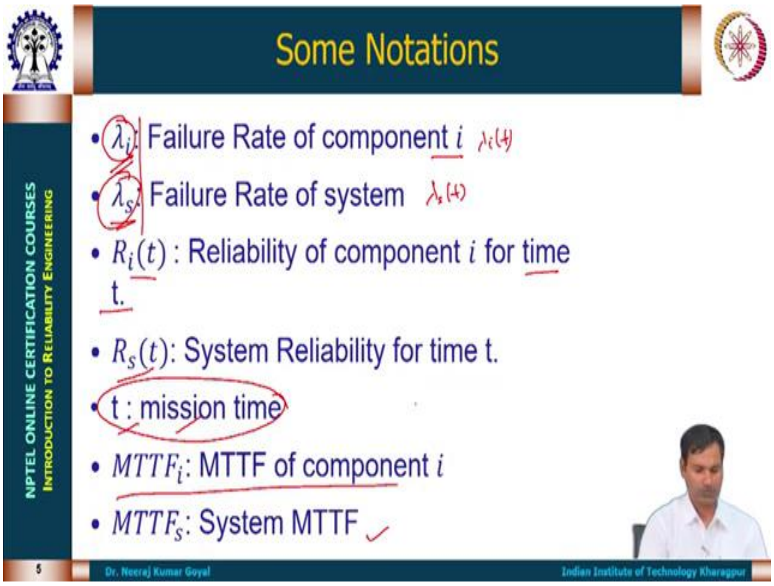

Let us see some notations which we will be using throughout, we will try to stick to this lambda i lambda i is the failure rate of component i. So, each component i will be having a failure rate. So, whenever we are using exponential distribution, we know it is having the constant failure rate. So, each component failure rate we will be representing by lambda i. So, if I say i equal to 1 this will become lambda 1 equal to 2 it will become lambda 2 like that.

Lambda s is the failure rate of the system. So, for overall system how much will be the system failure rate that is given as a lambda s. Generally, whenever we use these terms in terms of exponential distribution these are supposed to be irrespective of time, but many times they may be function. So, we may use lambdas s t we may use lambda i t as per the applicability.

Reliability is to be calculated for time t this definition etcetera we have already discussed R s t is for the system, t is the mission time. So, t is the time for which we want that system should be working and should not fail. MTTF i is the MTTF of component i, MTTF s is the MTTF of system.

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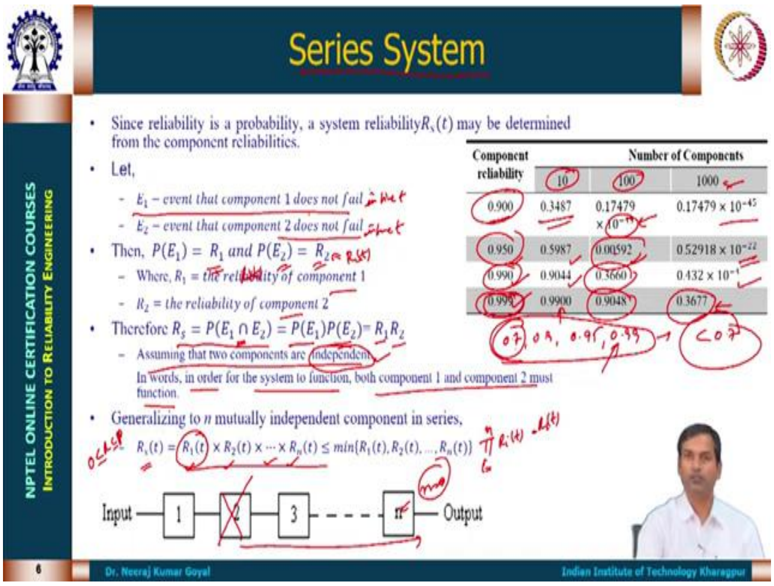

All the terms we have already discussed and we have also seen how these can be evaluated for the component level. We will now see that if we know the component level values how do we get the system level values. So, first system which is the very common way that is the series system. In series system what happens that whatever we are using everything is important, anything fails the system will fail if let us say component 2 is failed my system will not work the I will not get the required output.

Any component fails my system will not work. So, every component has to work for series system. So, let us say if we say that we are concerned about two events, event E1 and event E2. Now, what happens that event E1 means that component 1 does not fail, event E2 means component 2 does not fail. So, probability of event 1 will be reliability R1, probability of event 2 means reliability R2.

So, R1 is the probability that component 1 does not fail R2 is the probability that component 2 does not fail. Generally, it is a function of time. So, we can say if I say in time t in time t, then I can also write it as R1 t and R2 t. So, R1 is the reliability of component 1, R2 is the relative component 2. So, I want to know the reliability of system. What is the system reliability? System will only be reliable when component 1 does not fail and component 2 does not fail, since it is and relationship we use the intersection here.

So, I am interested in that what is the probability that neither that both event occurs that means, event E1 also is true event E2 is also true because in that case only my system will be reliable. So, systems reliability probability that E1 intersection E2, E1 and E2. That is if as we discussed earlier if the events are independent then this probability is equal to probability of E1 into probability E2 and that is equal to R1 into R2.

$$R_s = P\left(E_1 \cap E_2\right) = P\left(E_1\right) P\left(E_2\right) = R_1 R_2$$

This is only applicable when components are independent in general during the reliability evaluation in it is rarely assume that events are dependent almost all the time we assume the independency among the events, if there is a visible or there is identifiable dependency then only we use them as dependent events. So, another way if we see the series system is for that for system to function both component has to function if you have n component then I can make this like this.

So, all n components has to work that means reliability of system is multiplication of reliability of the n components because all has to work so E1 E2 like that En. So, R1 into R2 into R3 like that we will have the reliability system has to be multiplication of individual reliability. So, I can also write it as multi pi of i equal to 1 to n R i if t is involved, I can write R s t.

$$R_s(t) = R_1(t) \times R_2(t) \times \cdots \times R_n(t) \le \min\left\{R_1(t), R_2(t), \ldots, R_n(t)\right\}$$

Now, this reliability as we know reliability are the probability values so the reliability are always somewhere between 0 to 1. Now, if you see if I multiply reliabilities then the reliability will always become smaller reliability cannot become higher because I am going to multiply with a value less than 1. So, anything which gets multiplied with a value between 0 to 1 it is always going to be reduced value.

So, this value if we assume then whatever is the minimum of this reliability my system reliability is going to be lesser than that because this will be further reduced by multiplication by the other terms. So, let us say if I am having reliability is like 0.7, 0.9, 0.95, 0.99. So, here, my system reliability is going to be less than 0.7 this because I if I multiply this time, this is 0.7 into 0.9 into 0.95 into 0.99 that is definitely going to be less than 0.7.

Because of that, we can say that for a series system whatever is the minimum reliability component that is going to decide the system reliability because system reliability is going to be lesser than that. So, in us, in such a system, even if I am using some components, which are very highly reliable, but if even if one component is there which is less reliable my system reliability will fall down.

So, the weakest component will generally define the reliability of such systems. So, wherever we find weakest components we have to improve them so that we are able to improve the system reliability. Here if you see I have given one example here that if component will have this 0.9, in that case, if I am using 10 components in series n is equal to 10 then my reliability turns out to be 0.3487 that means though my component will have these 90 percent the reliability for system is only around 35 percent.

Here I have around 65 percent chances of failure while this was all component level I having only 10 percent chance of failure, if let us say I was using 100 components such components, then my reliability would be of the order of 10 to the power minus 45 so low that means I can assume that this is always going to fail almost 0. If I am going to use 1000 then minus 45. If reliability this 0.95 then these values turned out to be 0.9 8 5 9 8 7 0.00592 like even for 100 again the reliability values have become very small this is 0.5 percent that means if you practically see that this system is not going to work.

And this is almost 0 for 0.99 reliability that means each component has only 1 percent chance of failure. In that case, I am having around 90 percent reliability for 10 components, and I am having around 36 or 37 percent reliability for the 100 components. So this is just surviving it is sometimes survive sometimes not most of the time it is failed state. And this is failed only when I say 0.999 component to reliability than 10 such components.

When I am using in series my reliability turns out to be 0.99 that means my system is quite reliable for 10, even for 100 component the system is around 90 percent reliable but for 1000 then this will again fall to the around 36 percent. So, as we see here that in case of series system whenever we are using multiple components we have to make sure that our components are very, very reliable.

If any few components are unreliable that will bring down our system reliability to a very low level and the because any component fails, my system will fail. So, whatever is the reason my system is not working. So, in such cases we have to be careful and we have to make sure that our component reliabilities are very, very high.

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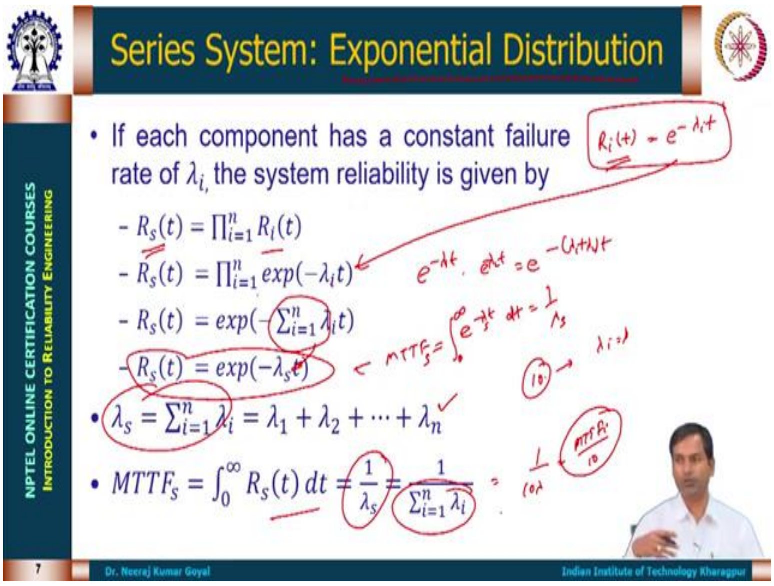

Now, let us discuss that how to calculate system reliability if component reliabilities are following the exponential distribution. So, we already know for exponential distribution R i t is equal to e to the power minus lambda i t this we have already discussed in exponential distribution then when we discussed so in that case, if I want to know system reliability we know multiplication of R i t gives us the system reliability for series system. So, that says multiply of i equal to 1 to n R i t.

$$R_S(t) = \prod_{i=1}^{n} R_i(t)$$

$$R_S(t) = \prod_{i=1}^{n} \exp(-\lambda_i t)$$

$$R_S(t) = \exp\left(-\sum_{i=1}^{n} \lambda_i t\right)$$

$$R_S(t) = \exp(-\lambda_s t)$$

Now, this system reliability if I replace this formula, then this becomes R s t is equal to pi of i equal to 1 this we have already validated this is a repetition of what we discussed earlier. So, R s t becomes e to the power minus because multiplication exponential of each exponential will lead to the, like e to the power minus lambda 1 t into e to the power minus lambda 2 t is nothing but e to the power minus lambda 1 plus lambda 2 t.

So, exponential of summation of lambda i t. Now, this exponential of summation of lambda we can call it as lambda s. So, R s t becomes e to the power minus lambda s t where lambda s is nothing but the summation of individual failure rates lambda i. So, this is how we can calculate the reliability and we know if reliability is this then MTTF for e to the power minus lambda t is 0 to infinity dt this turned out, turned out to be 1 upon lambda this we have already evaluated.

$$\lambda_s = \sum_{i=1}^{n} \lambda_i = \lambda_1 + \lambda_2 + \cdots + \lambda_n$$

$$MTTF_s = \int_0^\infty R_s(t)dt = \frac{1}{\lambda_s} = \frac{1}{\sum_{i=1}^{n} \lambda_i}$$

So, I am not going to repeat that this is the same. So, here I want to know the for system this will become 1 upon lambda s and what is lambda s? Lambda s the summation of individual lambda. So, in this case if we see, if we take the individual of failure rates lambda and if you sum it up we get the system failure rate. And if you want to know the MTTF of the system.

That is going to 1 upon summation of failure rate let us say if we take the previous example the 10 components are there in series then and all are same, all are having lambda equal to lambda in that case what will be the lambda, 1 upon 10 lambda or we can say this is equal to MTTF i divided by 10 as you see if I make it 100 this will become MTTF i divided by 100 MTTF i means the same in the component MTTF. So, as we see that more components as we use MTTF is reduced by the same proportion.

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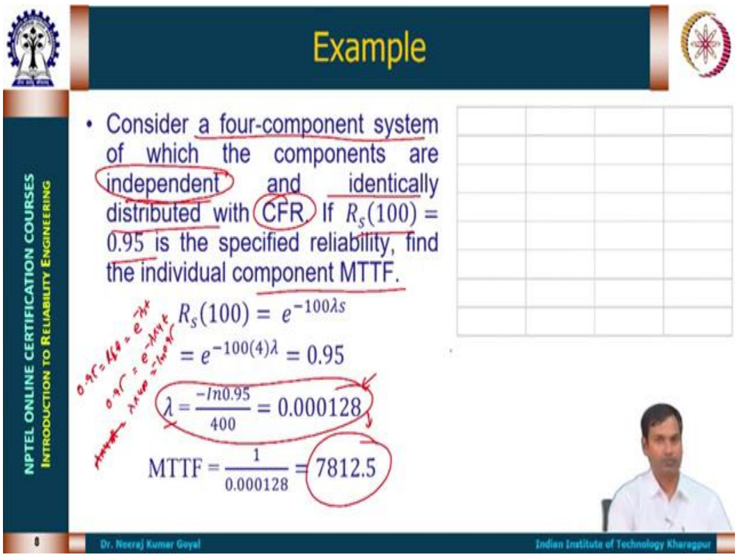

Let us take one example. Let us say we have a four-component system where components are independent and identically distributed. So, we are assuming they are independent. So, we can use our laws and identically distributed means everyone has the same distribution and that is also constant failure rate CFR. So, that means now the value given to me is that in that case my system reliability R s 100 is equal is 0.95.

So, I can write it that 0.95 is equal to R s t what is my R s t? That is e to the power minus lambda s t. And what is my lambda s that is e to power minus individual lambda into 4 because four components are there so that we can 4 lambda into t that is equal to 0.95 so I can reverse evaluate so lambda into 4 into t will be equal to t is also given to as 100. So, I can use lambda into 400 is equal to l n minus l n of 0.95.

$$R_S(100) = e^{-100\lambda s}$$
$$= e^{-100(4)\lambda} = 0.95$$
$$\lambda = \frac{-In0.95}{400} = 0.000128$$
$$\text{MTTF} = \frac{1}{0.000128} = 7812.5$$

So, lambda will become minus l n of 0.95 divided by 400 so this becomes my failure rate for each component. I want to know the MTTF for the component so MTTF is nothing but 1 upon lambda so MTTF is inverse of this which comes out to be 7812.5. So, as we see here we are able to reverse calculate the component reliability or component MTTF or component failure rate then we know the system reliability knowing that all components are identical.

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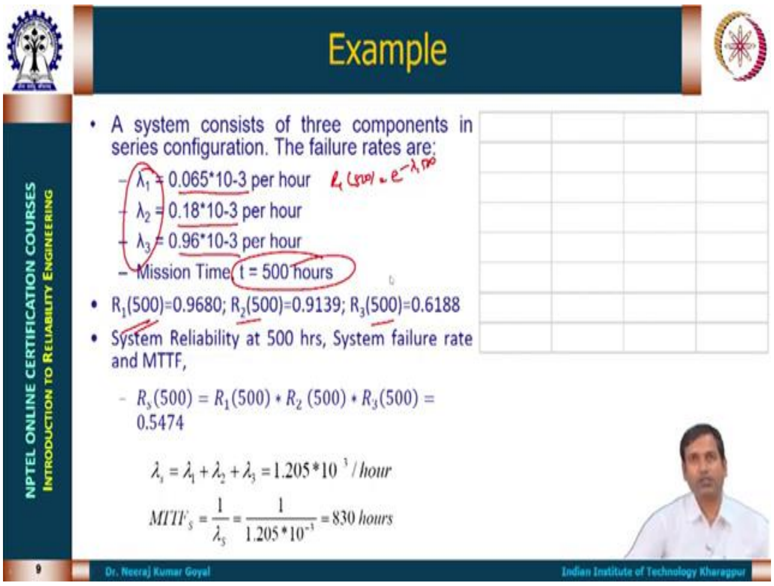

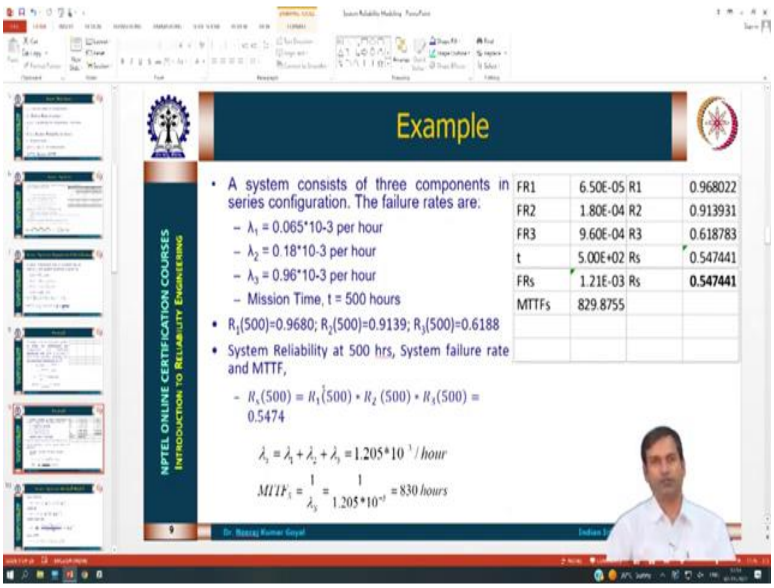

$$R_S(500) = R_1(500) * R_2(500) * R_3(500) = 0.5474$$

$$\lambda_s = \lambda_1 + \lambda_2 + \lambda_3 = 1.205 * 10^{-3} / \text{ hour}$$

$$\text{MTTF}_S = \frac{1}{\lambda} = \frac{1}{1.205 * 10^{-3}} = 830 \text{ hours}$$

Let us take one more example, let us see that we have four, we have three components here, lambda 1, lambda 2, lambda 3 their failure rates are given lambda 1, lambda 2, lambda 3 and our mission time is 500 hours. So, I want to know the reliability, to know the reality I can do various ways either I can calculate individual component reliability. So, I can get R1 500 that is e to the power minus lambda 1 500.

Similarly, I can get R2 500, R3 500 this I will show you in Excel sheet calculations so that you will be able to follow when you are solving how this we are solving. So, I will just put the lambda values here 0.0. So, lambda let us say I will call it as failure rate 1 that is 0.065 e minus 3 sorry 0.065 e minus 3 then my failure rate 2 is 0.18 e minus 3 then my failure rate 3 is 0.96 e minus 3 I have got my lambda 1, lambda 2, lambda 3.

From here I can get R1 R2 R3 if I want, R1 is equal to exponential minus lambda into, if I want I can write 500 directly or I will put t value here and I will use it. So, I will put time t equal to 500 here. So, my reliability 1 turns out to be 0.9680, my R2 turns out to be if I use the same formula, but the time has to be changed, same thing I can do for third as you see I can calculate R1 R2 R3.

Now I want to know what is my system reliability. So, system reliability as we know R s that is equal to multiplication of so product of all the above terms by multiplying the reliability you get the system reliability. So, my system reliability comes out to be 0.5474. If I want to know the failure rate for system as we know that system failure rate is summation of component failure rates. So, that is equal to sum of these failure rates, three failure rates.

So, as you know if I want I can also calculate reliability again, if I try to do this by the system failure rate. So, that is nothing but exponential of, exponential of minus t into this summation of failure rate as you see whether I use multiplication of reliability or whether I got it by direct formula both reliabilities are same because both represent the same thing, either you sum up the failure rate and then take exponential minus lambda t or you individually calculate reliability and then multiply both will give you the same answer.

MTTF, we want to know for system, MTTF of system as we know that is equal to 1 divided by lambda. We get this MTTF here because this is coming as the exponential scientific I want general cycle I can use the general formula here. So, as you see this turns out to be approximately 830 hours, same thing we have done here and you can see here so we will stop it here today and we will continue our discussion further in next lecture. Thank you