Introduction to Reliability Engineering
Professor Neeraj Kumar Goyal
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Indian Institute of Technology, Kharagpur
Lecture 12
Weibull Distribution

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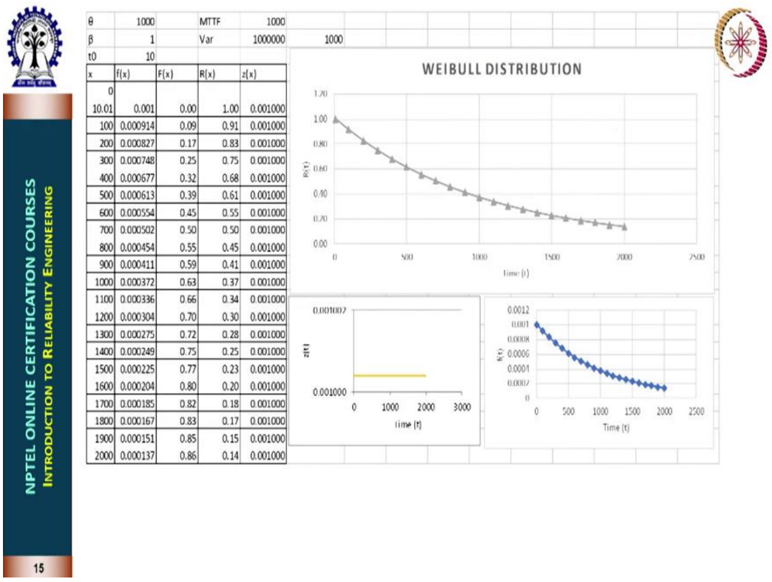
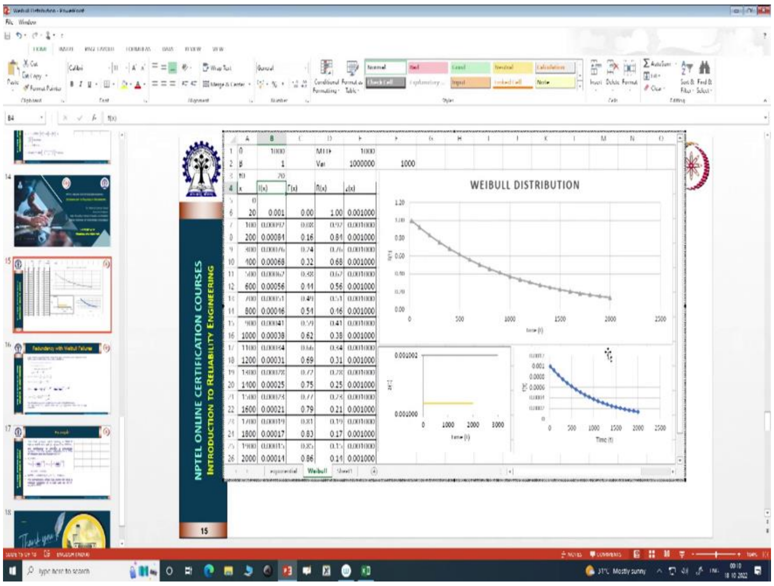


θ	1000		MTTF	1000
β	1		Var	1000000
t0	10			
x	f(x)	F(x)	R(x)	z(x)
0				
10.01	0.001	0.00	1.00	0.001000
100	0.000914	0.09	0.91	0.001000
200	0.000827	0.17	0.83	0.001000
300	0.000748	0.25	0.75	0.001000
400	0.000677	0.32	0.68	0.001000
500	0.000613	0.39	0.61	0.001000
600	0.000554	0.45	0.55	0.001000
700	0.000502	0.50	0.50	0.001000
800	0.000454	0.55	0.45	0.001000
900	0.000411	0.59	0.41	0.001000
1000	0.000372	0.63	0.37	0.001000
1100	0.000336	0.66	0.34	0.001000
1200	0.000304	0.70	0.30	0.001000
1300	0.000275	0.72	0.28	0.001000
1400	0.000249	0.75	0.25	0.001000
1500	0.000225	0.77	0.23	0.001000
1600	0.000204	0.80	0.20	0.001000
1700	0.000185	0.82	0.18	0.001000
1800	0.000167	0.83	0.17	0.001000
1900	0.000151	0.85	0.15	0.001000
2000	0.000137	0.86	0.14	0.001000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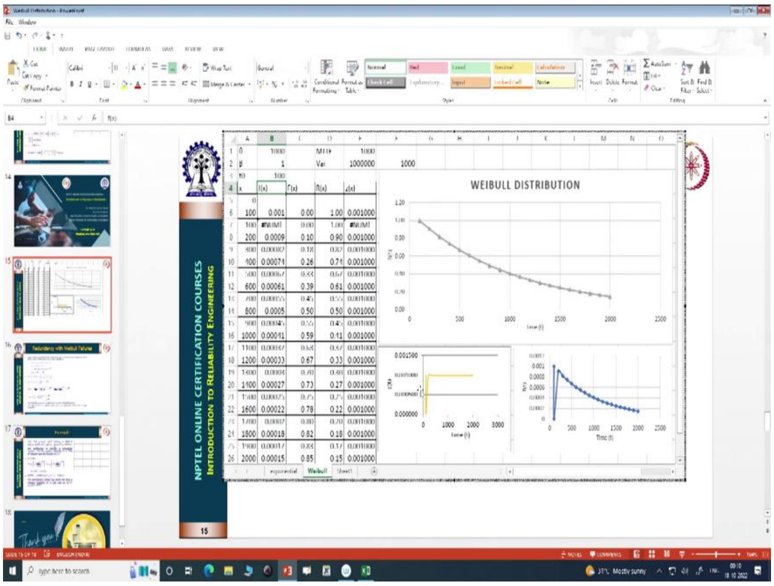
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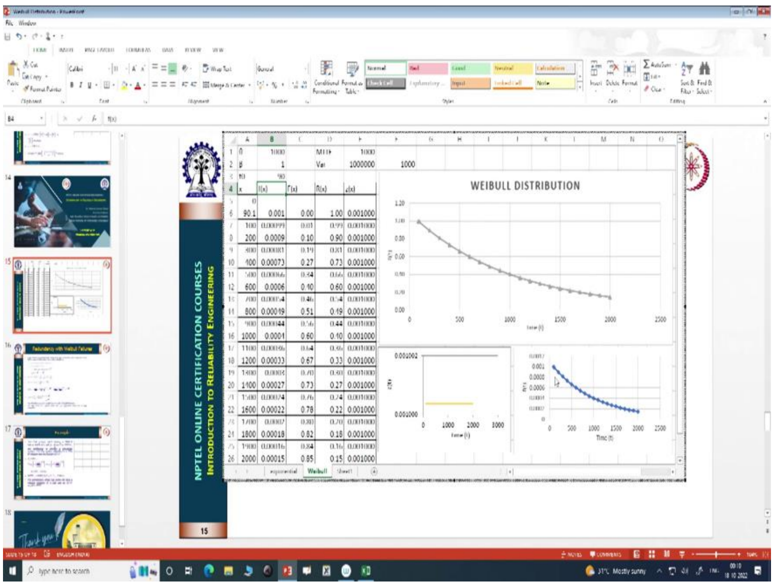
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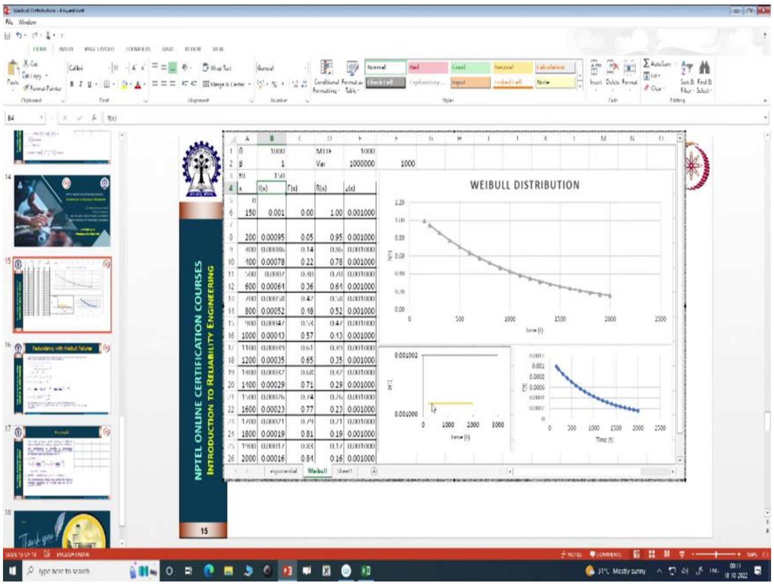
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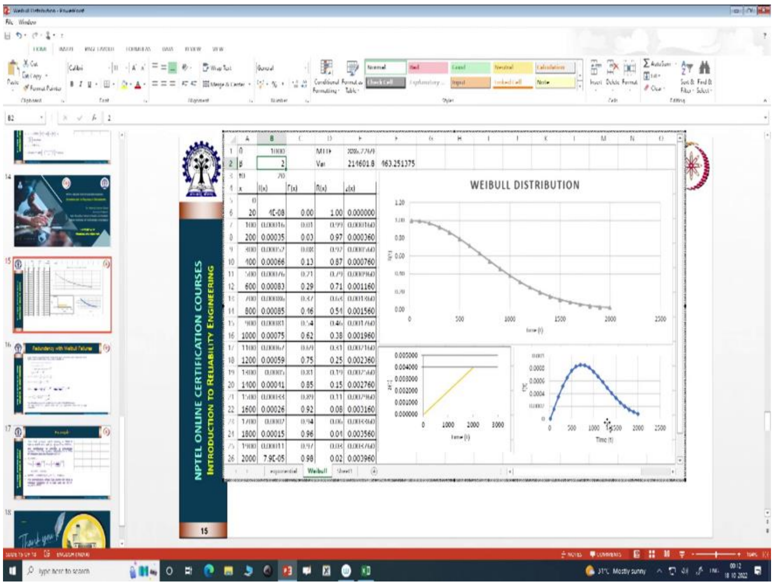
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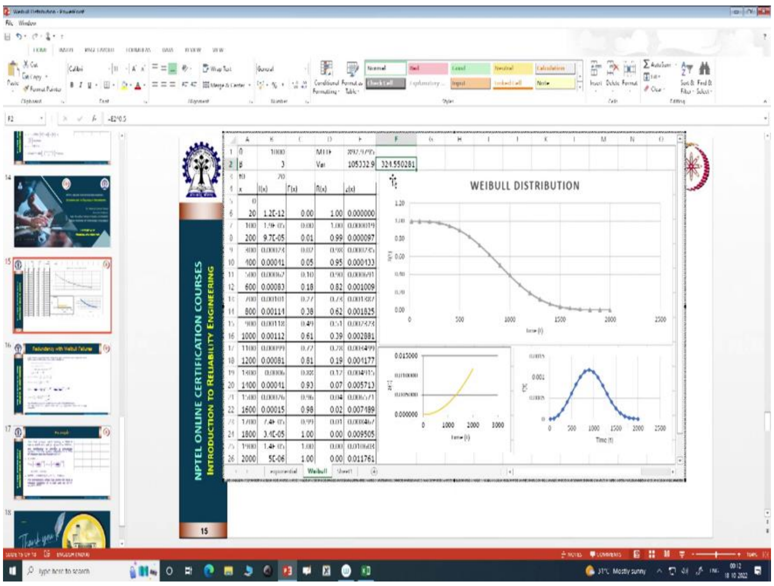
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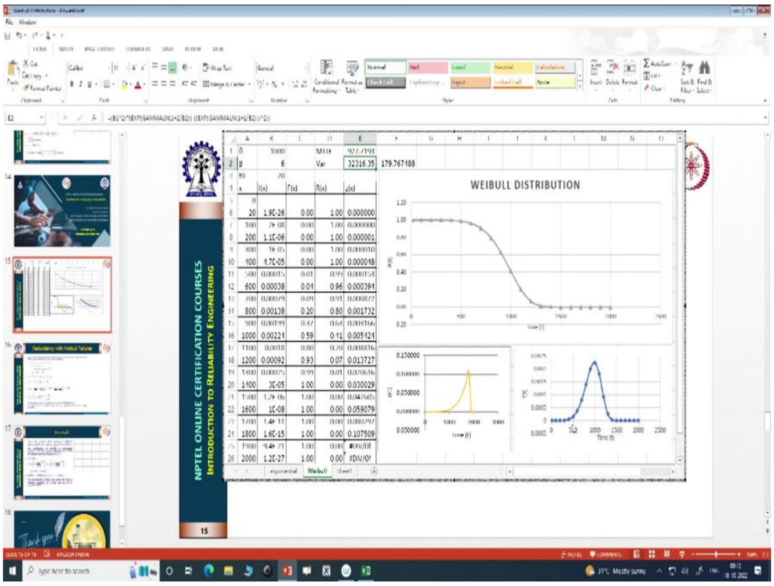
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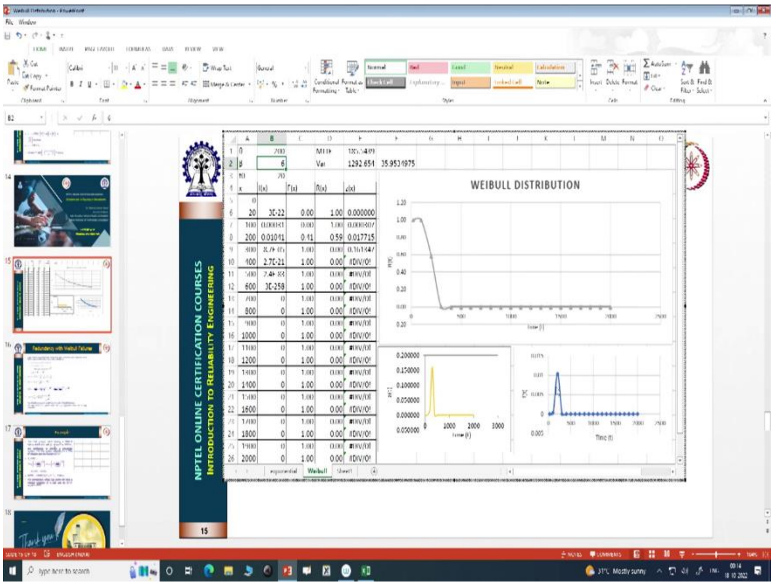
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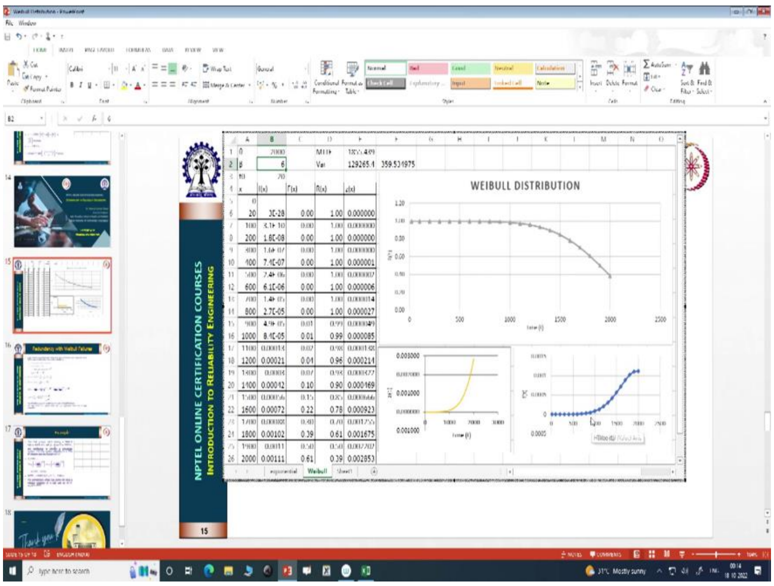
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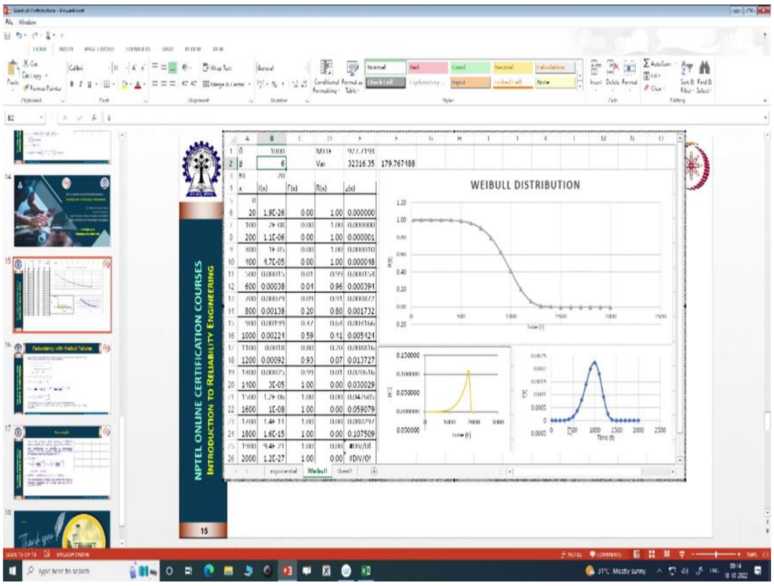

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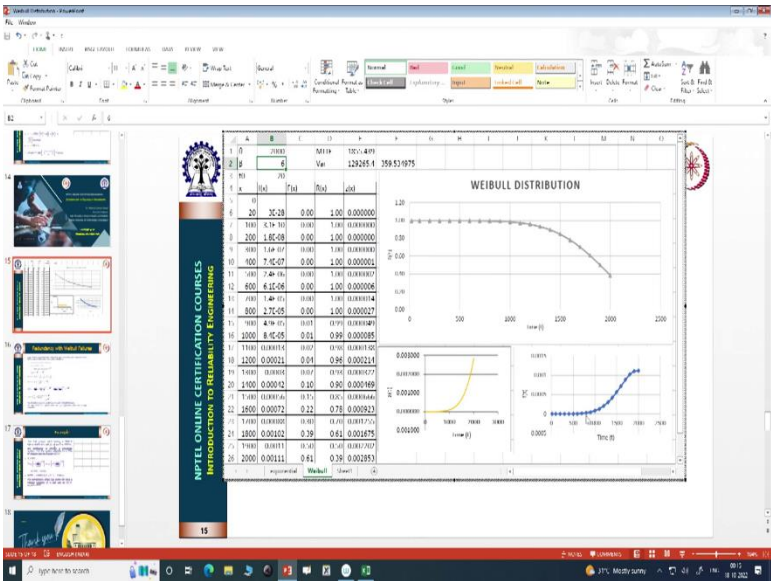

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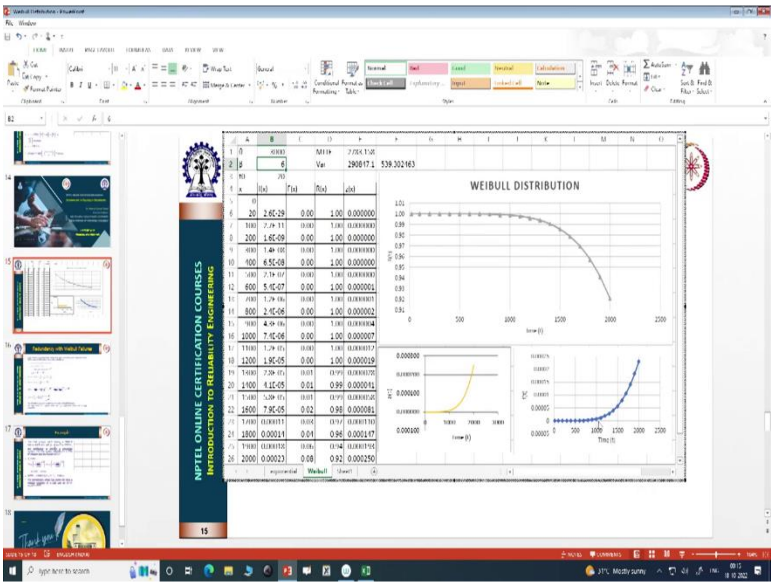

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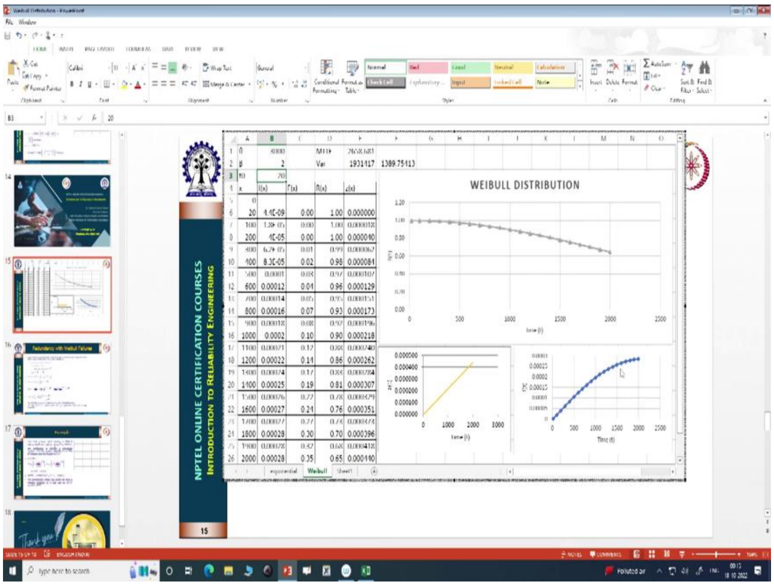
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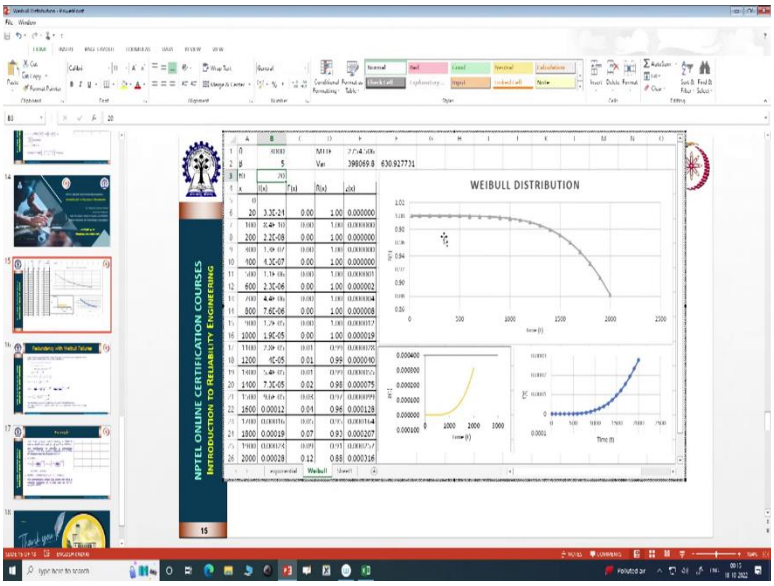
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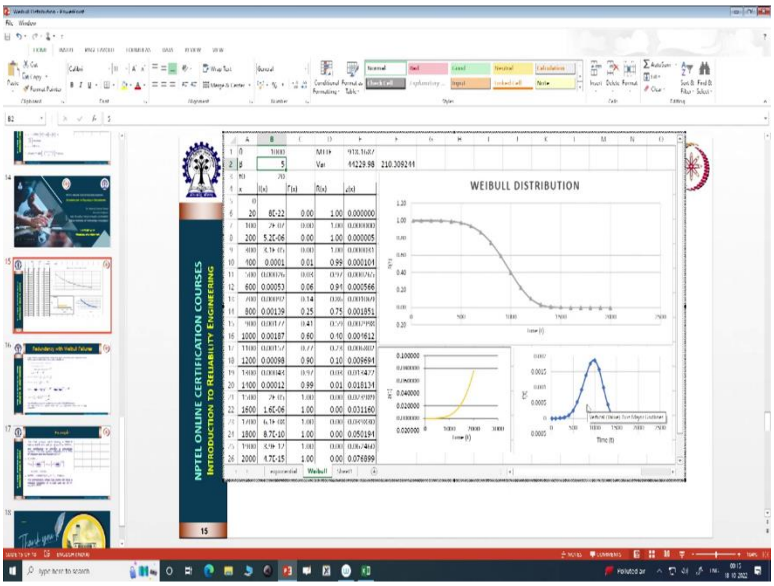
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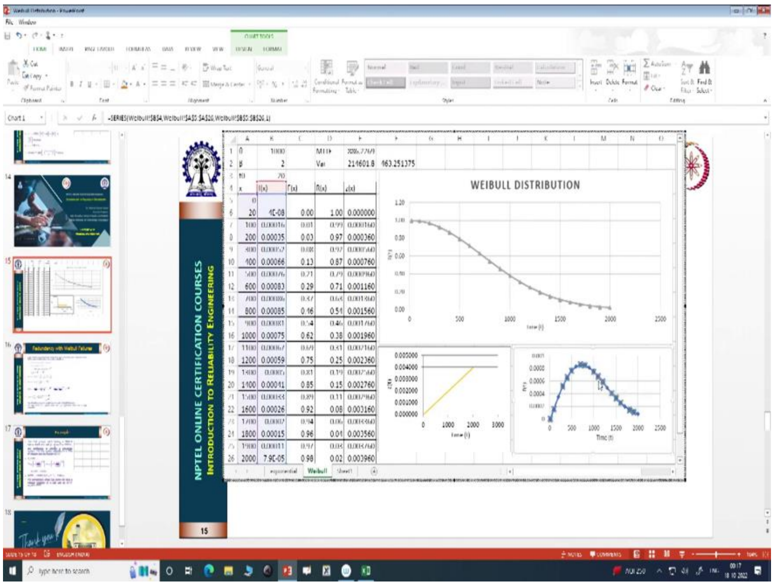

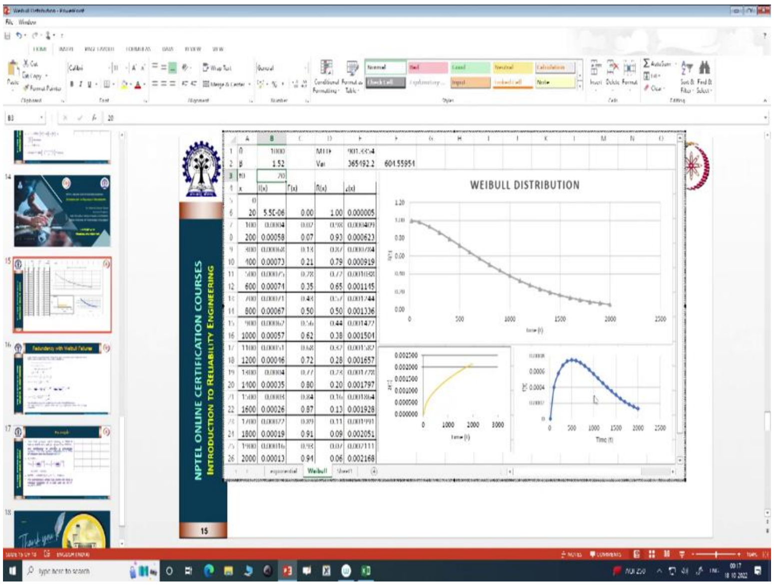

Hello everyone. So, we have been discussing about Weibull Distribution. So, we have now moved to lecture number 12. We will discuss more about three parameter Weibull distribution as well as one more application of Weibull distribution here. So, as we see here, let us see, we have discussed what is three-parameter distribution, I have built Excel sheet here there all those formulas I have already put up here what we discussed with the three parameter Weibull distribution as we see here, I have included the third parameter here that is let us say t0. So, if we take t0 equal to 10, what will the impact here.

So, I have started this calculation from t0, so, t0 plus certain a little bit higher value so, that you will be able to calculate because at t equal to same as 10, it will be undefined. Now, here if you see if I increase let us say if I change the location parameter then what will happen?

So, if I make it let us say 20, what will happen? The visible changes not there let us say I will make it 100, if I make it 100, you see that the values are changed or if I make it 90.

So, this simply shifted nothing much changes there the Weibull or the distribution is only shifted. So, let us see if I make it from 200 or I will say I will do not take this, let us make it 150, then what will happen it will be only shifting towards right nothing much changes happening, because up to this value the reliability is equal to 1 then now, let us say if we consider that t is 20 here only.

But let us say if we take the change in beta, if I make it beta change it is becoming more and more modal. If you see there is another thing which you can see here, let us say if we talk about the is this is small ft distribution small ft is the PDF. What does PDF tell? PDF is telling us where the failures are concentrated. So, if we have a full population, then from the population most of the failures are populated in this region while here still some equipment's continue to work for a longer period, most of the failures are happening within this period. Now, for this if I make let us say beta equal to 3, what happens here the this is becoming much sharper, that this peakedness is increasing.

So, failure concentration is also becoming in less area this is also visible, if you look at the standard deviation, standard deviation is 3 to 4, but when I took beta equal to 2 standard deviation was 463, when I made beta equal to 3 standard deviation has become 324, that means the standard deviation has reduced or the variability has reduced or the distribution has become peaked.

Now, let us see if I make this more let us say I make this 6 then my standard deviation is further decrease it is becoming only 180 and this very sharp, sharp distribution here means that most of my failures are going to be in small smaller region. So, for a company it is desirable, if you have the higher beta it will have the less variance and because of the less variance or because of the less standard division what is happening you are having larger failure free period if you see that up to almost 400 hours 500 hours the chances of failure are very small.

But when we move to this area, most of the failure occurs and nothing is surviving very long. So, very few units will survive very low. So, failure concentration is becoming you know, smaller zone. And that is that would be the desirable situation for the manufacturer because this is going to help him understand that where the failures are occurring, but this is only happening when there is a deterioration.

So once we know that deterioration kind of failures are there, we have much more certainty about it like if you see, like if we simply change theta here, what will happen, this will be this is supposed to stretch, but not much. If you see most of the failures are concentrated in here, sorry, this is to 2000.

Failures are starting to happen after a longer period of around 1000, like when we took 1000 here than most of the failures were happening somewhere around 600, failures were starting to happen, the earlier failure probability was very less, we are happening somewhere around 400 or 300 around, but when we made it 2000. Then we see that failure, the probability of failure was very small up to somewhere around 800, when we made this 3000 then probability of failure was small almost up to 1100 or 1000.

As we see here, as we are having high characteristic life, it is means that my failures are starting late to happen when beta is high, but when beta is small, then in that case, it does not make much difference, the number of the probability of failure is still existing here it is comparable here, but when beta is high, then chances of early failures would be less, most of the failures would be concentrated in a smaller zone here.

So, because of that, this gives us an understanding that the system, if we know more about the system, if we know the degradation mechanism, if we are able to design the system better, then we are expecting that beta should be larger. So that variability is lesser, variability about the failure time is lesser. So that will help us to decide our policies better, our surprises will be lesser and but the chance failures cannot be ignored, chance failures which are there due to the random failures, they may still happen. So that may further give rise to the ft value.

So those because of that, early failures may be more but if all, if those can be removed, then you may have a very less chance of failure till you are having a certain life period around missed in that case this empty tf is having much more meaningful 918. If we look at it 918. That is the central almost central where the failures are happening when beta is large, but when beta is small, then it is not having much impact because variability is high. So, at 1000 if you look then it is already past much after the PQ, if we take 1.5 if you see that further 1000 values coming in too far towards the, towards the peak, away from the peak.

So, here, if we look at the Weibull distribution, it is it can show you various factors, it can show you the various ways of the failures are happening and will help you to model various scenarios, which are happening in the field or with the manufacturing. And if you are able to understand this better, it will give you very much insight into your design process and the manufacturing processes.

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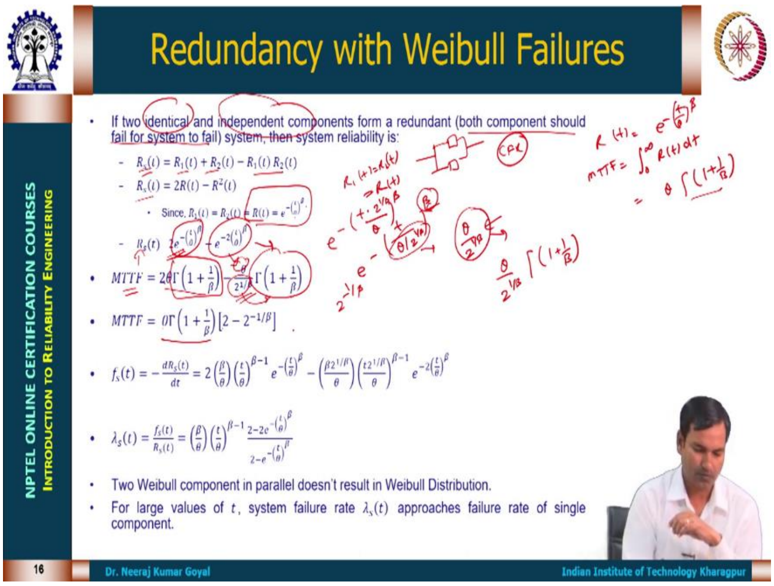

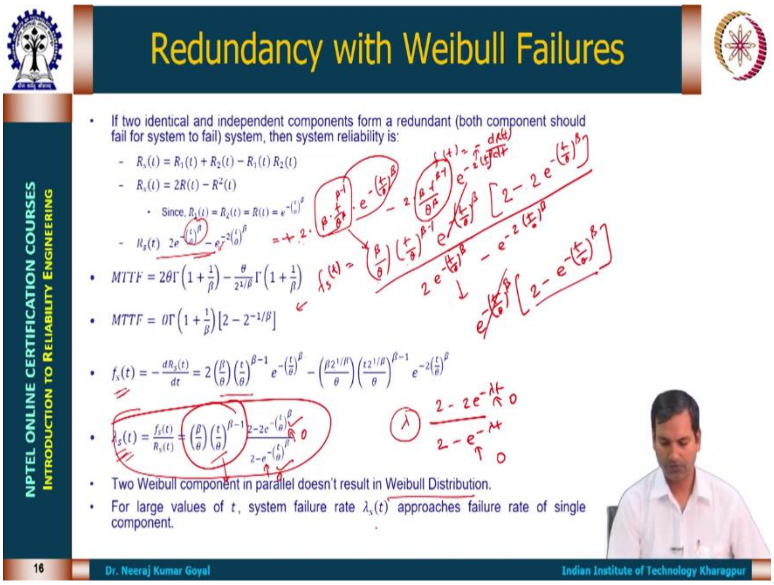

There is a like we discussed for exponential distribution, if you put two components in parallel or redundant fashion that means if one component fail, the system will still continue to work. So, for system failure, both components should fail. So, we have the parallel redundancy here, that means two out of the two components should fail here.

$$R_s(t) = R_1(t) + R_2(t) - R_1(t)R_2(t)$$
$$R_s(t) = 2R(t) - R^2(t)$$
$$\text{Since, } R_1(t) = R_2(t) = R(t) = e^{-\left(\frac{t}{\theta}\right)^\beta}$$
$$R_s(t) = 2e^{-\left(\frac{t}{\theta}\right)^\beta} - e^{-2\left(\frac{t}{\theta}\right)^\beta}$$
$$\text{MTTF } = 2\theta\Gamma\left(1 + \frac{1}{\beta}\right) - \frac{\theta}{2^{1/\beta}}\Gamma\left(1 + \frac{1}{\beta}\right)$$
$$MTTF = \theta\Gamma\left(1 + \frac{1}{\beta}\right)\left[2 - 2^{-1/\beta}\right]$$
$$f_s(t) = -\frac{dR_S(t)}{dt} = 2\left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta-1} e^{-\left(\frac{t}{\theta}\right)^\beta} - \left(\frac{\beta 2^{1/\beta}}{\theta}\right)\left(\frac{t2^{1/\beta}}{\theta}\right)^{\beta-1} e^{-2\left(\frac{t}{\theta}\right)^\beta}$$
$$\lambda_s(t) = \frac{f_s(t)}{R_s(t)} = \left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta-1} \frac{2 - 2e^{-\left(\frac{t}{\theta}\right)^\beta}}{2 - e^{-\left(\frac{t}{\theta}\right)^\beta}}$$

So, we have already seen this in the when we discuss the CFR model, constant failure rate model that when we put two into parallel then system reliability becomes R1 plus R2 minus R1, R2. If both are identical, we are assuming that both are identical and independent. So, since both are identical R1 t is equal to R2 t which should be equal to R, Rt. So this can be written as Rt plus Rt minus Rt square.

So this will become 2 Rt minus Rt square. Where what is Rt, Rt is equal to single component reliability which is e to the power minus t upon theta raise to the power beta. So, our system reliability in this case will become 2 into e to the power minus t upon theta raised to the power beta multiply by R square, can I take a square of this this will become multiply by power will be multiplied by 2. So, e to the power minus 2 into t upon theta raised to the power beta.

As we have seen this quantity I can write it is e to the power minus t upon into 2 raised to the power 1 upon beta divided by theta raised to the power beta or I can write it as e to the power minus t upon theta divided by 2 to the power 1 upon beta whole raised to the power beta, this is same as what we did earlier or in case of multiple failure modes that our theta is changed to theta divided by 2 raised to the power 1 upon beta.

So, this is a this becomes a standard form for Weibull distribution reliability formula. So, there beta is same and theta has become theta divided by 2 raised to the power 1 upon beta. Now, we know this is the standard that we know that when Rt is equal to e to the power minus t upon theta raised to the power beta, then my MTTF, MTTF which is equal to integration from 0 to infinity Rt dt, this comes out to be theta gamma function of 1 plus 1 upon beta.

So, same thing if we apply here, then 2 will remain same, and then we take integration of this from 0 to infinity this will give me gamma function of theta into gamma function of 1 plus 1 upon beta same thing, what we have got here, for this when we do the integration, this will be theta is this and beta is this.

So, when I apply the formula this will become theta upon 2 raised to the power 1 upon beta gamma function of 1 plus 1 upon beta. Same thing I have written here theta upon 1 2 raised to power 1 upon beta gamma function of 1 plus 1 upon beta. So, this becomes my new MTTF, MTTF of this system.

So, this MTTF of the system and what I can do I can take gamma function of 1 plus 1 upon beta into theta as common from here also from here also, then what is remaining, 2 is remaining from here and 1 upon 2 raised to the power 1 upon beta is remaining here. So, taking common theta gamma function of 1 plus 1 upon beta the remaining is 2 minus. Now, this I can take it up this will become 2 to the power minus 1 upon beta.

So, MTTF becomes theta gamma function of 1 plus 1 upon theta multiplied by 2 minus 2 raised to the power minus 1 upon beta. Now, if we want to calculate. So, MTTF calculation is clear now, let us say if you want to calculate lambda s or if you want to calculate fs. So, we know that f t is equal to minus dR t over dt. So, differentiating this function with respect to t will give me the fs t, if I differentiate this, then we know that 2 will remain as it is if we differentiate this then we have to differentiate this.

So, this differentiation will give beta into t upon theta raised to the power beta t raised to the power beta minus 1 into e to the power minus t upon theta raised to the power beta minus, if I differentiate this I will get so, minus sign was also there, but minus sign will be cancelled with this minus. So, that is why I am not putting minus, minus minus will become plus here.

So, minus of this which is coming down that will be compensated by this minus so, this will again remain, so, sign remains same here. So, differentiation of this will produce 2 into again beta t to the power beta minus 1 divided by theta raised to the power beta. Once you take this then if I take common that is beta I can write this as beta upon theta into t upon theta raised to the power beta minus 1.

If I take this as common this is this, then what is remaining and I can also take e to the power minus t upon theta raised to the power beta as common. Then what is remaining here is 2. So, this I have taken common and e to the power minus 2 into t upon theta raised to power beta is there. So, this is square of the t upon theta raised to the power beta. So, 2 is here, 2 is here also.

So, 2 e to the power minus t upon theta raised to the power beta will come from here, because this was 2. So, 1 has come out 1 will be remaining here, because it was square of this. So, this gives me the fs t. So, fs t becomes 2 into beta upon theta, I can take 2 also outside or I can take write in this fashion 2 upon 2 into beta upon theta t upon theta raised to the power beta minus 1 into e to the power minus t upon theta raised to the power beta and this is e to the power minus 2 upon 2 t upon theta raised to the power beta this is the remaining similar like 2 raised to the power 1 upon beta multiplied by 2 raised to 1 upon beta whole raised to the power beta minus 1 if we say then this will become 2 which is coming out.

So, this we have got, and this is my fs t to get the lambda t, lambda is equal to lambda t is equal to ft upon Rt. So, this is my ft already available here. Now, what I have to do I have to divide it by the Rt, what is my Rt here? Rt is 2 e to the power minus t upon theta raised to the power beta if I take minus e to the power minus 2 t upon theta raised to the power beta, if I take e to the power minus t upon theta as raised to the power beta as common than this will be equal to e to the power minus t upon theta raised to the power beta into 2 minus e to the power minus t upon theta raised to the power beta.

If you see it here, this will get cancelled with this and what is remaining is beta upon theta into t upon theta raised to the power beta minus 1 multiplied by 2 minus 2 e to the power minus t upon theta raised to the power beta divided by 2 minus e to the power minus t upon theta raised to the power beta.

So, this gives me this is very similar to what we have got in the constant failure rate. The only difference is beta was 1 there so, if beta we put 1 there then this was coming as the lambda, lambda into 2 minus 2 e to the power minus lambda t and this was 2 minus e to the 1 minus lambda t similar. So, this also have the similar function that when t is tending to infinity what will happen when t is tending to infinity then this value tend to be 0 this value tend to be 0 when time is large, this will become negligible This will reduce. So, this will become 2 upon 2 and the failure rate was lambda as we discussed earlier, when two components are in parallel, then as time becomes large the system failure rate reaches the single component failure rate lambda.

Same is applicable here when time is large what will happen this will tend to 0, this will also tend to 0, this will become 2 upon 2 that will be equal to 1, and what is remaining is beta upon theta into t upon theta raise to the power beta minus 1 which is the failure rate for one component as we have considered earlier.

So, when time becomes high, time tends to infinity then our failure rate for the system is tending to be the same as the failure rate of one component. But as we discussed earlier neither for exponential, neither for Weibull distribution, if we have components in parallel, then it will not result in a Weibull distribution.

So, if two components Weibull components are in parallel, we are not able to get the same as the Weibull distribution as you see here, this distribution cannot be represented in a standard form of the Weibull distribution. And as we discussed for larger value of t the system failure rate approaches the failure rate of single component.

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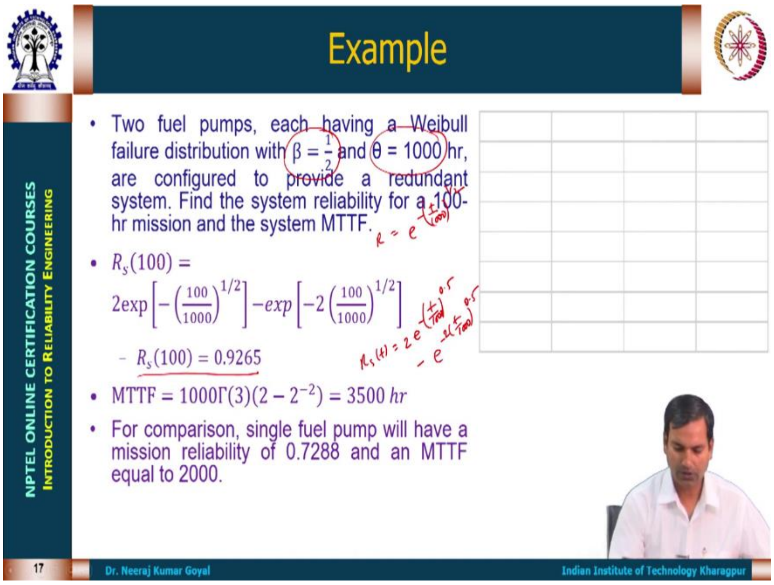

Let us take one example, that we have two fuel pumps and both are having failure distribution with beta equal to 1 by 2 and theta equal to 1000 hours. So, that means, my reliability for each component is e to the power minus t upon theta. So, that is t upon 1000 raised to the power beta 1 by 2.

Now, we want to know the relative 100-hour mission. So, we know that Rt is equal to Rs t will be equal to twice of e to the power minus t upon theta raised to the power beta 0.5 minus square of this that is minus twice of t upon theta raised to the power beta. Once we apply this formula here, we will get the reliability we want to know 100-hour reliability. So, we can calculate the 100-hour liability here.

$$R_S(100) = 2\exp\left[-\left(\frac{100}{1000}\right)^{1/2}\right] - \exp\left[-2\left(\frac{100}{1000}\right)^{1/2}\right]$$

$$R_S(100) = 0.9265$$

$$\text{MTTF} = 1000\Gamma(3)(2 - 2^{-2}) = 3500hr$$

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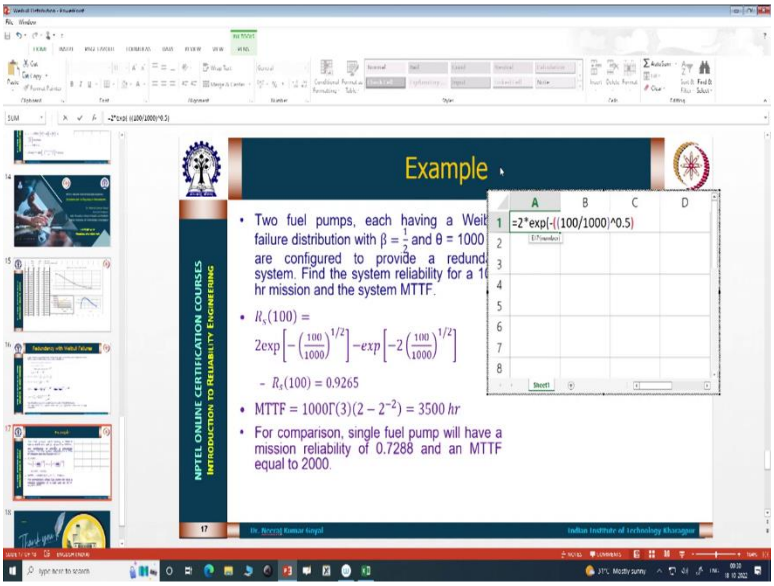

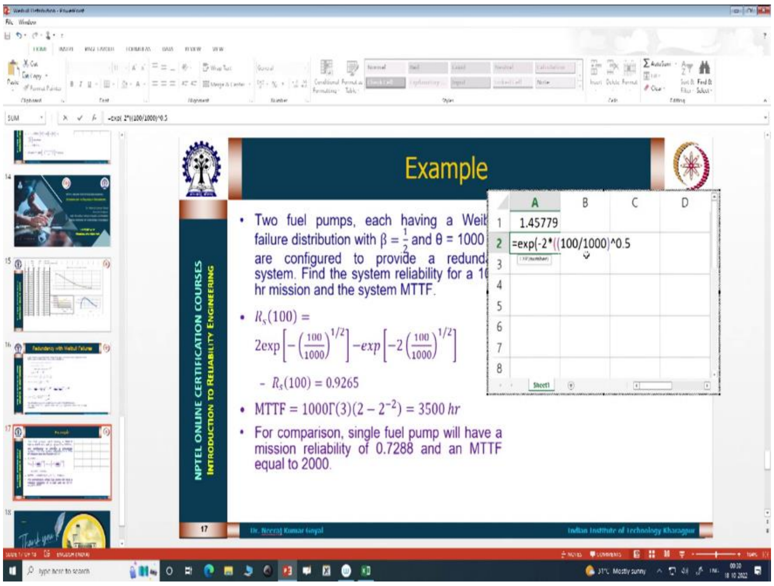
Example
Two fuel pumps, each having a Weib failure distribution with β = 1/2 and θ = 1000 are configured to provide a redund system. Find the system reliability for a 10 hr mission and the system MTTF.
$R_s(100) =$
$2\exp\left[-\left(\frac{100}{1000}\right)^{1/2}\right] - exp\left[-2\left(\frac{100}{1000}\right)^{1/2}\right]$
- $R_s(100) = 0.9265$
$\text{MTTF} = 1000\Gamma(3)(2-2^{-2}) = 3500\ hr$
For comparison, single fuel pump will have a mission reliability of 0.7288 and an MTTF equal to 2000.
A B C D
1 1.45779
2 =exp(-2*((100/1000)^0.5
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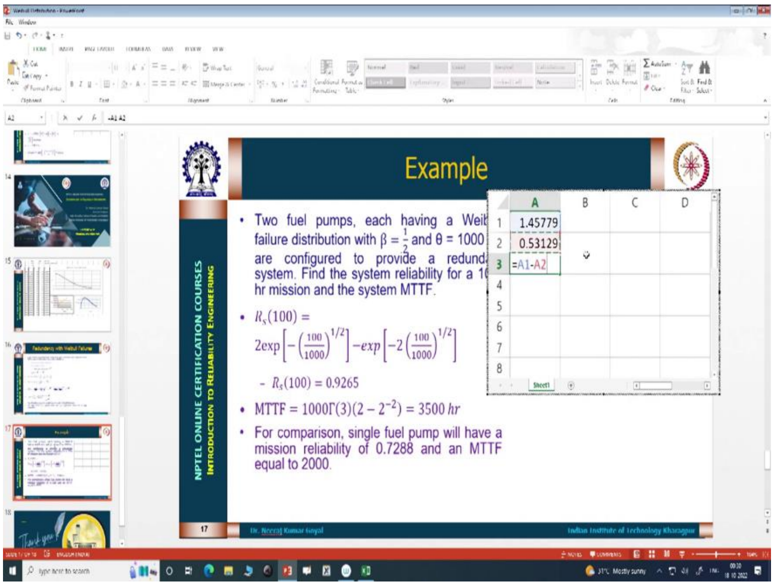
Dr. Neeraj Kumar Goyal
Indian Institute of Technology Kharagpur


Example
Two fuel pumps, each having a Weib failure distribution with β = 1/2 and θ = 1000 are configured to provide a redund system. Find the system reliability for a 10 hr mission and the system MTTF.
$R_s(100) =$
$2\exp\left[-\left(\frac{100}{1000}\right)^{1/2}\right] - exp\left[-2\left(\frac{100}{1000}\right)^{1/2}\right]$
- $R_s(100) = 0.9265$
$\text{MTTF} = 1000\Gamma(3)(2-2^{-2}) = 3500\ hr$
For comparison, single fuel pump will have a mission reliability of 0.7288 and an MTTF equal to 2000.
A B C D
1 1.45779
2 0.53129
3 =A1-A2
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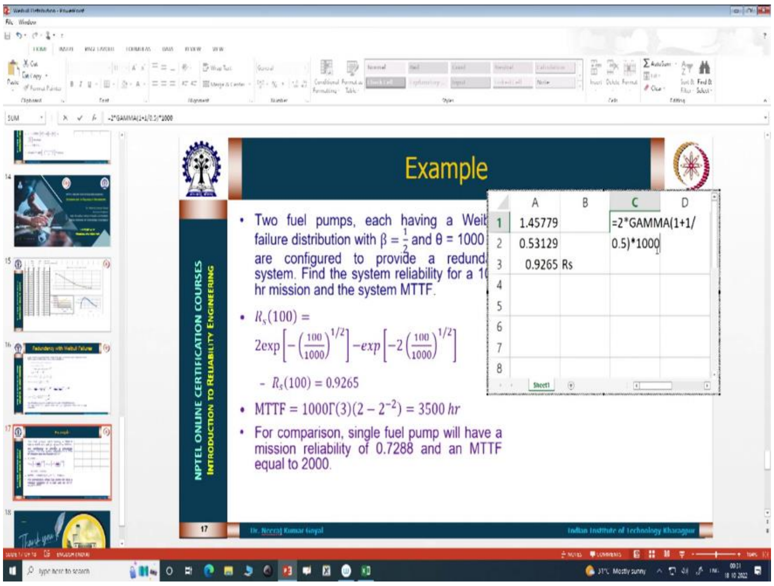
Example
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$2\exp\left[-\left(\frac{100}{1000}\right)^{1/2}\right] - exp\left[-2\left(\frac{100}{1000}\right)^{1/2}\right]$
- $R_s(100) = 0.9265$
$\text{MTTF} = 1000\Gamma(3)(2-2^{-2}) = 3500\ hr$
For comparison, single fuel pump will have a mission reliability of 0.7288 and an MTTF equal to 2000.
A B C D
1 1.45779 =2*GAMMA(1+1/
2 0.53129 0.5)*1000
3 0.9265 Rs
Sheet1
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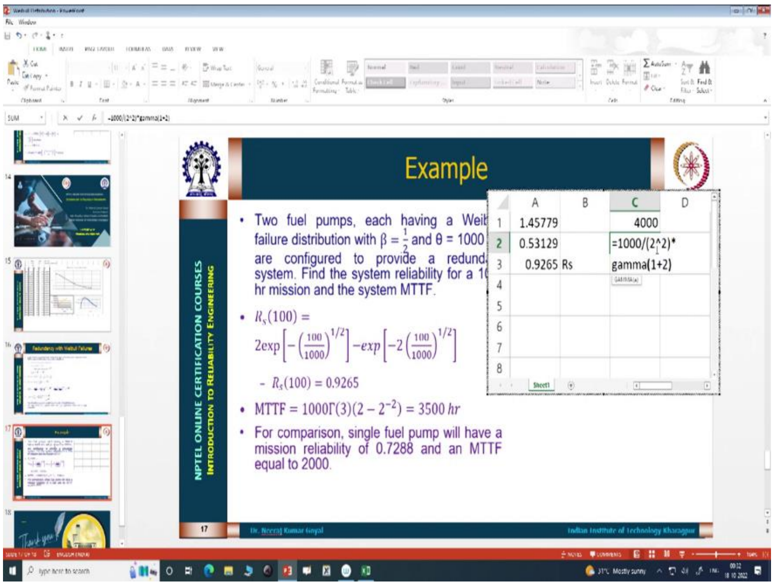
Example
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MTTF = 1000Γ(3)(2 − 2^{−2}) = 3500 hr
For comparison, single fuel pump will have a mission reliability of 0.7288 and an MTTF equal to 2000.
A
B
C
D
1.45779
4000
0.53129
=1000/(2^2)*
0.9265 Rs
gamma(1+2)
Sheet1
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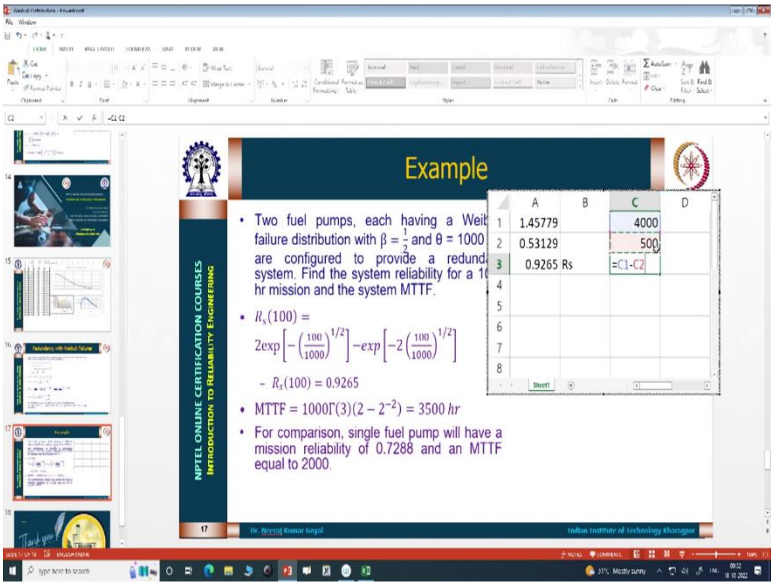
Example
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A
B
C
D
1.45779
4000
0.53129
500
0.9265 Rs
=C1-C2
Sheet1
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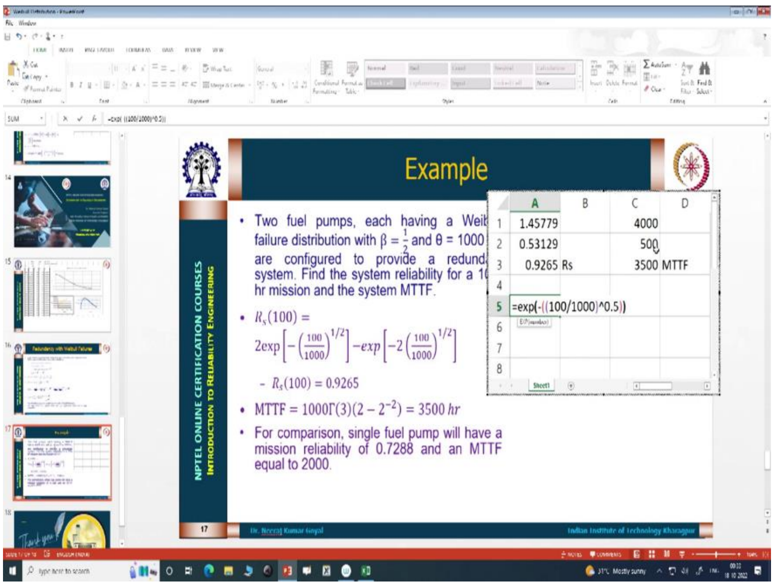
Example
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A
B
C
D
1.45779
4000
0.53129
500
0.9265 Rs
3500 MTTF
=exp(-((100/1000)^0.5))
Sheet1
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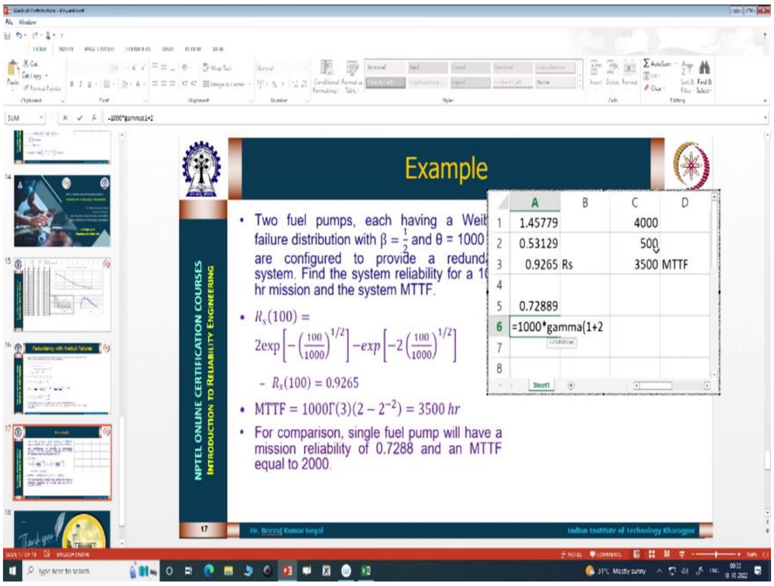
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A
B
C
D
1.45779
4000
0.53129
500
0.9265 Rs
3500 MTTF
0.72889
=1000*gamma(1+2
Sheet1
17
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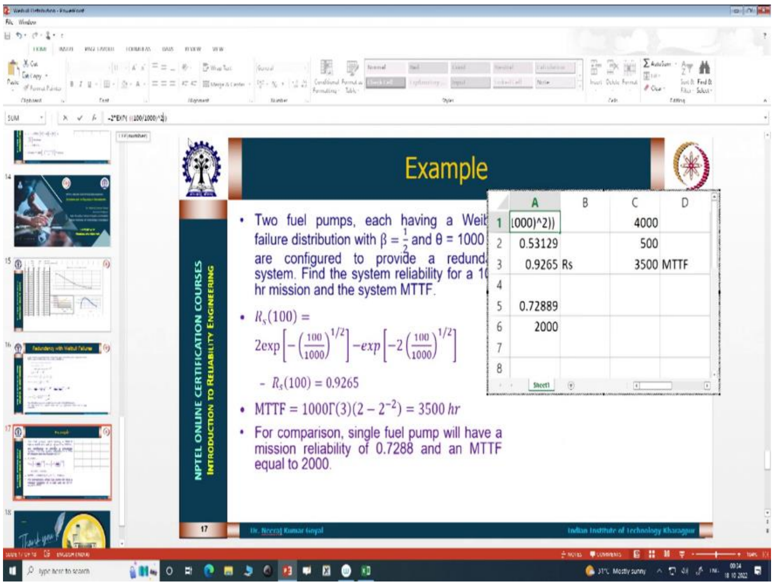
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A
B
C
D
000)^2))
4000
0.53129
500
0.9265 Rs
3500 MTTF
0.72889
2000
Sheet1
17
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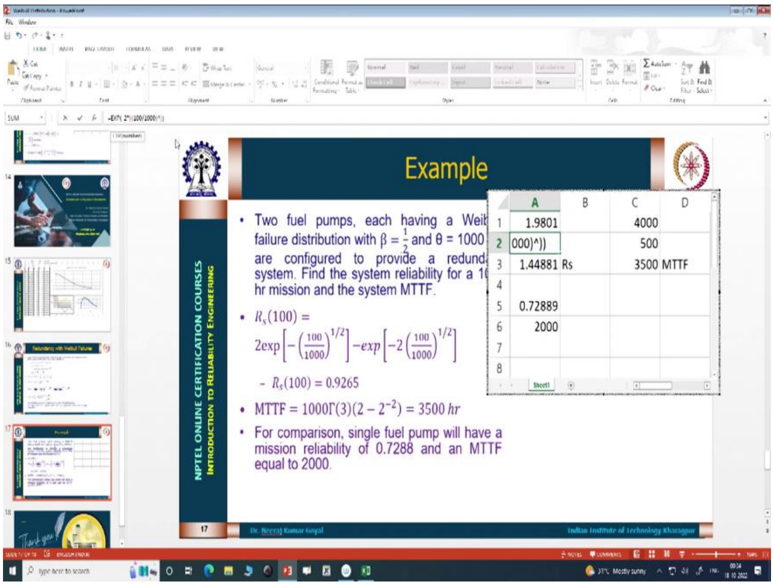
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For comparison, single fuel pump will have a mission reliability of 0.7288 and an MTTF equal to 2000.
A
B
C
D
1.9801
4000
000)^))
500
1.44881 Rs
3500 MTTF
0.72889
2000
Sheet1
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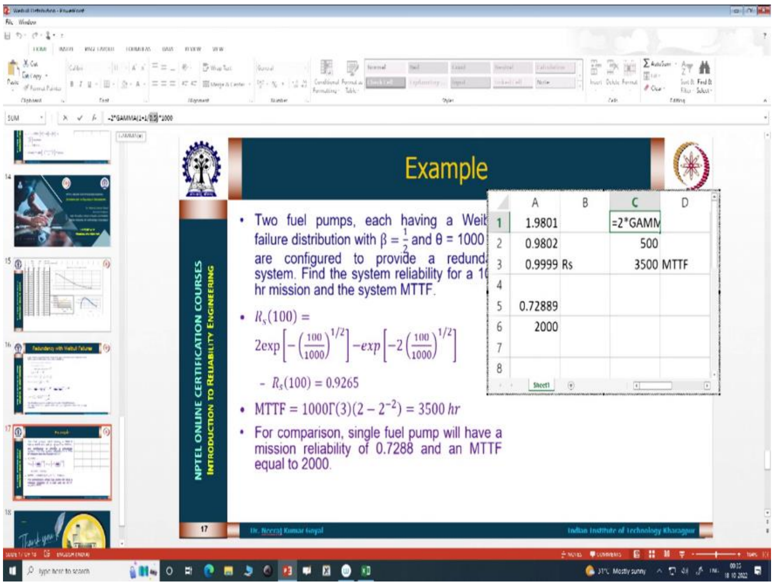
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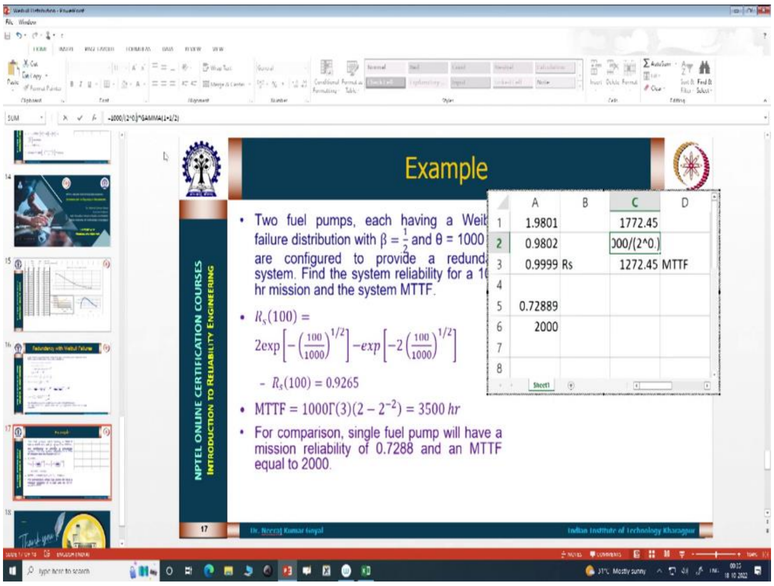
A
B
C
D
1.9801
=2*GAMM
0.9802
500
0.9999 Rs
3500 MTTF
0.72889
2000
Sheet1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17
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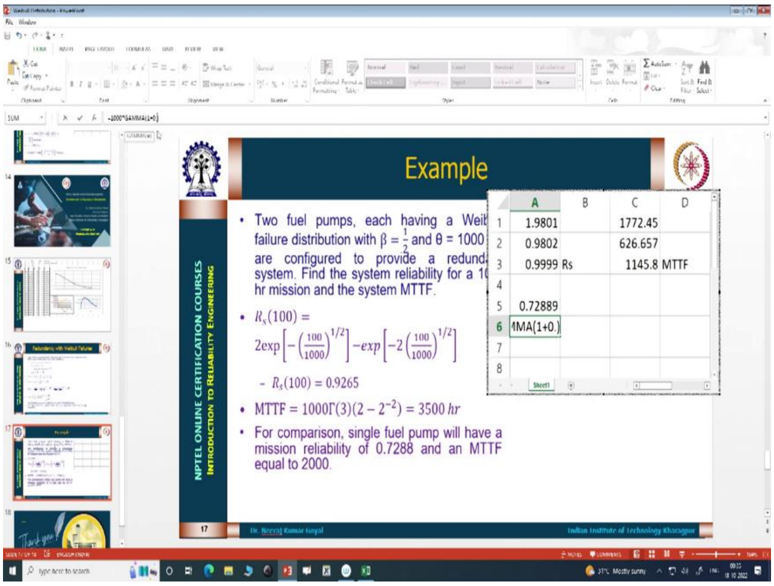
A
B
C
D
1.9801
1772.45
0.9802
)00/(2^0.)
0.9999 Rs
1272.45 MTTF
0.72889
2000
Sheet1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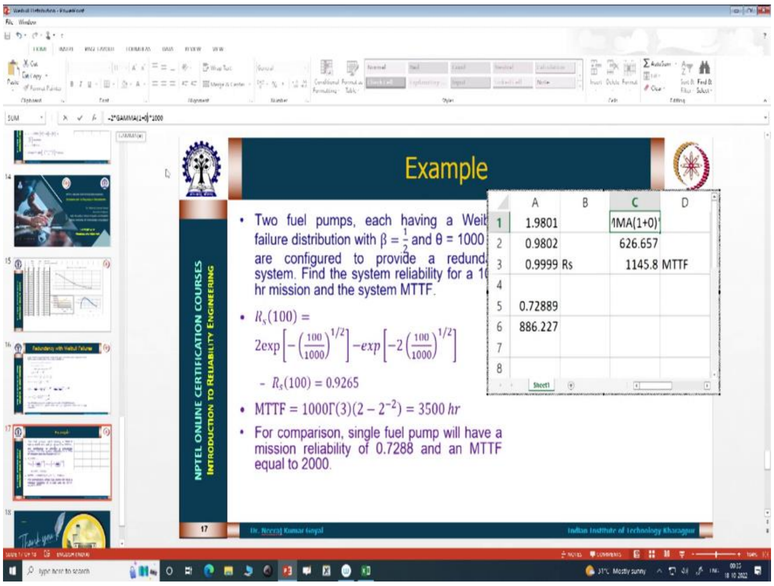
A
B
C
D
1.9801
1772.45
0.9802
626.657
0.9999 Rs
1145.8 MTTF
0.72889
MMA(1+0.)
Sheet1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17
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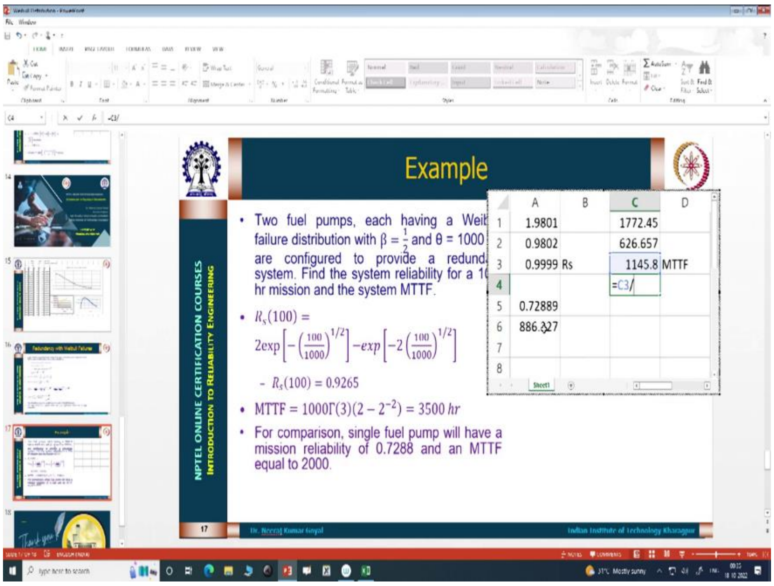
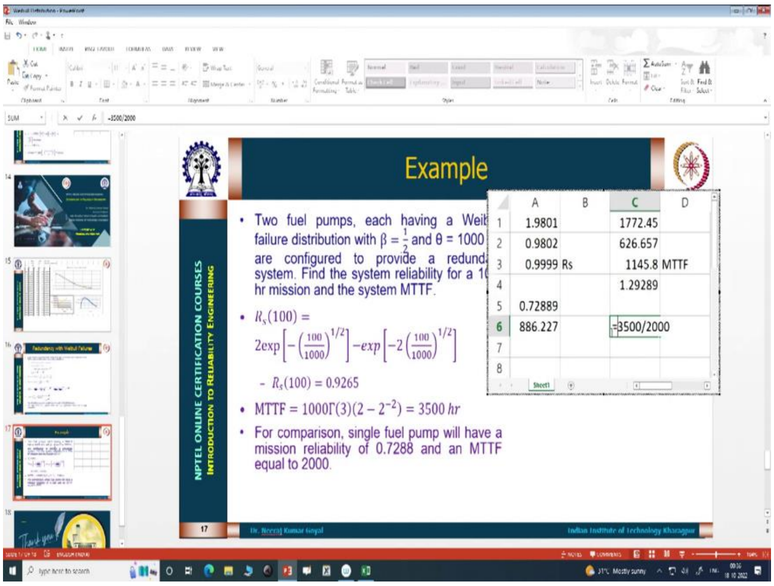
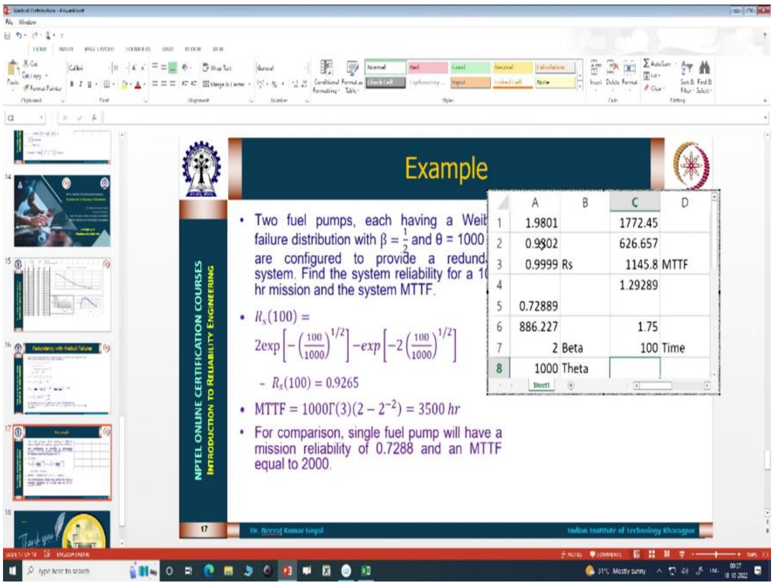
A B C D
1 1.9801 MA(1+0)
2 0.9802 626.657
3 0.9999 Rs 1145.8 MTTF
5 0.72889
6 886.227
Sheet1


Example
A B C D
1 1.9801 1772.45
2 0.9802 626.657
3 0.9999 Rs 1145.8 MTTF
4 =C3/
5 0.72889
6 886.227
Sheet1


Example
A B C D
1 1.9801 1772.45
2 0.9802 626.657
3 0.9999 Rs 1145.8 MTTF
4 1.29289
5 0.72889
6 886.227 =3500/2000
Sheet1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1.9801
0.9802
0.9999 Rs
0.72889
886.227
2 Beta
1000 Theta
1772.45
626.657
1145.8 MTTF
1.29289
1.75
100 Time
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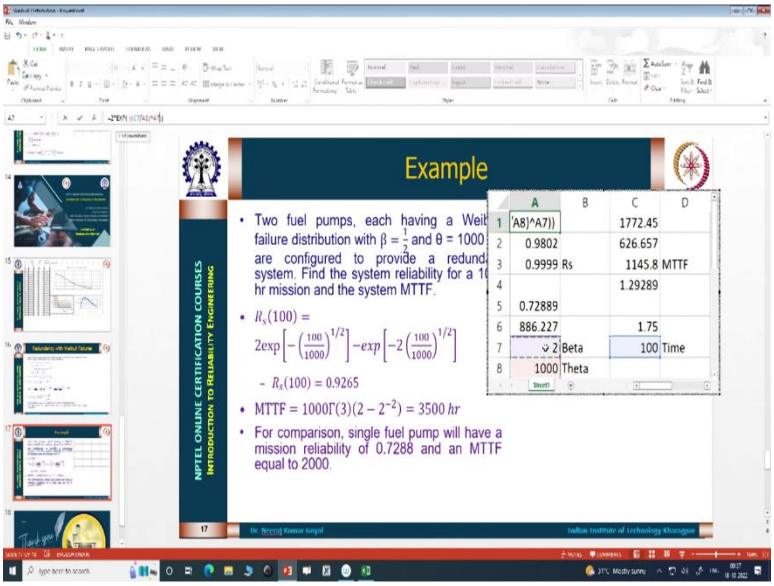
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A8)^A7))
0.9802
0.9999 Rs
0.72889
886.227
2 Beta
1000 Theta
1772.45
626.657
1145.8 MTTF
1.29289
1.75
100 Time
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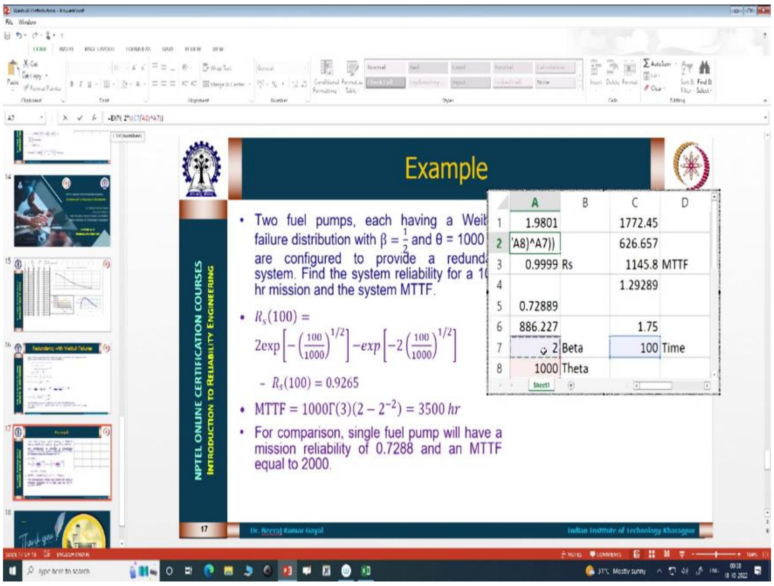
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1.9801
A8)^A7))
0.9999 Rs
0.72889
886.227
2 Beta
1000 Theta
1772.45
626.657
1145.8 MTTF
1.29289
1.75
100 Time
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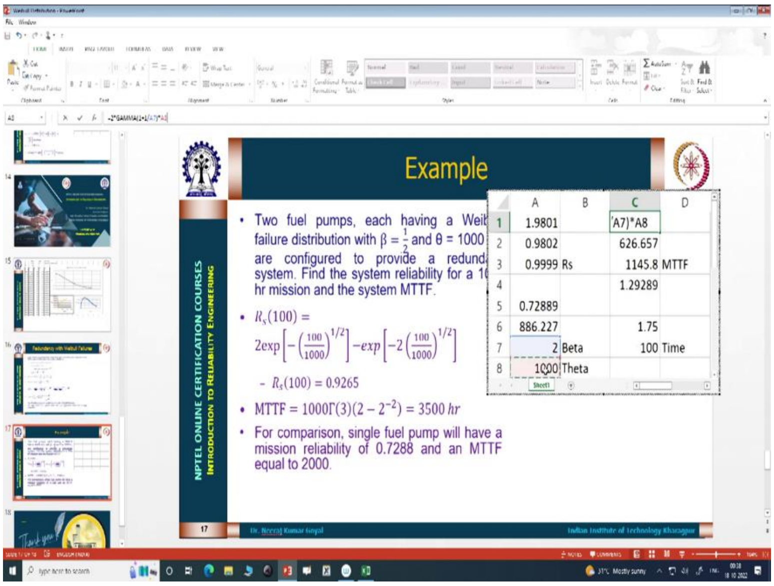
Example
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failure distribution with $\beta = \frac{1}{2}$ and $\theta = 1000$
are configured to provide a redund
system. Find the system reliability for a 1(
hr mission and the system MTTF.
$R_s(100) =$
$2\exp\left[-\left(\frac{100}{1000}\right)^{1/2}\right] - exp\left[-2\left(\frac{100}{1000}\right)^{1/2}\right]$
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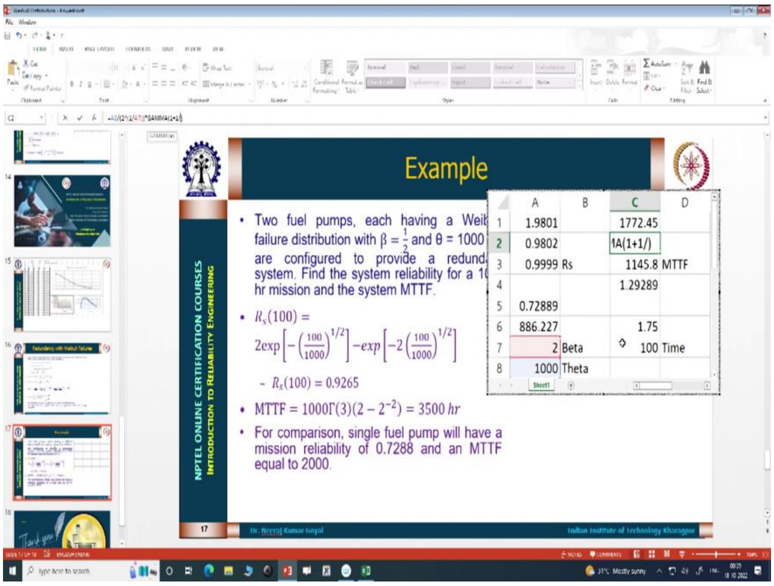
A B C D
1 1.9801 A7)*A8
2 0.9802 626.657
3 0.9999 Rs 1145.8 MTTF
4 1.29289
5 0.72889
6 886.227 1.75
7 2 Beta 100 Time
8 1000 Theta
Sheet1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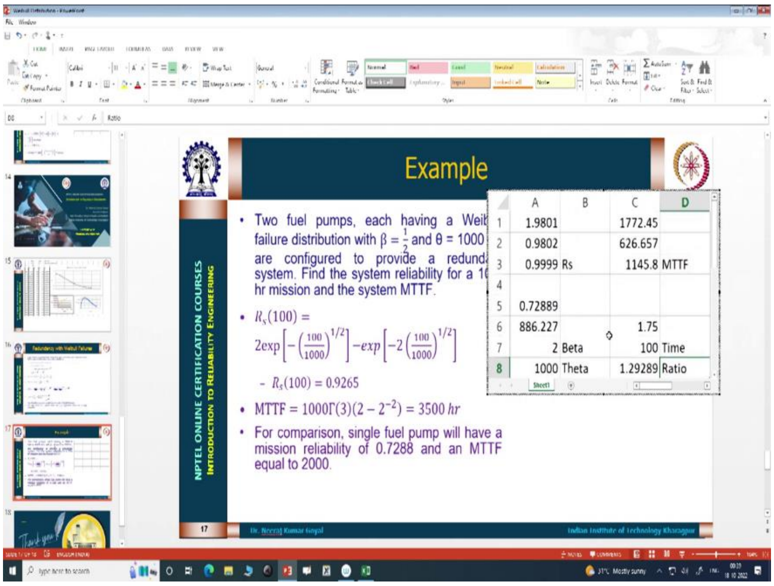
A B C D
1 1.9801 1772.45
2 0.9802 1A(1+1/)
3 0.9999 Rs 1145.8 MTTF
4 1.29289
5 0.72889
6 886.227 1.75
7 2 Beta 100 Time
8 1000 Theta
Sheet1


Example
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hr mission and the system MTTF.
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$2\exp\left[-\left(\frac{100}{1000}\right)^{1/2}\right] - exp\left[-2\left(\frac{100}{1000}\right)^{1/2}\right]$
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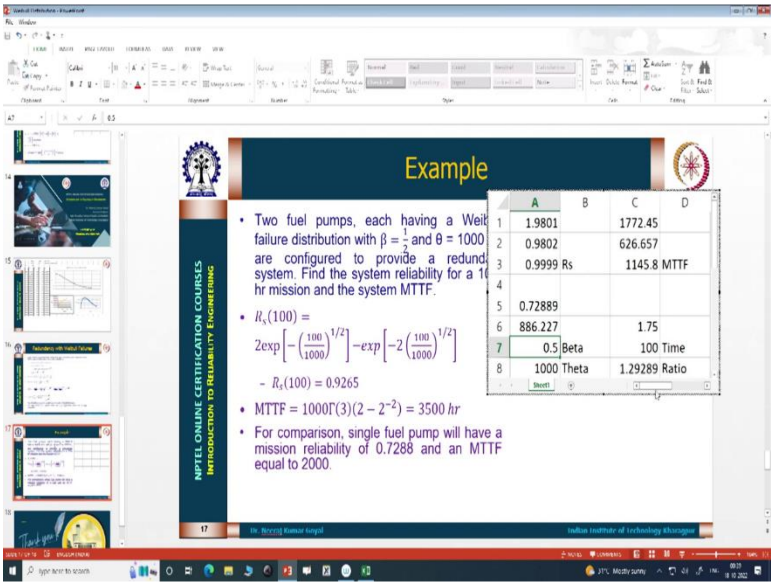
A B C D
1 1.9801 1772.45
2 0.9802 626.657
3 0.9999 Rs 1145.8 MTTF
4
5 0.72889
6 886.227 1.75
7 2 Beta 100 Time
8 1000 Theta 1.29289 Ratio
Sheet1



Example
Two fuel pumps, each having a Weib[ull] failure distribution with $\beta = \frac{1}{2}$ and $\theta = 1000$ are configured to provide a redund[ant] system. Find the system reliability for a 1[00] hr mission and the system MTTF.
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A B C D
1 1.9801 1772.45
2 0.9802 626.657
3 0.9999 Rs 1145.8 MTTF
5 0.72889
6 886.227 1.75
7 0.5 Beta 100 Time
8 1000 Theta 1.29289 Ratio
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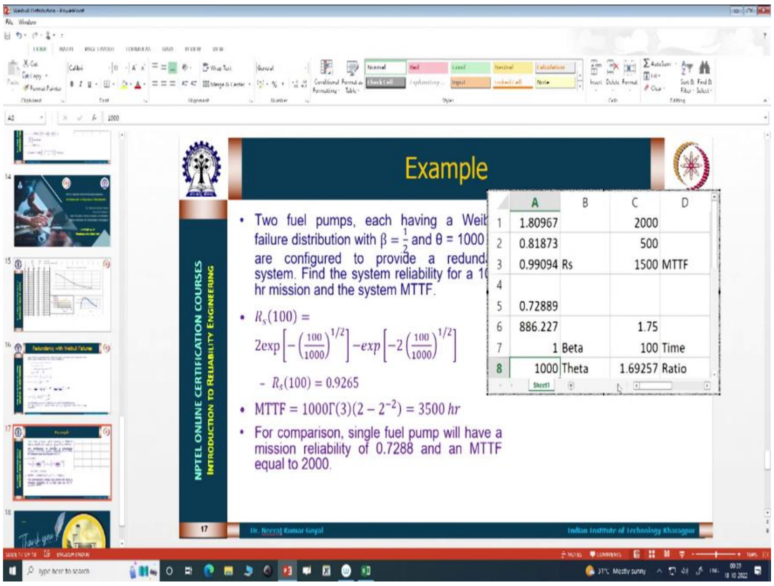

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A B C D
1 1.80967 2000
2 0.81873 500
3 0.99094 Rs 1500 MTTF
5 0.72889
6 886.227 1.75
7 1 Beta 100 Time
8 1000 Theta 1.69257 Ratio
17
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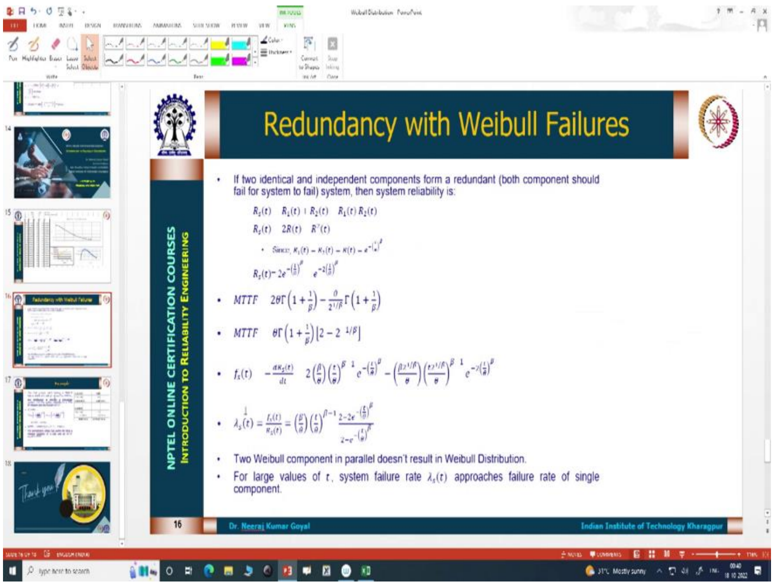

Redundancy with Weibull Failures
If two identical and independent components form a redundant (both component should fail for system to fail) system, then system reliability is:
$R_s(t) = R_1(t) + R_2(t) - R_1(t)R_2(t)$
$R_s(t) = 2R(t) - R^2(t)$
Since, $R_1(t) = R_2(t) = R(t) = e^{-\left(\frac{t}{\theta}\right)^\beta}$
$R_s(t) = 2e^{-\left(\frac{t}{\theta}\right)^\beta} - e^{-2\left(\frac{t}{\theta}\right)^\beta}$
$MTTF = 2\theta\Gamma\left(1+\frac{1}{\beta}\right) - \frac{\theta}{2^{1/\beta}}\Gamma\left(1+\frac{1}{\beta}\right)$
$MTTF = \theta\Gamma\left(1+\frac{1}{\beta}\right)\left[2 - 2^{-1/\beta}\right]$
$f_s(t) = -\frac{dR_s(t)}{dt} = 2\left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta-1}e^{-\left(\frac{t}{\theta}\right)^\beta} - \left(\frac{\beta 2^{1/\beta}}{\theta}\right)\left(\frac{t 2^{1/\beta}}{\theta}\right)^{\beta-1}e^{-2\left(\frac{t}{\theta}\right)^\beta}$
$\lambda_s(t) = \frac{f_s(t)}{R_s(t)} = \left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta-1}\frac{2-2e^{-\left(\frac{t}{\theta}\right)^\beta}}{2-e^{-\left(\frac{t}{\theta}\right)^\beta}}$
Two Weibull component in parallel doesn't result in Weibull Distribution.
For large values of t, system failure rate $\lambda_s(t)$ approaches failure rate of single component.
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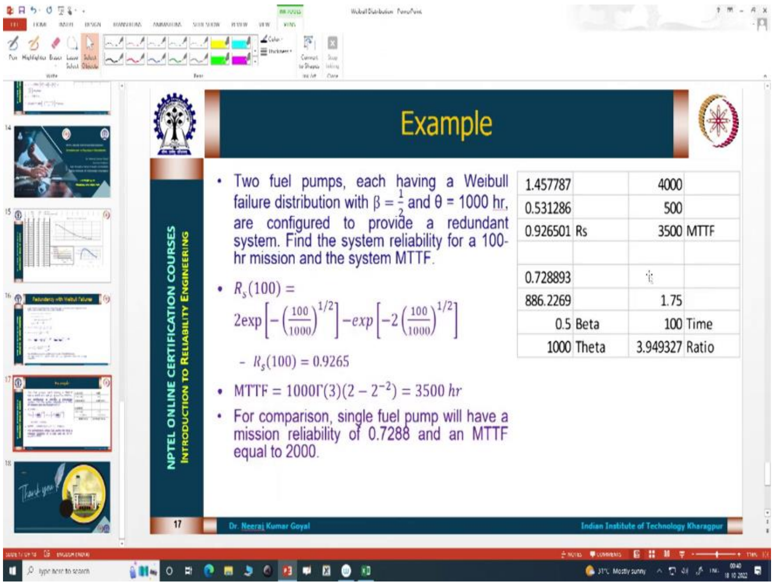
INTRODUCTION TO RELIABILITY ENGINEERING


Example
Two fuel pumps, each having a Weibull failure distribution with β = 1/2 and θ = 1000 hr, are configured to provide a redundant system. Find the system reliability for a 100-hr mission and the system MTTF.
$R_s(100) =$
$2\exp\left[-\left(\frac{100}{1000}\right)^{1/2}\right] - \exp\left[-2\left(\frac{100}{1000}\right)^{1/2}\right]$
$R_s(100) = 0.9265$
$MTTF = 1000\Gamma(3)(2-2^{-2}) = 3500\ hr$
For comparison, single fuel pump will have a mission reliability of 0.7288 and an MTTF equal to 2000.
1.457787 | 4000
0.531286 | 500
0.926501 Rs | 3500 MTTF
0.728893
886.2269 | 1.75
0.5 Beta | 100 Time
1000 Theta | 3.949327 Ratio
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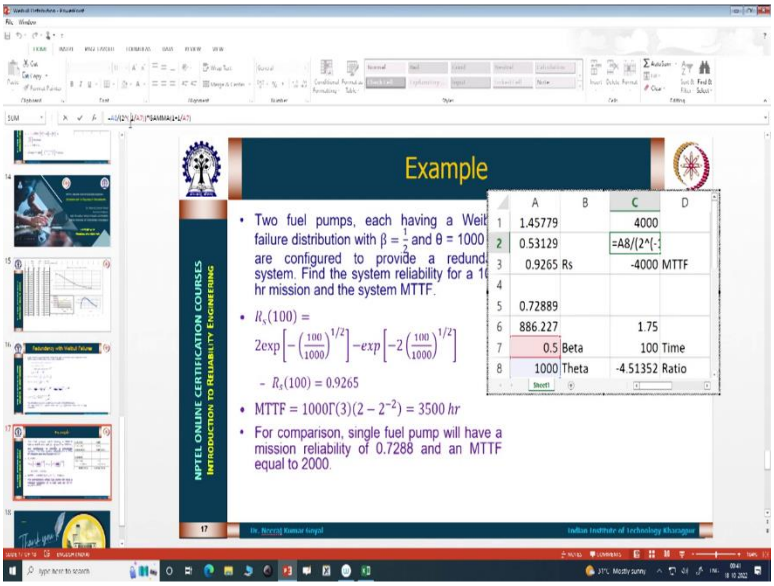
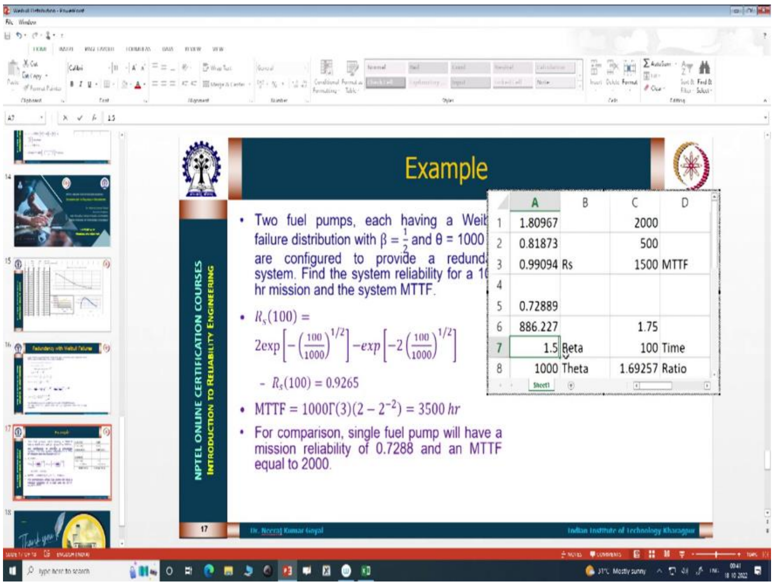
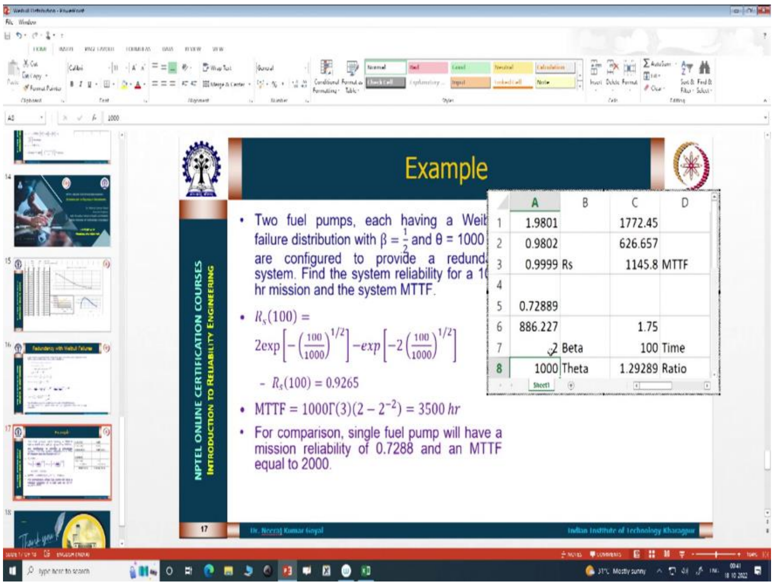
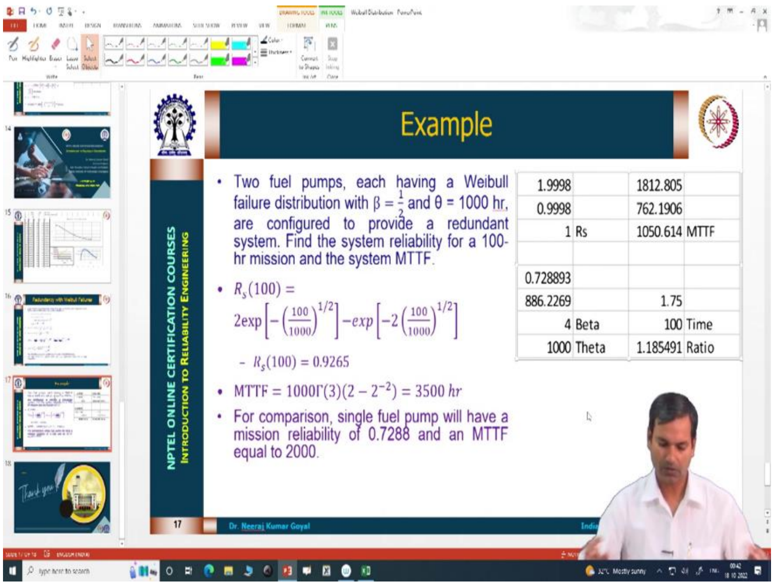

Example
1.45779 | 4000
0.53129 | =A8/(2^(-
0.9265 Rs | -4000 MTTF
0.72889
886.227 | 1.75
0.5 Beta | 100 Time
1000 Theta | -4.51352 Ratio


Example
1.80967 | 2000
0.81873 | 500
0.99094 Rs | 1500 MTTF
0.72889
886.227 | 1.75
1.5 Beta | 100 Time
1000 Theta | 1.69257 Ratio





Let us see how we can get this here. So, we can take first term, first term is equal to 2 into exponential of minus of t is 100 here divided by theta is 1000 here whole the raised to the power, beta is 1 by 2 0.5, second term is equal to exponential minus t minus 2 into or we can say minus 2 itself I can write minus 2 into t divided by theta whole raised to the power beta, beta is 0.5 and my reliability is this minus this.

So, this becomes my system reliability Rs 0.9265. What is my MTTF? As we discussed earlier MTTF like for first term if I want to calculate MTTF that will be equal to 2 into gamma function of 1 plus 1 divided by beta, beta is of 0.5 and for this multiply sorry multiply by this multiply by the theta is 1000 and this will be equal to as we discussed earlier this will be equal to theta, theta is change to theta divided by 2 raised to the power 1 divided by beta.

So, 0.1 by 0.5, 1 by 1 by 2 if I do this will become 2, multiply by gamma of 1 plus 1 divided by 1 by 2 that will become 2 and answer would be equal to this minus this which comes out to be 3500 hours. So, my MTTF also we are able to calculate this is my MTTF. As we know if I, if I did not have this if I want to know for single components, for single component this was exponential minus t was let us say 100 divided by 1000 whole raised to the power 0.5. This was my single component reliability that is 0.7288 and MTTF for the same is 1000 into gamma function of 1 plus 3, 1 plus 2, 1 plus 1 by 2 is 2 that comes out to be 2000.

So, if we see it here that by having two component by reliabilities change to from 2000 it is changed to 3500. But, in case of CFR the relative is 1.5 of this that means, if I had taken that would have been 3000. But because of different beta, beta is 0.5 here the reliability improvement has been like has been like if we see here, it is almost double 2000 has become 3500 the MTTF has increased to that, while reliability is changed from 0.72 to 0.92 when we use components in parallel. Similar thing if we want to investigate let us say if you take beta is was equal to 2 let us hypothetically assume this question is solved let us say if we assume beta equal to 2.

So, in that case let us see how it impacts rather than 0.5, I will write wherever beta have taken I will take 2, then my reliability was this. And my MTTF, I will take 2 beta is 2. So this will become 1 by 2 and this will also become 0.5, I will write 0.5 here. If you see here that here the ratio is 1.29, but when beta was 0.5, the ratio was 3500 divided by 2000, which was around 1.75.

If you see that here or depending on the value of beta you may have a different improvement. So, when we changed when beta was less than the improvement was more in MTTF because of the redundancy, but when beta was higher than it became 2 my improvement has become 1.75 times. So, similarly we will be able to, we can see that or whether for different values of beta we may have a different improvement in MTTF when we are having the redundancy in the components.

So if I had made it properly I could have shown like we have shown with beta different values or if I take let us say beta value if I write separately, then it will be much easy. Let us say beta I am writing here 2. So, then we can investigate that how this happens. And theta is 1000, now I will write the standard formula here. So that is twice of exponential. Let us say time, time also I will write. Time is 100 here. So, when time is 100 then this will become 2 into time divided by theta raised to the power beta. This will become time divided by theta raised to the power beta.

This will become 2 into gamma 1 plus 1 upon beta plus 1 plus 1 divided by beta multiplied by theta this is theta divided by 2 raised to the power 1 by beta multiply with gamma 1 plus 1 divided by beta, this we can take it here as the ratio so, now, let us say, if we investigate that when let us say beta is 0.5, then my ratio was 3.94, when I took beta equal to 1, 1.69 265 gamma of this 2 to the power minus 2 only or minus 2 is minus beta, just let me just check this formula once again that we have MTTF 2 to the power minus 1 upon beta. So this will I took a division then minus sign will be adjusted here.

So, same thing, as we see here when we put 0.5, if you put 1.69. When beta is around 1.5, I will have 1.39, I am making 2 1.29 then I am making it 4. If we see that, as beta is increasing the effect of having redundancy may not be that much in improvement in MTTF as it was there for the lower values of the beta. We can do this exercise you can try this exercise once again to see that how the change of beta is giving you the improvement in MTTF, but reliability improvement can already be, can also be seen here. And this will give you an idea that when you improve redundancy, whether it is effective or not, that can also depend on the value of the beta.

So, by doing this exercise, you can do more intensive exercise by taking different values of beta for different theta’s also, theta’s will actually not change much in MTTF, that will be similar proportion will be there. But reliability also you can look into that how reliability ratios are being changed. So, this all you can do and or you can do much more exercise which will be given to you in as assignment. So thank you. Next time we will continue our discussion with normal distribution. Thank you.