Introduction to Reliability Engineering
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Lecture 11
Burn-In Screening for Weibull

Hello everyone. So, now, we are moving to lecture number 11 which is in continuation with our previous lecture on Weibull distribution.

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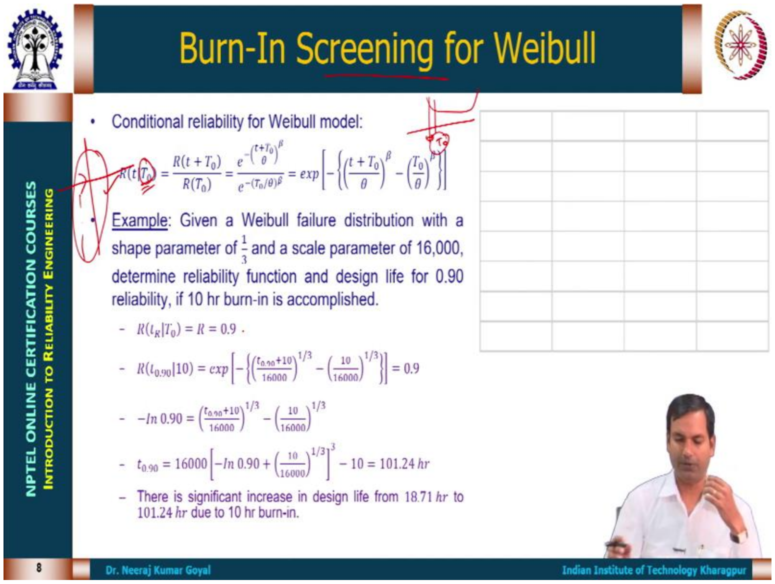

In previous lecture we discussed about the basic equations used in Weibull distribution, calculation of design life, variance, MTTF et cetera. Now, let us discuss that burn in. So, in previous example, when we discussed that there was a decreasing failure rate. So, whenever we have a decreasing failure rate, case of decreasing failure rate and in those cases, we use the burn in so, that out of box failure or the immediate failures are not experienced by the customer.

So, customer is able to have a good and observe the useful life period he is not having the infant mortality period. So, what we are doing here we are having a burn in period for that. So, how much period will be good enough for us in which we will be able to ensure that customer is getting good reliability.

So, because this is going to be costly for the company, a company has to keep a product for certain period t0 here within their premises under stress conditions. So, once they apply the stresses and these many times there is a high temperature applied around 60 to 70 degrees celsius as well as all electrical stresses are applied to the product. For non electrical product they will be like for bikes et cetera they will run the bike for let us say 4 hours 5 hours or for a higher terrain, the terrain will not be a smooth terrain, they will have a rough terrain and the which will produce more load on the system.

So, if there is any weakness in the product or if there is any manufacturing defect or other kinds of design defect, then what will happen, the product will fail faster. And so, the product which are failing will be removed they will not be sent to the customer. So, customer will only be sent those products which pass this this burn in screening. So, we call this screen because this is screening there is this kind of screen from which the faulty products are dropped here and only good products are passed.

So, screening strength, the strength of this screen that means how much is the capability of this screen to stop bad product from reaching to the customer. That screening strength will depend on the stresses which we are choosing for this screen as well as the time how long we are going to keep the product for these stresses. So, generally if we assume here that general case that the same stresses as the normal use conditions are applied here.

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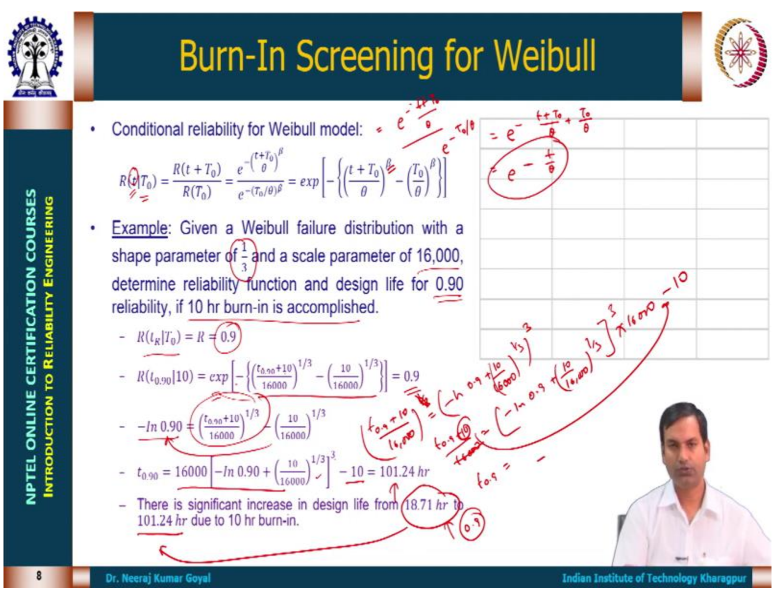

If we do that, in that case we are seeing that we are keeping the product for time t equal to t0 within the premises then we are sending the product to the customer for and we want to know the reliability for time small time t which is the time observed by the customer. So, that means we know from the condition reliability formula Rt given T0 is the reliability at time t plus T0 divided by reliability at time T0. And we know the reliability formula that is e to the power minus t upon theta raise to the power beta here t is t plus T0. So, this will become t plus T0 divided by theta raise to the power beta divided by e to the power minus T0 upon theta raise to the power beta.

$$R(t \mid T_0) = \frac{R(t+T_0)}{R(T_0)} = \frac{e^{-\left(\frac{t+T_0}{\theta}\right)^{\beta}}}{e^{-(T_0/\theta)^{\beta}}} = \exp\left[-\left\{\left(\frac{t+T_0}{\theta}\right)^{\beta} - \left(\frac{T_0}{\theta}\right)^{\beta}\right\}\right]$$

The same if we take a this will become this. We knew that for exponential distribution makes no difference the burn in will have no difference. We can see it here, if beta is equal to 1 this will become e to the power minus t plus T0 divided by theta divided by e to the power minus T0 divided by theta.

If we take it up, this will become e to the power minus t plus T0 divided by theta plus T0 divided by theta. This will be equal to e to the power minus T0 T0 will get cancelled minus t upon theta. So burn-in has no impact on the exponential distribution or the constant failure, burn-in is only effective for the decreasing hazard rate. For increasing hazard rate the reliability will decrease.

So, here the beta if we take the previous example, which we discussed in previous lecture, that there is a Weibull failure distribution with parameter beta equal to 1 by 3, 1 by 3 means decreasing failure rate less than 1 and scale parameter is 16,000. We want to determine the reliability function and design life for 0.90 reliability. Given that we are doing the 10 hours burn-in So, if we do 10 hours burn-in then reliability will become tR given T0 that is our target value 0.9. And so, this will become reliability of t 0.9 that means 90 percent reliability is aim given 10 hours are already spent.

So, exponential minus of we use this formula t is equal to t 0.9 T0 is 10 hours divided by 16,000 hours minus 10 divided by 16,000 hours whole raise to power 1 by 3, this our target value is 0.9. So, if we reverse all this we take exponential here. So, this will become ln, so, minus ln of 0.90 will be equal to this value inside value t 0.9 plus 10 divided by 16,000 power 1 by 3 minus this.

Now here this value we can take right side. So, this will become minus ln of 0.90 plus 10 divided by 16,000 raise to the power 1 by 3 this will be equal to this. Now, if I want to take this then this will become t 0.9 plus 10 divided by 16,000 whole raise to the power 1 by 3 will be equal to minus ln 0.9 plus 10 divided by 16,000 whole raise 16,000 raise to the power 1 by 3. Now, this 1 by 3 we can take it here this will become 3.

So, t 0.9 plus 10 divided by 16,000 will be equal to minus ln 0.9 plus 10 divided by 16,000 whole raise to the power 1 by 3 and this whole raise to the power 3. So, t 0.9 will be equal to we can take multiply here this 16,000, we can multiply here and this 10 we can take it here as a subtraction.

So, this will be equal to minus same what you have written here that will be 16,000 into minus ln of 0.9 plus 10 divided by 16,000 raise to the power of 1 by 3 whole raise to the power 3 minus 10. This if we solve we get the 101.24 hours. So, if we as we saw earlier in previous example, then we did not do the burn-in then for 0.9 reliability my system was having only 18.71 hours as a life that means 10 percent failures for happening only in 18 hours or around 19 hours.

$$R\left(t_R \mid T_0\right) = R = 0.9$$

$$R\left(t_{0.90} \mid 10\right) = \exp\left[-\left\{\left(\frac{t_{0.90}+10}{16000}\right)^{1/3} - \left(\frac{10}{16000}\right)^{1/3}\right\}\right] = 0.9$$

$$-In0.90 = \left(\frac{t_{0.90}+10}{16000}\right)^{1/3} - \left(\frac{10}{16000}\right)^{1/3}$$

$$t_{0.90} = 16000\left[-In0.90 + \left(\frac{10}{16000}\right)^{1/3}\right]^3 - 10 = 101.24\text{hr}$$

But if we see that if we applied 10 hours burn in, then the customer will see a life of 101 hours. If you see it is a significant increase in the life seen by the customer, because what is happening the products which were failing fast in a shorter period, those products are not sent to the customer those products are kept within the premises they are repaired and then sent to the customer.

So, the problem which customer was supposed to see has already been rectified by the manufacturing unit and this results in a significant increase in the life of the product. So, customer is for 100 hours almost for 100 hours in place of 18 hours they are getting the point reliability. So, 10 percent failures are happening in 100 hours approximately. So, this becomes quite useful here. And as we see that burn-in is used when there is a decrease in failure rate.

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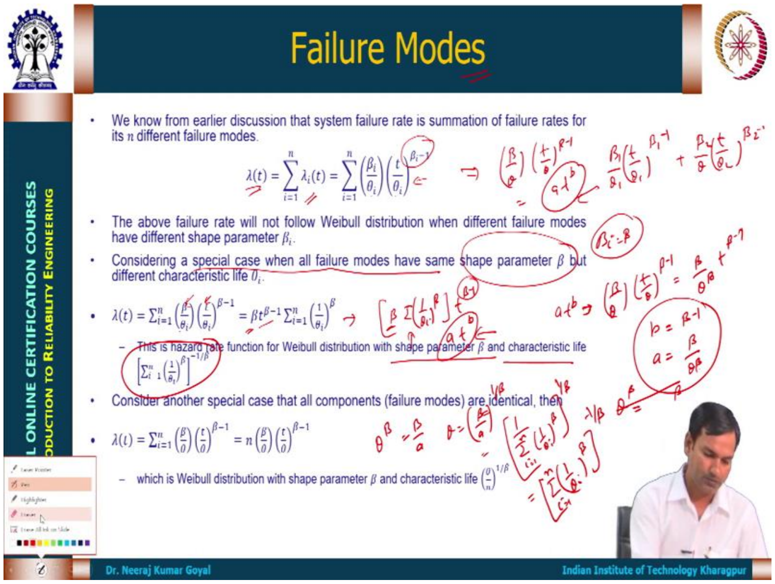

Like we discussed earlier, multiple failure modes in case of exponential distribution, let us discuss the same when we have the Weibull distribution. So, we know from earlier discussion that system failure rate is summation of failure rate of the failure modes. So, lambda t is equal to summation of lambda i t. So, if there are n failure modes it is i equal to 1 to n. Failure modes and if components are in series both have the same effect. So, failure modes can be considered as the series failures or series component.

$$\lambda(t) = \sum_{i=1}^{n} \lambda_i(t) = \sum_{i=1}^{n} \left(\frac{\beta_i}{\theta_i}\right)\left(\frac{t}{\theta_i}\right)^{\beta_i - 1}$$

So, here what is lambda i t? Lambda i t is beta i upon theta i upon t upon theta i raise to power beta i minus 1. Same formula we have kept it here. Now, this formula if we see we cannot convert this formula into the standard beta upon theta and t and t upon theta raise to power beta minus 1 or we can say a t to the power b.

We are not able to convert this into this formula because this is the summation of this we cannot like if you say beta 1 upon theta 1 to t upon theta 1 raise to the power let us say beta 1 minus 1 plus beta 2 upon theta 2 t upon theta 2 raise to the power beta 2 minus 1. Now, I cannot convert this into the standard form.

So, that is why this failure rate when we sum this failure rate, it may not be following the Weibull distribution. So, in case of when components are failing, Weibull distribution then we cannot say even though they are in series, they may not the system may not follow the Weibull distribution. But, in case of exponential distributions this first true if components or the multiple failure modes are having the exponential distribution resultant distribution was also the exponential distribution, but same is not true for the Weibull distribution, when components are in series following Weibull distribution, it may not result in a Weibull distribution for the system.

But there is a condition that when all come if we consider special case when all failure modes are having same shape parameter that means, beta is same for all, all beta i is equal to beta. In that case, if we see this lambda t lambda t is changed to a another that is beta upon theta i t upon theta theta is different for different failure modes but beta is same so, this will become beta upon theta i t upon theta i raise to the power beta minus 1.

$$\lambda(t) = \sum_{i=1}^{n} \left(\frac{\beta}{\theta_i}\right)\left(\frac{t}{\theta_i}\right)^{\beta-1} = \beta t^{\beta-1} \sum_{i=1}^{n} \left(\frac{1}{\theta_i}\right)^{\beta}$$

So, time dependent parameters are beta and t. So, I can take beta outside from the summation sign. Similarly, t raise to the power beta minus 1 also I can take it outside and this will become 1 upon theta i raise to the power beta. Now, this if we see this becomes kind of similar approach as we have taken earlier, that I can say it that this is beta summation of 1 upon theta i raise to the power beta this will be beta minus 1 not sorry beta sorry this will be beta only beta into t to the power beta minus 1 this I can say this is a t to the power b where a is this and t to the power b.

So, and we know that if I compare a t to the power b and if I compare an equivalent here that is beta upon theta and t upon theta raise to the power beta minus 1 if I convert this here then this will be equal to beta upon theta raise to the power beta into t raise to the power beta minus 1. So, if we equivalent here then b is equal to beta minus 1 and a is equal to beta upon theta raise to the point beta.

So, here I can get the so, if I equivalent here, so, beta minus 1. So, here we are able to have the standard form which is standard form is similar to the Weibull distribution form. So, I can convert this into the standard form and that is standard form if you look into here, then this is having the same beta value, if I say b is equal to beta minus 1, so, in that case beta will be equal to b plus 1.

So, I am getting here beta minus 1. So, here the shape parameter will be same as beta because t to the power beta and beta is known to me, so, my I can get the theta here. So, theta will be equal to if I take it here that will be equal to theta raise to the power beta I will take theta raise to the power beta will be equal to beta upon a and this if I solve further than theta will be equal to beta upon a raise to the power 1 upon beta.

So, same formula if I apply here then my a here is this and if I divide beta upon this will be 1 upon summation 1 upon theta i raise to power beta i equal to 1 to n this whole raise to the power 1 upon beta. Because beta and beta will get cancelled this beta and this beta will get cancelled and this becomes summation i equal to 1 to n 1 upon theta i whole raise to power beta and this whole raise to the power minus 1 upon beta.

So, we are able to get the same so, that means, we are able to convert when beta is same the system is also following the Weibull distribution Weibull distribution which is having shape parameter which is the same as the component shape parameter which is beta and characteristic life is having this formula that is summation i equal to 1 to n summation of 1 upon theta i whole raise to the power minus 1 upon beta. So, this becomes this gives that when components are in series or when there are multiple failure modes, the result can also be Weibull distribution, if all the components which are falling Weibull distribution has the same beta otherwise it will not be Weibull distribution.

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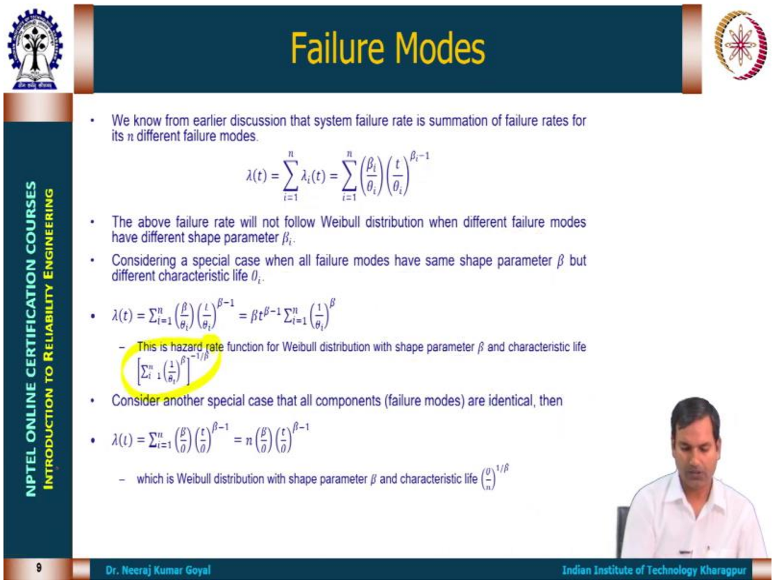

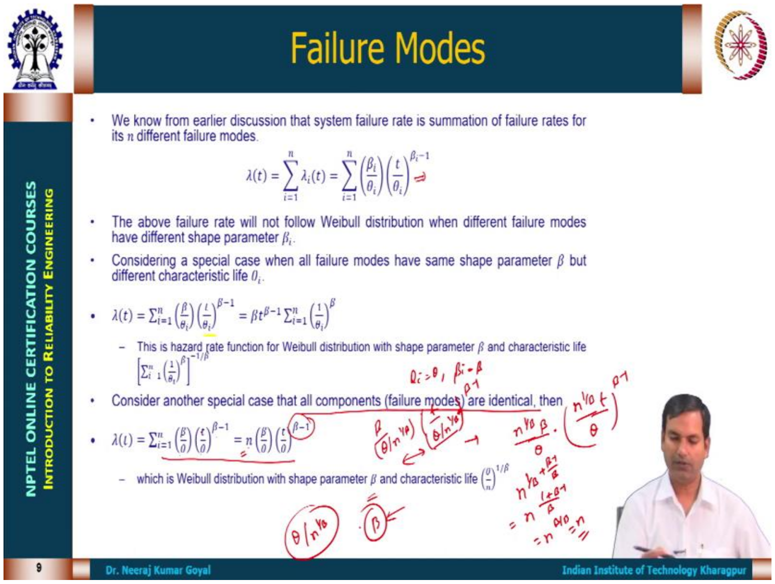

There can be another special case that can be sorry another special case can be that we have all components are identical, all components are identical means theta i is also equal to theta and beta i is also equal to beta all are same for everyone. So, then what will happen this becomes very simple that I have to simply sum it up n number of times, because this will become summation of beta upon theta t upon theta raise to beta minus 1. So, this become n beta upon on theta upon t upon theta raise to power beta minus 1. If we see here, this again has the same shape parameter as beta.

$$\lambda(t) = \sum_{i=1}^{n}\left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta-1} = n\left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta-1}$$

If I compare with this like earlier formula, which we have got. So, in this case, if I put n so, this is beta then we can say theta and if I make this n here in a manner that theta raise to the power n raise to the power 1 upon beta, then t upon theta divided by n raise to the power 1 upon beta minus 1 1 upon beta sorry and whole raise to the power beta minus 1.

In that case once we multiply these bottom terms what will happen, this I can write it as a n can be taken up. So, this will become n raise to the power of 1 upon beta upon into beta divided by theta into n raise to the power of 1 upon beta divided into t divided by theta raise to the power beta minus 1.

So, here if I am taking multiplying this this will become n raise to the power 1 upon beta plus beta minus 1 upon beta, if we sum it up, we get n n raise to the power beta 1 plus beta minus 1 that will be equal to n raise to the power beta upon beta that will be equal to n which is same this one.

So, here we are able what we are able to do we are able to convert this where theta is changed into theta divided by n raise to the power 1 upon beta and beta remains same. So, we are able to transform and we are able to get the system failure rate or system will follow the Weibull distribution which is having same shape parameters as beta and which is having characteristic life which is equal to theta upon n raise to the power 1 upon beta n is the number of failure modes or number of components here.

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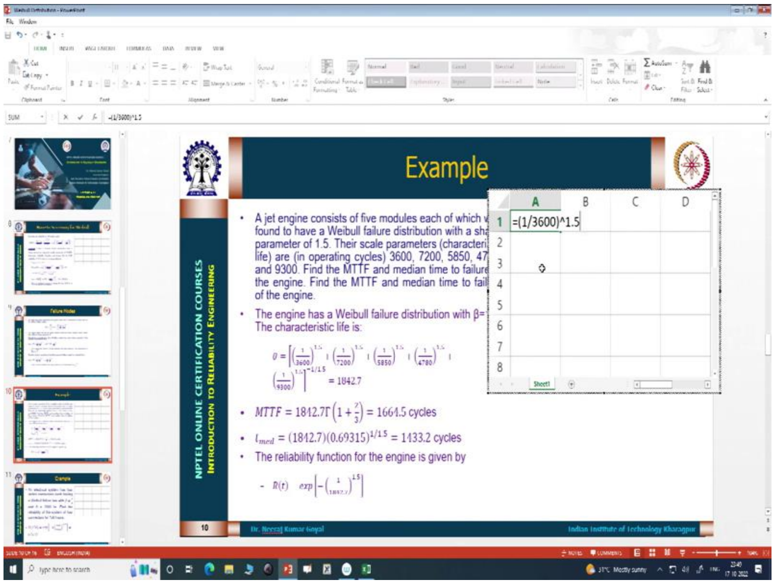

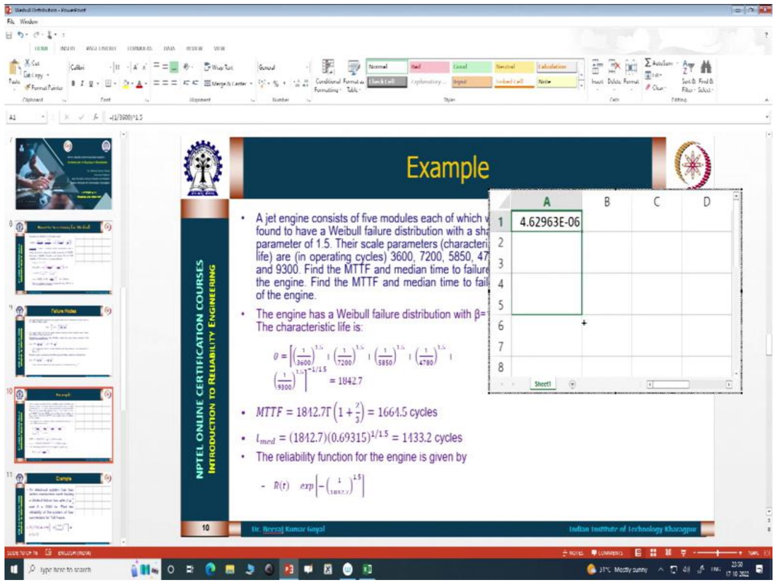
=(1/3600)^1.5
Example
A jet engine consists of five modules each of which
found to have a Weibull failure distribution with a sha
parameter of 1.5. Their scale parameters (characteri
life) are (in operating cycles) 3600, 7200, 5850, 47
and 9300. Find the MTTF and median time to failure
the engine. Find the MTTF and median time to fail
of the engine.
The engine has a Weibull failure distribution with β=
The characteristic life is:
= 1842.7
MTTF = 1842.7Γ(1 + 2/3) = 1664.5 cycles
t_{med} = (1842.7)(0.69315)^{1/1.5} = 1433.2 cycles
The reliability function for the engine is given by
A
B
C
D
4.62963E-06
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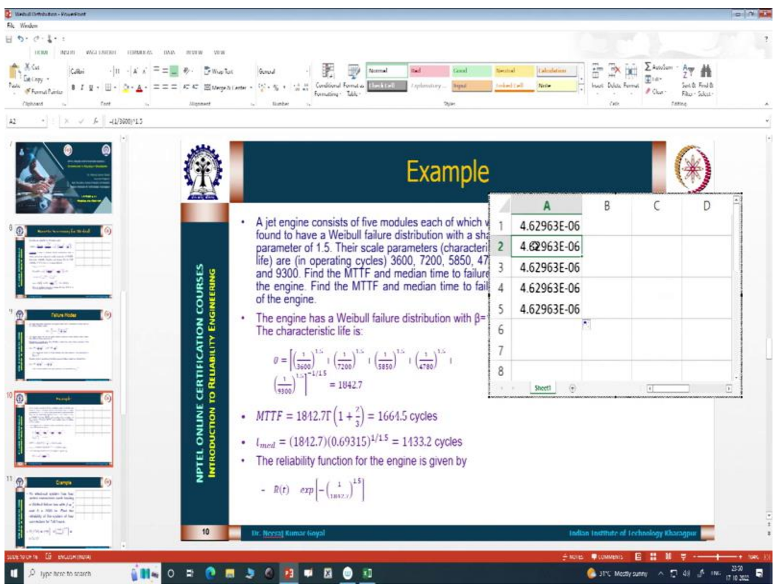
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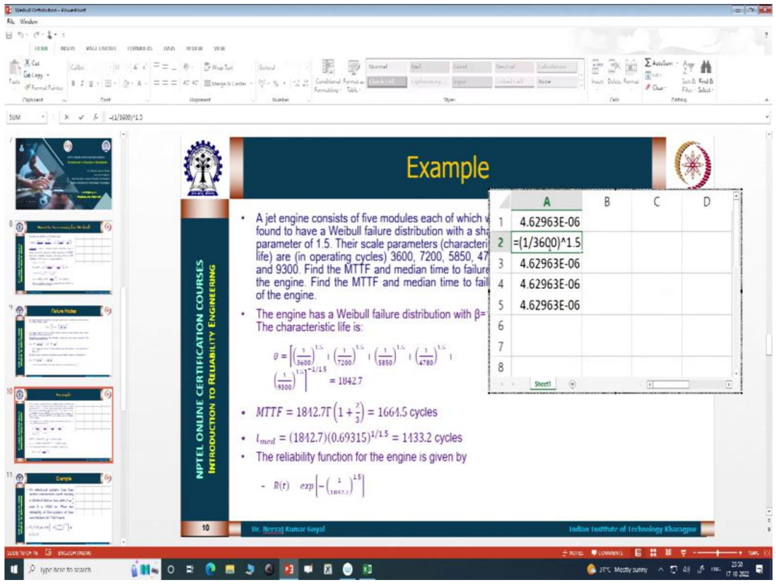

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A B C D
1 4.62963E-06
2 =(1/3600)^1.5
3 4.62963E-06
4 4.62963E-06
5 4.62963E-06
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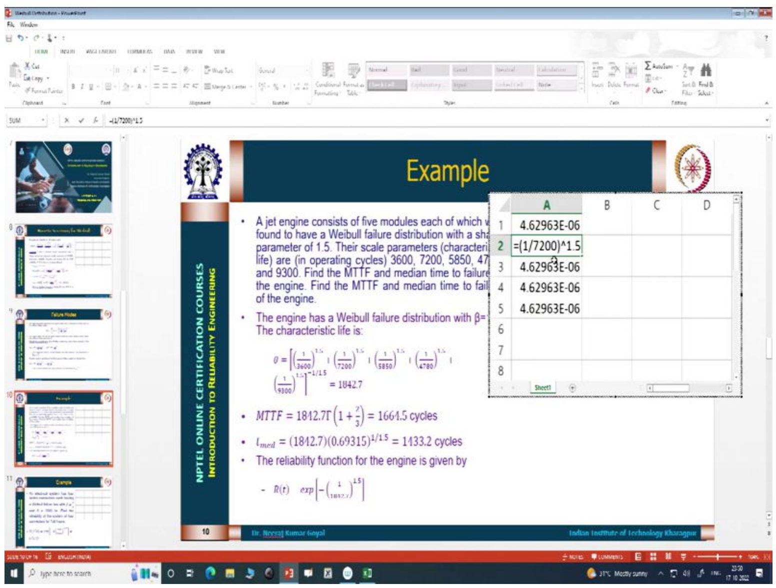

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(1842.7)(0.69315)^1/1.5 = 1433.2 cycles
The reliability function for the engine is given by
A B C D
1 4.62963E-06
2 =(1/7200)^1.5
3 4.62963E-06
4 4.62963E-06
5 4.62963E-06
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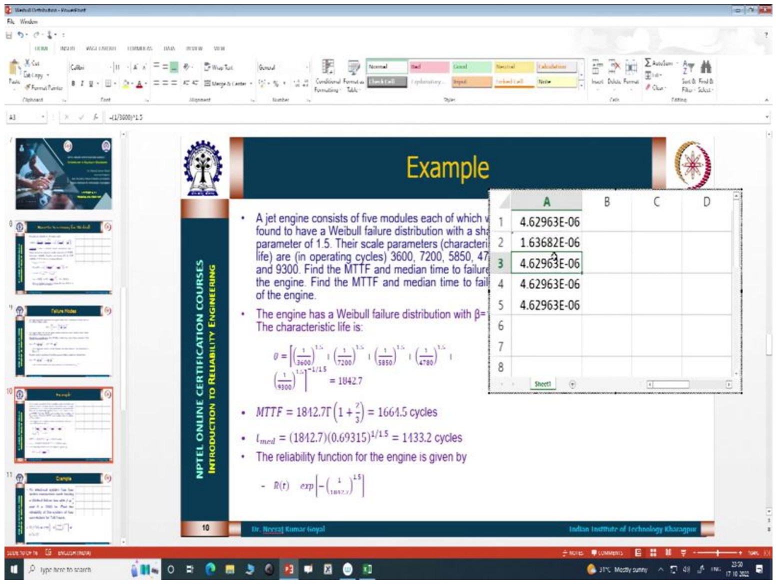
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t_med = (1842.7)(0.69315)^1/1.5 = 1433.2 cycles
The reliability function for the engine is given by
A B C D
1 4.62963E-06
2 1.63682E-06
3 4.62963E-06
4 4.62963E-06
5 4.62963E-06
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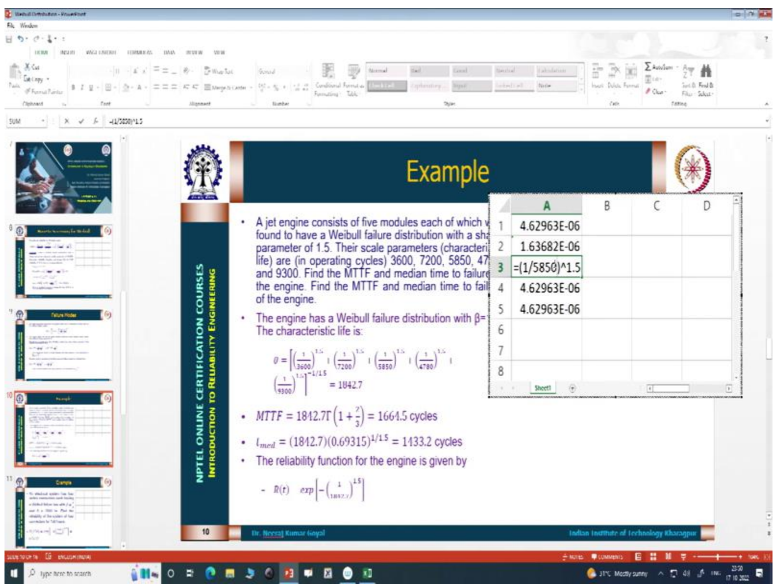
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t_med = (1842.7)(0.69315)^1/1.5 = 1433.2 cycles
The reliability function for the engine is given by
A B C D
1 4.62963E-06
2 1.63682E-06
3 =(1/5850)^1.5
4 4.62963E-06
5 4.62963E-06
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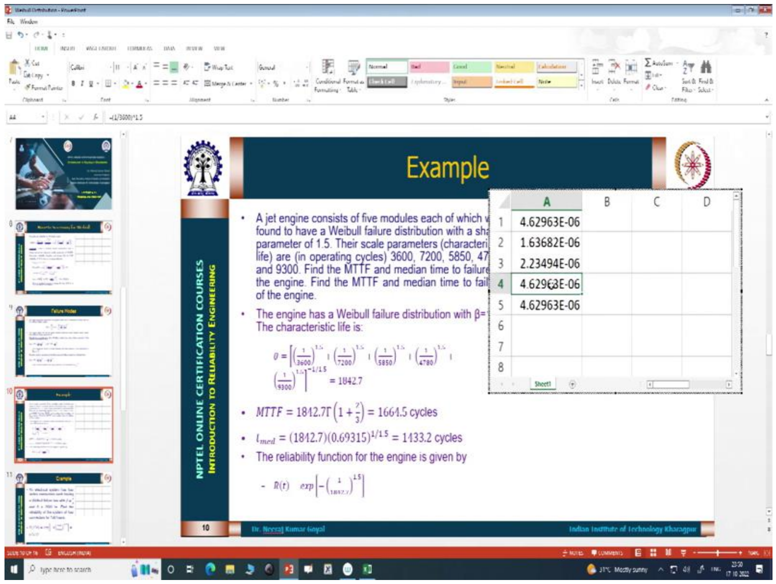
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The characteristic life is:
$\theta = \left[\left(\frac{1}{3600}\right)^{1.5} + \left(\frac{1}{7200}\right)^{1.5} + \left(\frac{1}{5850}\right)^{1.5} + \left(\frac{1}{4780}\right)^{1.5} + \left(\frac{1}{9300}\right)^{1.5}\right]^{-1/1.5} = 1842.7$
$MTTF = 1842.7\Gamma\left(1+\frac{2}{3}\right) = 1664.5$ cycles
$t_{med} = (1842.7)(0.69315)^{1/1.5} = 1433.2$ cycles
The reliability function for the engine is given by
$R(t) \quad exp\left[-\left(\frac{t}{1842.7}\right)^{1.5}\right]$
=(1/3600)^1.5
A B C D
1 4.62963E-06
2 1.63682E-06
3 2.23494E-06
4 4.62963E-06
5 4.62963E-06
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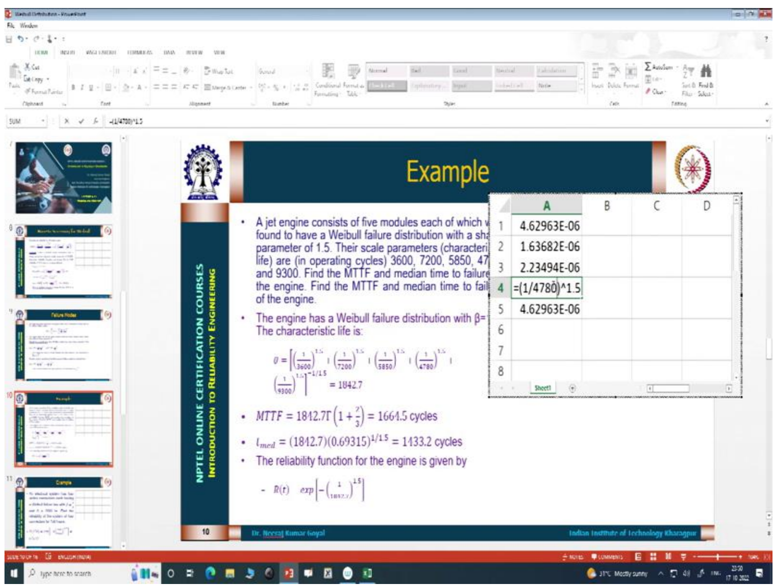
Example
=(1/4780)^1.5
A B C D
1 4.62963E-06
2 1.63682E-06
3 2.23494E-06
4 =(1/4780)^1.5
5 4.62963E-06
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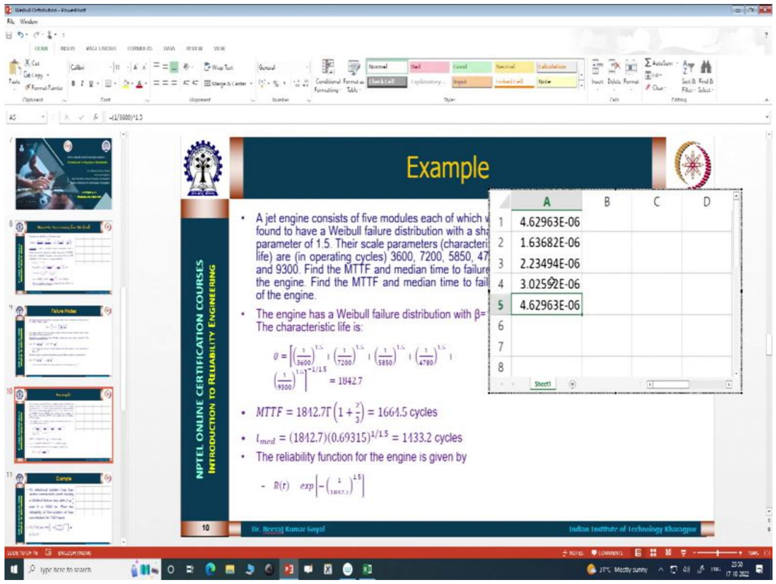

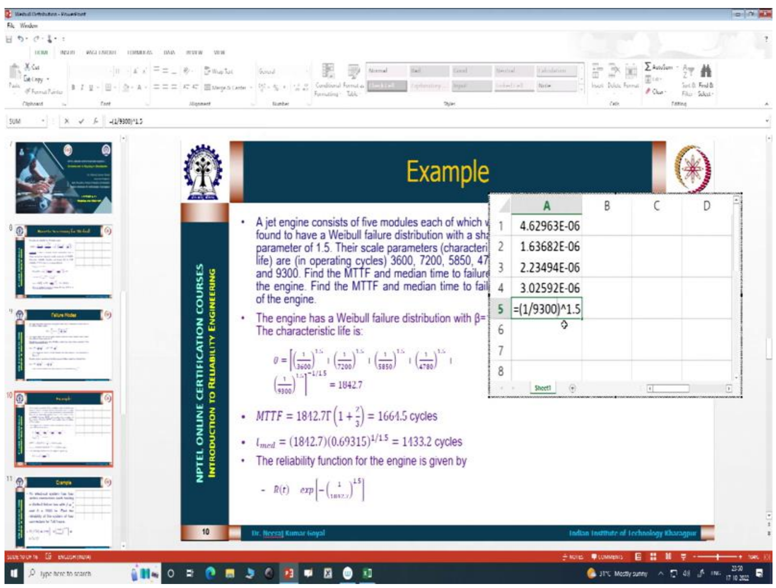

Let us see some example. Let us say jet. So, let us say if a engine is there, jet engine is there which is having 5 modules, these 5 modules are having the Weibull distribution following Weibull distribution and the shape parameter of all the Weibull distribution is 1.5. So, their scale parameters are 3600, 72,000, 5850, 4780 and 9300.

We want to find out the MTTF and median time to failure. So, here if you want we can use this I have just put it here Excel sheet here so, that we are able to see, now we know that how much is the failure rate for each one or how much is this. So, that is equal to 1 divided by first is 3600 whole power 1.5 beta is 1.5. Similarly, we will calculate others like here if you look into and 1 divided by 7200, then this is 1 divided by 5850, this is 4780 and this is 9300. So, what we have got here that is t upon theta sorry 1 upon theta raise to the power beta.

$$\theta = \left[\left(\frac{1}{3600}\right)^{1.5} + \left(\frac{1}{7200}\right)^{1.5} + \left(\frac{1}{5850}\right)^{1.5} + \left(\frac{1}{4780}\right)^{1.5} + \left(\frac{1}{9300}\right)^{1.5}\right]^{-\frac{1}{1.5}} = 1842.7$$

$$\text{MTTF} = 1842.7\Gamma\left(1 + \frac{2}{3}\right) = 1664.5 \text{ cycles}$$

$$t_{\text{med}} = (1842.7)(0.69315)^{\frac{1}{1.5}} = 1433.2 \text{ cycles}$$

$$R(t) = \exp\left[-\left(\frac{t}{1842.7}\right)^{1.5}\right]$$

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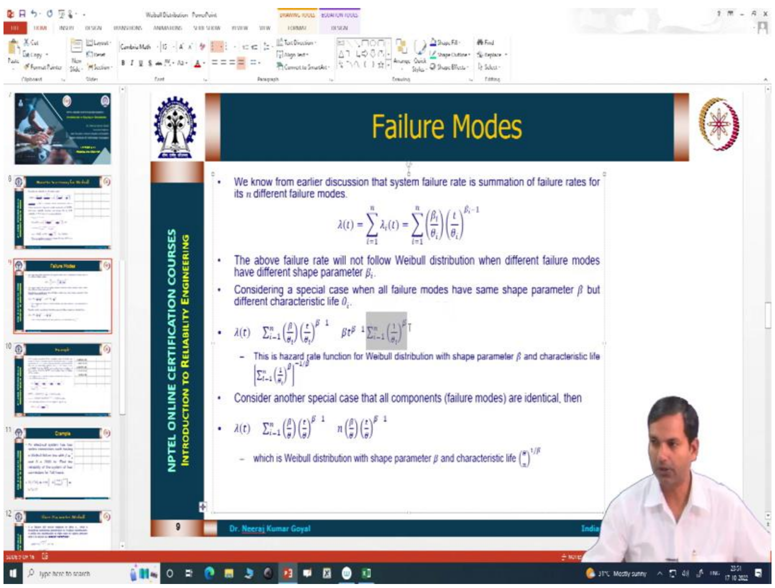

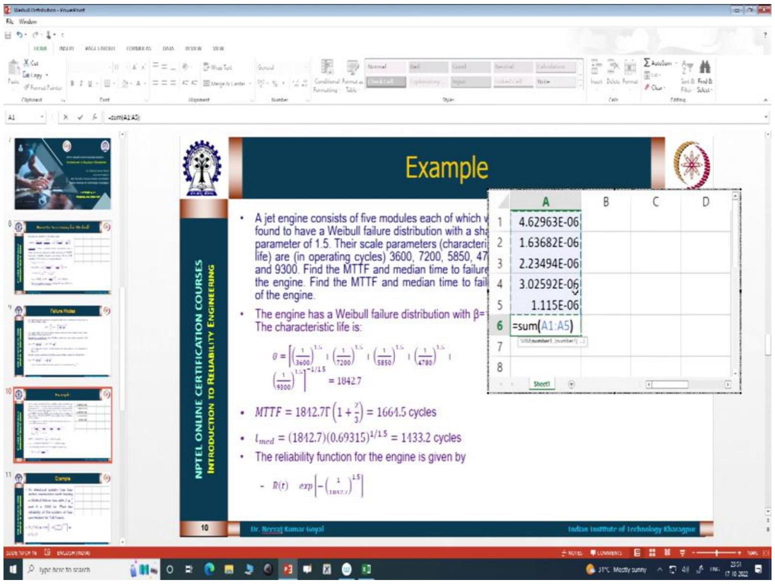

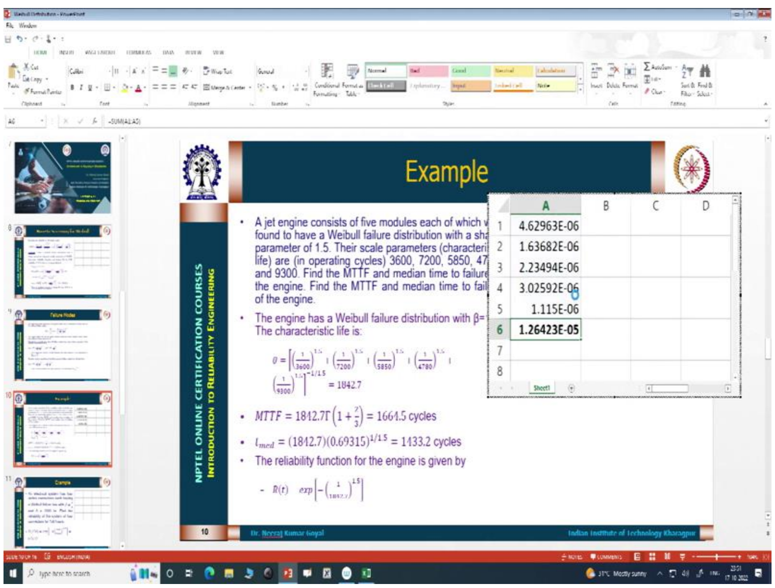

This if you look at it this is from our previous here here that when all are equal then what we have to do sorry when all are not equal then this is what we have calculated summation of 1 upon theta i raise to power beta. So, when we sum it up this is equal to sum of all these.

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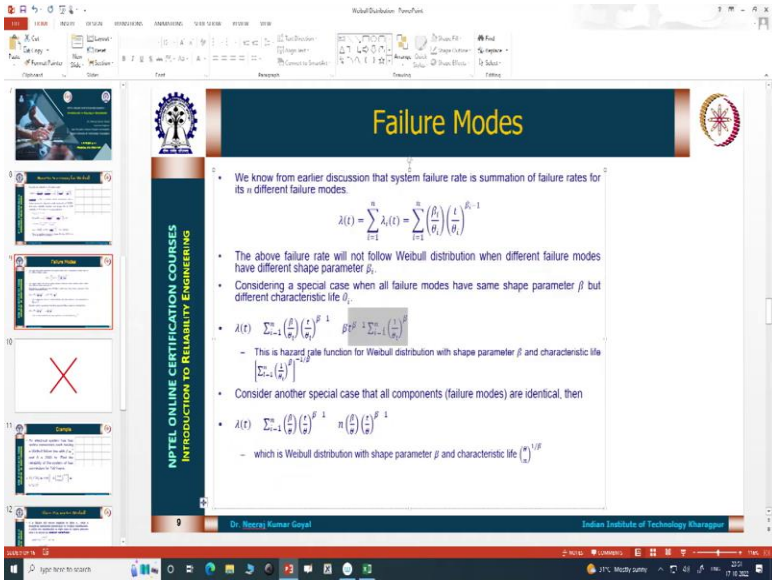
Failure Modes
We know from earlier discussion that system failure rate is summation of failure rates for its n different failure modes.
$\lambda(t) = \sum_{i=1}^{n} \lambda_i(t) = \sum_{i=1}^{n} \left(\frac{\beta_i}{\theta_i}\right)\left(\frac{t}{\theta_i}\right)^{\beta_i - 1}$
The above failure rate will not follow Weibull distribution when different failure modes have different shape parameter β_i.
Considering a special case when all failure modes have same shape parameter β but different characteristic life θ_i.
$\lambda(t) \quad \sum_{i=1}^{n}\left(\frac{\beta}{\theta_i}\right)\left(\frac{t}{\theta_i}\right)^{\beta \; 1} \quad \beta t^{\beta \; 1}\sum_{i=1}^{n}\left(\frac{1}{\theta_i}\right)^{\beta}$
This is hazard rate function for Weibull distribution with shape parameter β and characteristic life $\left[\sum_{i=1}^{n}\left(\frac{1}{\theta_i}\right)^{\beta}\right]^{-1/\beta}$
Consider another special case that all components (failure modes) are identical, then
$\lambda(t) \quad \sum_{i=1}^{n}\left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta \; 1} \quad n\left(\frac{\beta}{\theta}\right)\left(\frac{t}{\theta}\right)^{\beta \; 1}$
which is Weibull distribution with shape parameter β and characteristic life $\left(\frac{\theta}{n}\right)^{1/\beta}$
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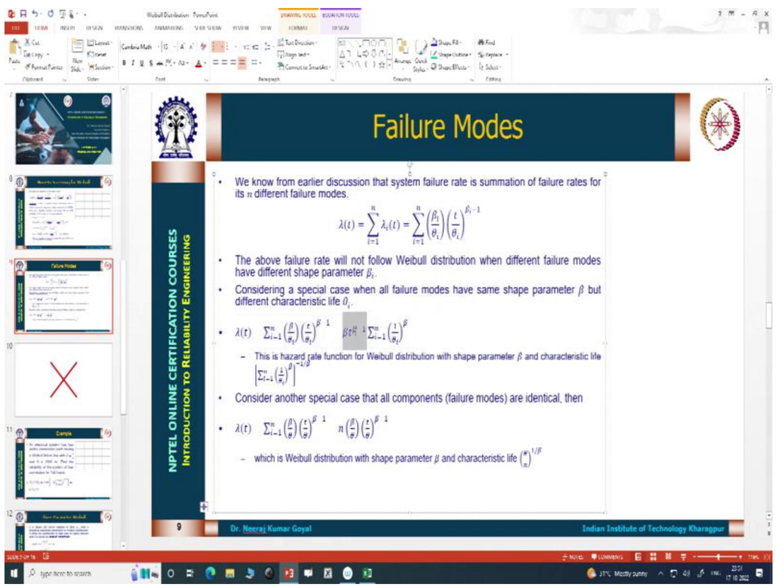
INTRODUCTION TO RELIABILITY ENGINEERING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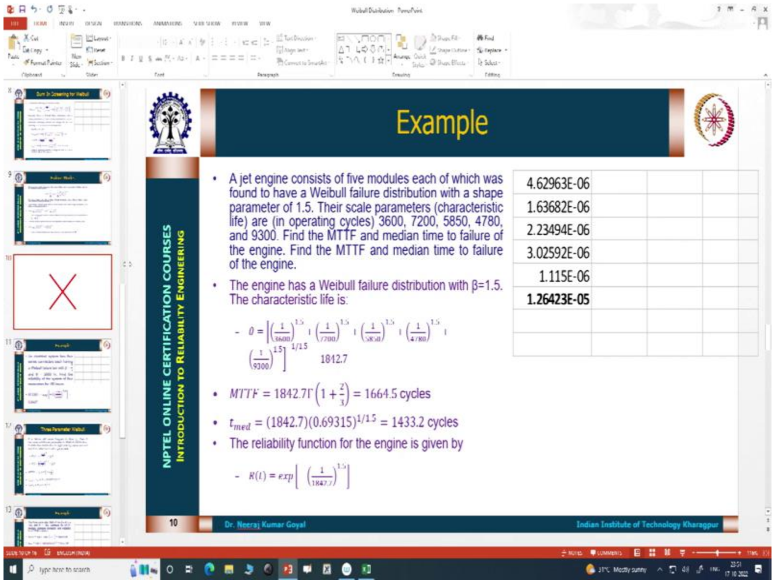

Now, this sum which we have got here I want to know the so, we have got this portion now, we have to multiply with beta into t raise to the power beta minus 1 then we will get the lambda, what we want to calculate here is, what we want to calculate here is theta.

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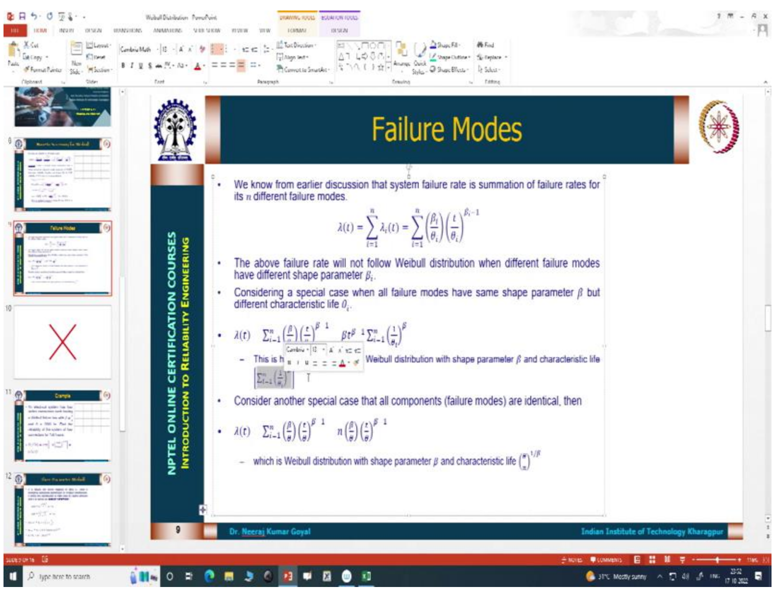

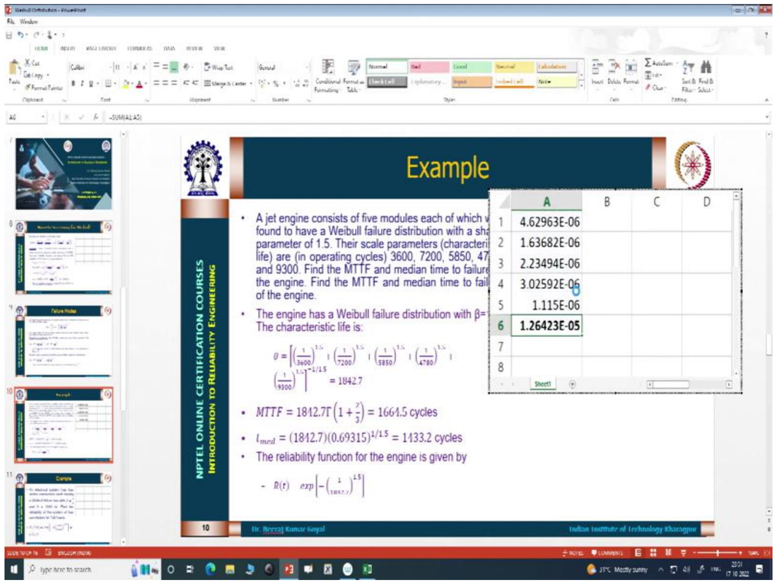
Example
A jet engine consists of five modules each of which
found to have a Weibull failure distribution with a sha
parameter of 1.5. Their scale parameters (characteri
life) are (in operating cycles) 3600, 7200, 5850, 47
and 9300. Find the MTTF and median time to failure
the engine. Find the MTTF and median time to fail
of the engine.
The engine has a Weibull failure distribution with β=
The characteristic life is:
= 1842.7
MTTF = 1842.7Γ(1 + 2/3) = 1664.5 cycles
= (1842.7)(0.69315)^1/1.5 = 1433.2 cycles
The reliability function for the engine is given by
A
B
C
D
4.62963E-06
1.63682E-06
2.23494E-06
3.02592E-06
1.115E-06
1.26423E-05
Sheet1
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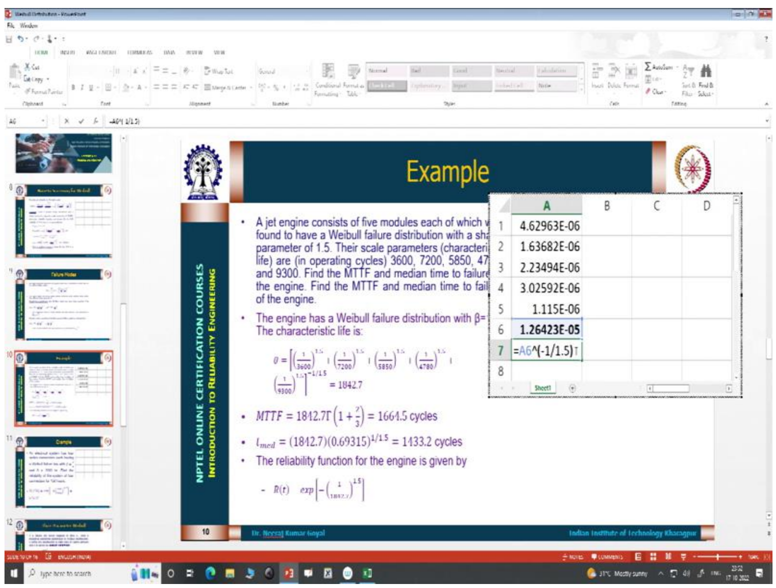
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10
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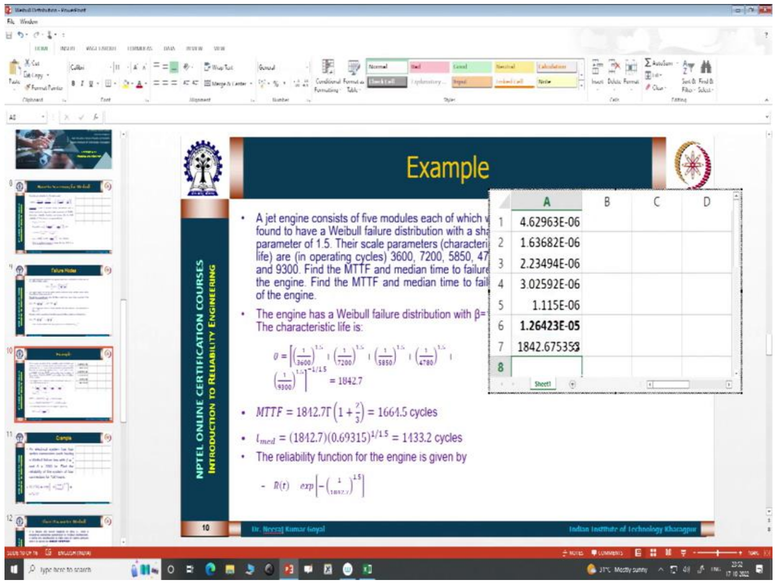

So, theta is summation we have calculated and now this raise to the power minus 1 upon beta. So, this is equal to summation whole to the power minus 1 divided by beta, beta is 1.5. So, my theta or my characteristic life comes out to be 1842.67.

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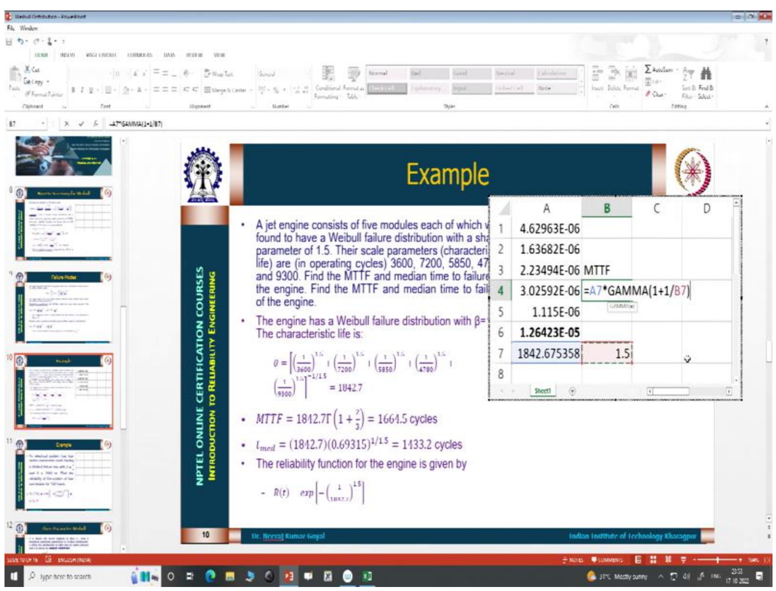

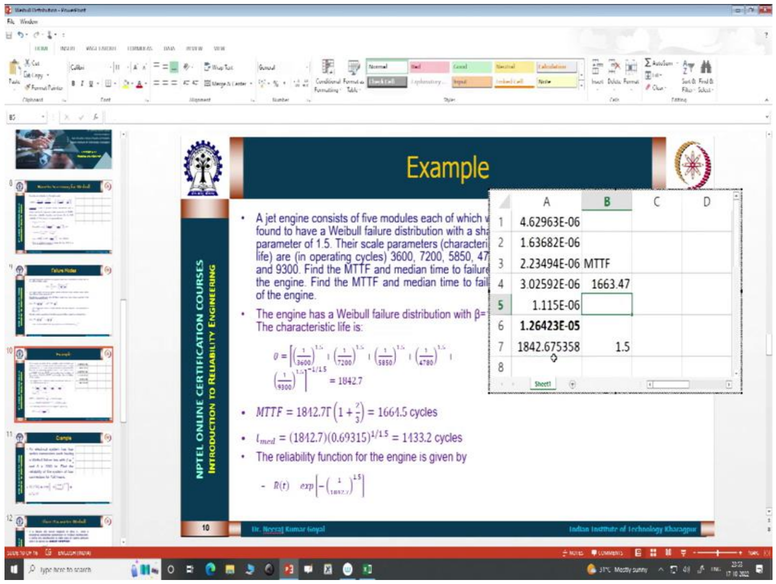

My beta is same as 1.5. Now, I want to do the MTTF. How do I calculate MTTF? For MTTF calculation that is equal to theta. I have got theta. This is my theta, multiply by gamma function of this is new version. So I am here I am directly having the gamma function gamma of 1 plus 1 divided by beta. What is my beta? This 1.5 is my beta. Once I do this, I will get the MTTF. So MTTF is approximately 1663.47.

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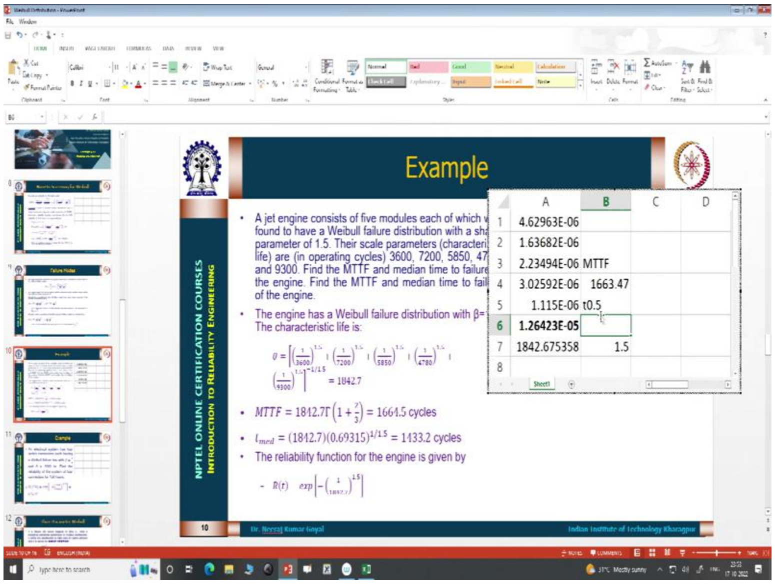

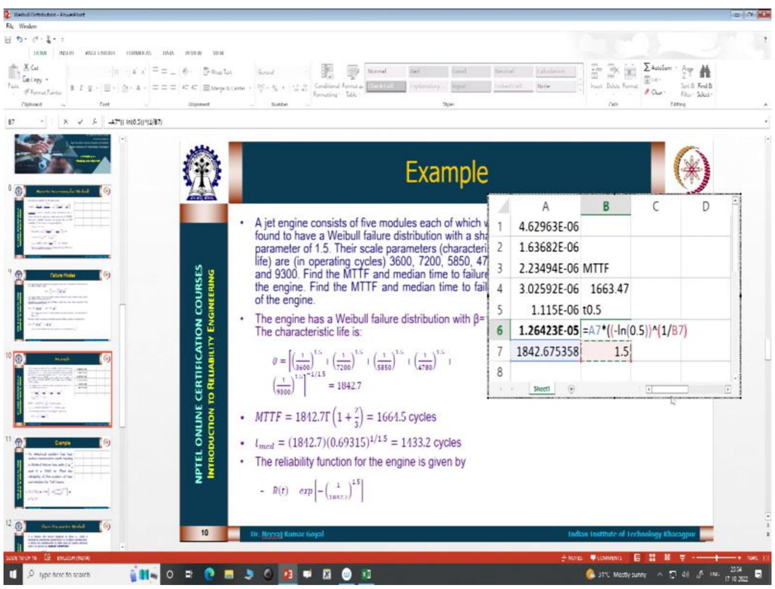

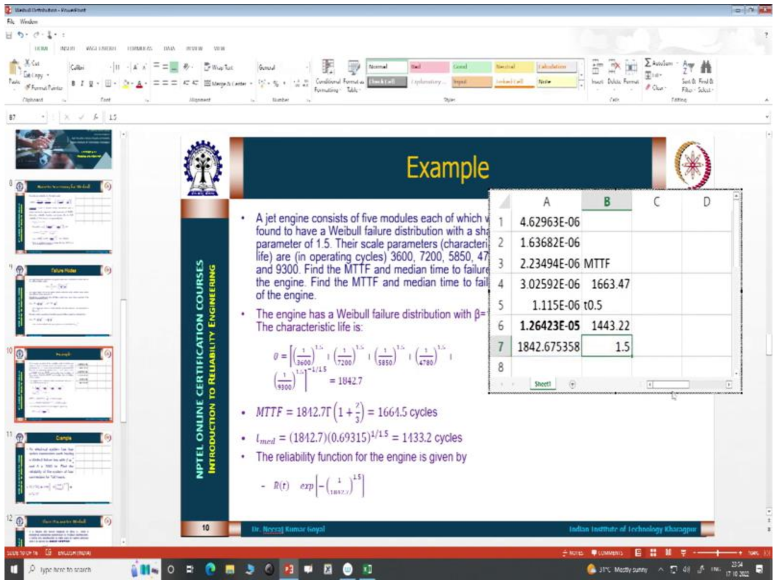

Similarly, I can get the MTTF, what is my MTTF? MTTF is equal to theta. This is my theta sorry MTTF we have already calculated t median. So t median if you want to calculate t median is equal to or we can say t 0.5. So for t 0.5. We are able to know that is theta into ln of minus ln of 0 this is equal to theta my theta is this one multiply by minus log of log of 0.5 whole raise to the power 1 divided by beta beta is this 1.5. And we get 1443 cycles.

And reliability function if we see that is exponential minus this is t upon theta raise to the power beta. If t is 1 then we will get that value. So, we have used Excel for calculation because it is faster and we can do by calculator also.

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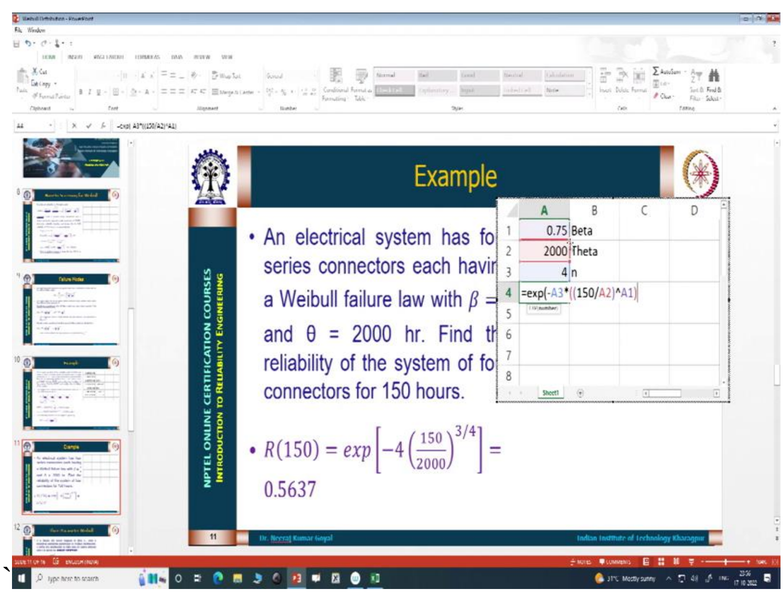

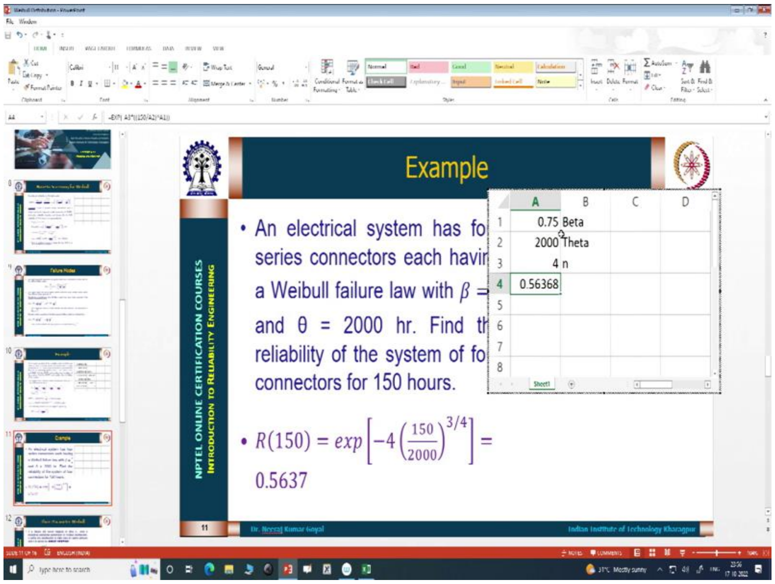

Let us take another example that a electrical system has 4 series connector each having Weibull failure rate beta is equal to 3.4 3 by 4. So, this is equal to 3 by 4, this is my beta. Now, theta is given as 2000. Now, I want to know that if I am having 4 connectors, so, n is 4, what will be my reliability?

As we know reliability is nothing but e to the power minus t upon theta raise to the power beta for 1. So, once we multiply for 4 this will become multiplication by 4. So, that is equal to exponential minus 4 into t, t is 150 hours 150 divided by theta raise to the power beta so that comes out to be 0.5637. So, we are able to as we see here we are able to calculate our reliability.

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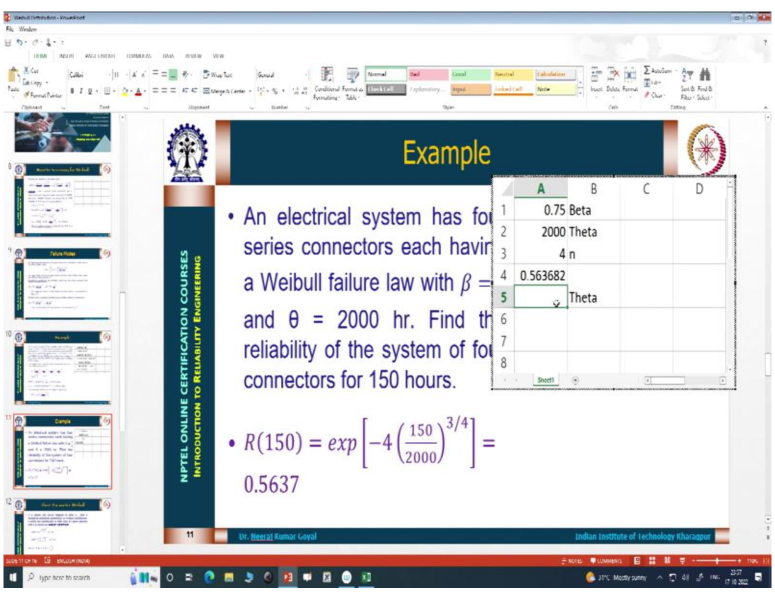

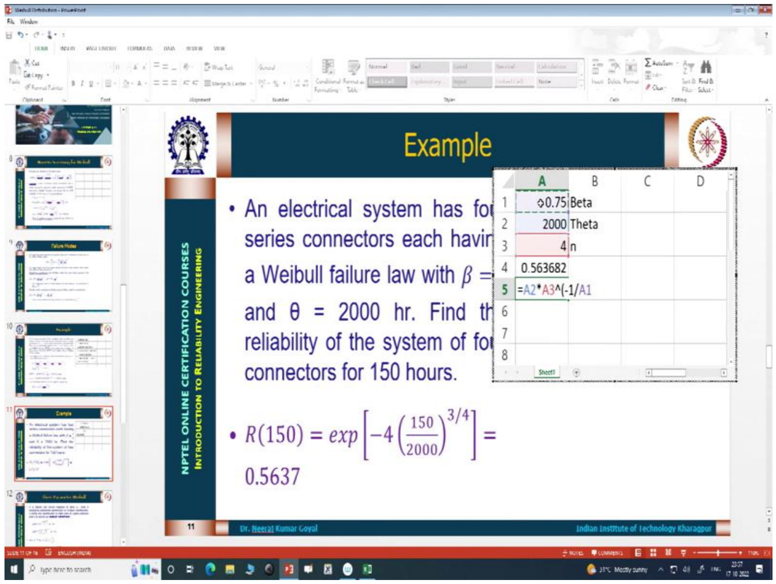

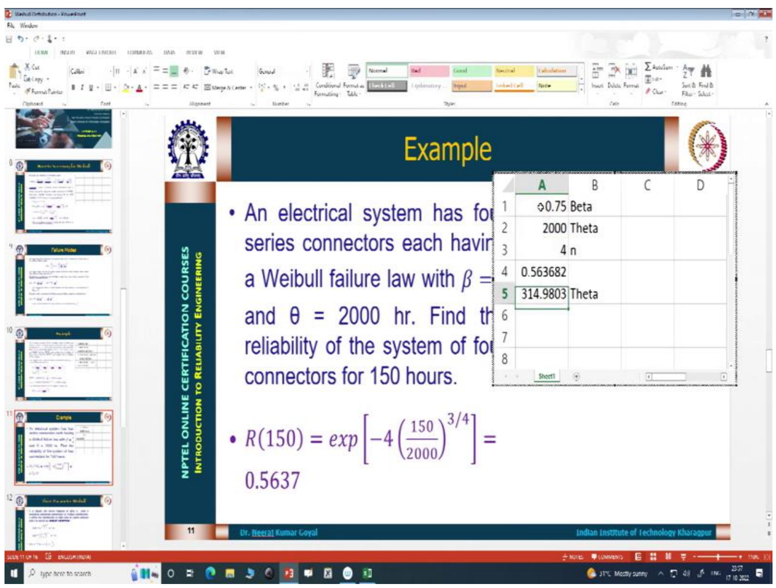

If we want to calculate let us failure rate here or the characteristic life here then we knew that characteristic life here or that beta will remain same because they are all same theta will be equal to theta divided by n to the power 1 upon beta. So, this will be equal to theta or multiply by n raise to the power minus 1 divided by beta. So, this becomes our new theta 314.98 for the system.

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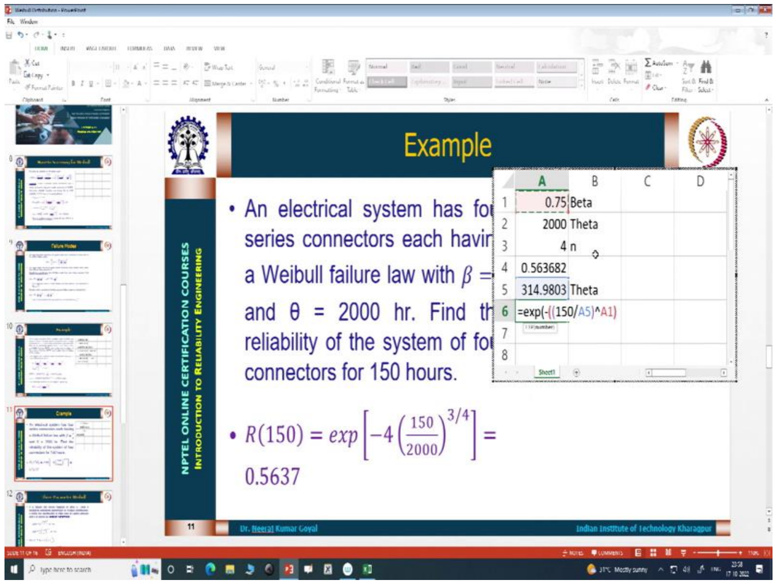

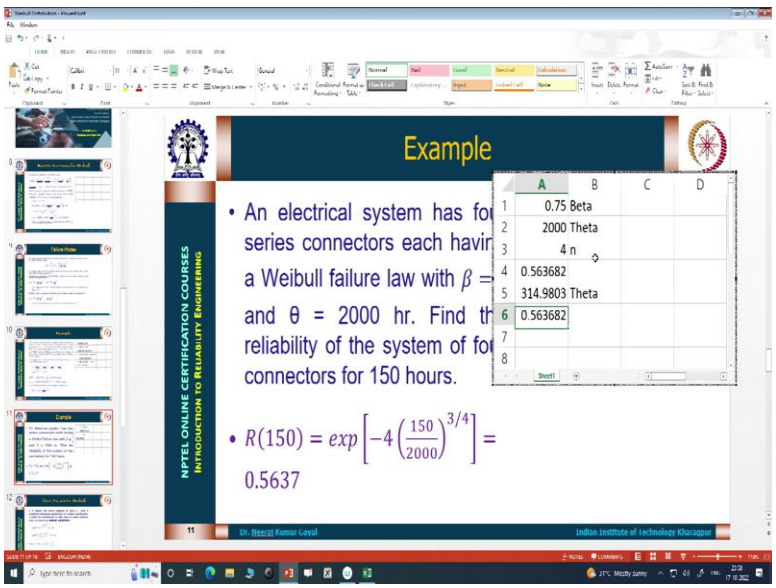

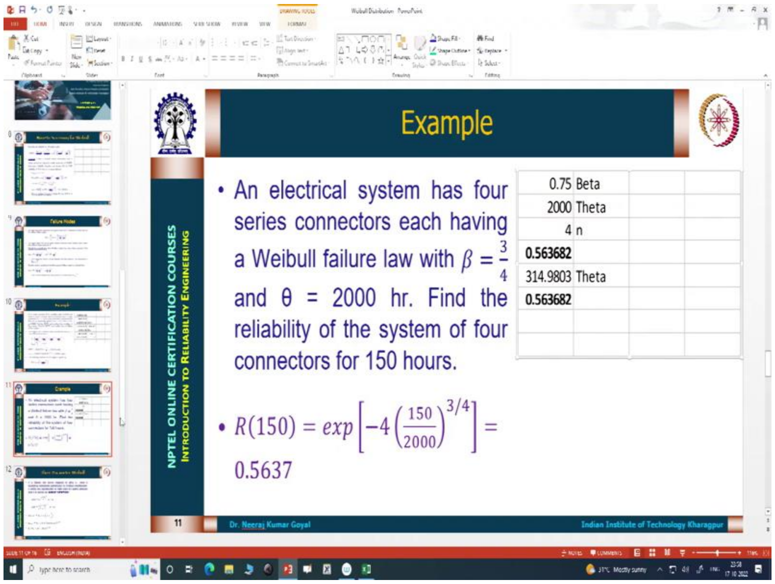

So, if we want we could have directly calculated here that is equal to reliability is exponential minus of 150 divided by theta. Now theta for system is this whole beta if you see we get the same reliability either we use this or we use this both will give you the same result. So, effectively we are able to calculate new theta and beta for the system.

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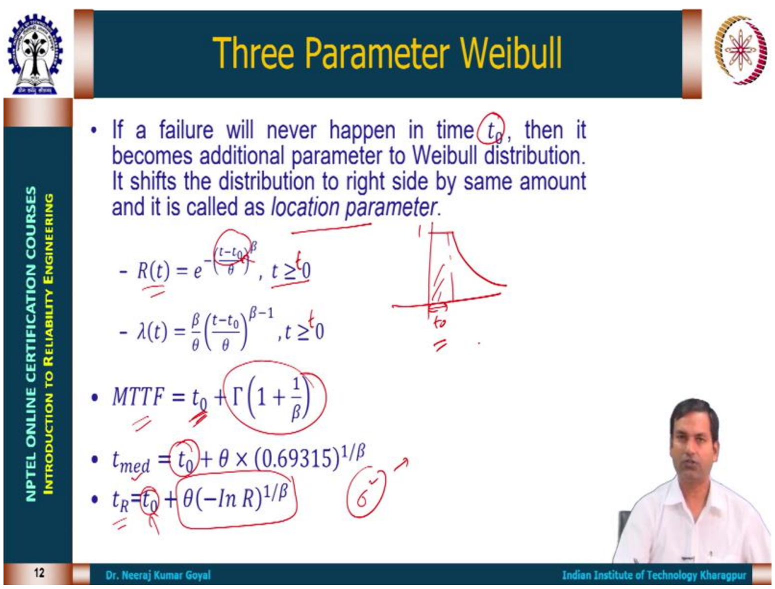

There is same what we have discussed can also be extended for the 3 parameter Weibull. As we discussed for the 2 parameter exponential distribution, one additional parameter t0 is added here, what is t0? t0 is the time for which we expect no failure here. So, in this case, if we replace t with t minus t0, the same thing happened whatever happened with the 2 parameters just we replace t with t minus t0 and this becomes the 3 parameter Weibull distribution.

The, here the third parameter is called location parameter because what will happen if we change the t0 value here there will be no change in the shape or there will no change in the scale the only change will be the distribution will be shifted. If I increase the value of t0 it will be shifting right if I decrease the value of t0 it will shift the towards left.

So, this is a location parameter because it is changing the PDF location only it is not changing the making any other changes PDF. But this is also the guaranteed life or we can say this is the life within which this failure will not happen. This should be t0 these values are defined for t greater than t0.

Now MTTF as we have calculated earlier that for exponential also what happened, whatever my reliability was here, this is 1 here. So, area under the curve t0 is added. This is 1 and this is t0 so 1 into t0 t0. So, whatever MTTF you got with the 2 parameter is this, if we simply sum up t0 with this we get the MTTF for the 3 parameter distribution.

Similarly, for t median also we have to add only t0 because distribution is simply sifted by the t0 towards the right and similarly for tR also we need to only add t0 with whatever formula we had earlier for the design life. The variance does not change variance remains same, because variability is not changing by the location.

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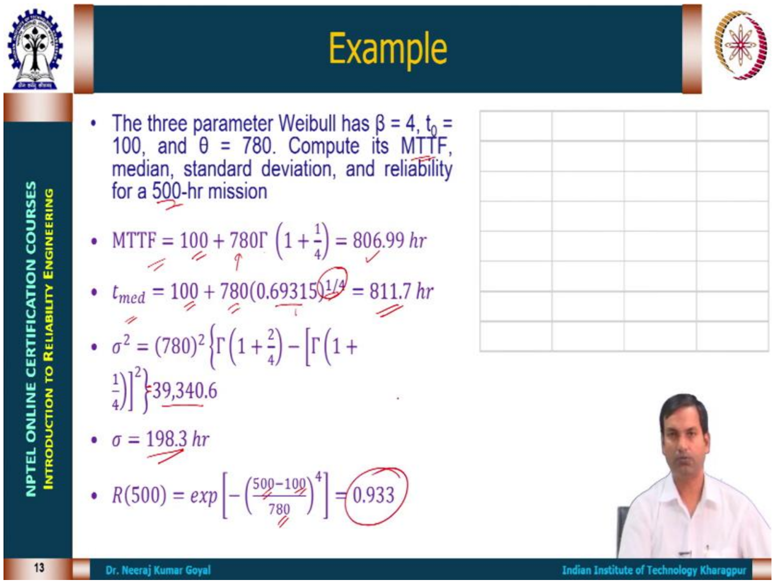

For an example, if we have the beta equal to 4 t0 is 100 and theta is 780. What is MTTF? MTTF is 100 plus that is 780 gamma of 1 plus 1 upon beta we can get a0 is 6.99 hour, t median is t0 plus theta into minus ln of 0.5 which is 0.6931 whole raise to the power beta. So, we get 100 into 780 into this to the power beta and this gives me sorry this to the power 1 upon beta.

So, this 811.7 and sigma square is theta square gamma of 1 plus 2 upon beta minus square of gamma of 1 plus 1 upon beta multiply with sorry this actually is equal to equal sign is missing. So, once we calculate this, we will get this and square root of this will produce 198. If I want to calculate reliability for 500 hours, where reliability is e to the power minus t minus t0 is 100 divided by 780 whole raise to the power 4. It comes out to be 0.933.

$$\text{MTTF} = 100 + 780\Gamma\left(1 + \frac{1}{4}\right) = 806.99\text{hr}$$

$$t_{\text{med}} = 100 + 780(0.69315)^{1/4} = 811.7\text{hr}$$

$$\sigma^2 = (780)^2\left\{\Gamma\left(1 + \frac{2}{4}\right) - [\Gamma(1 + \frac{1}{4})]^2\right\} 39{,}340.6$$

$$\sigma = 198.3\text{hr}$$

$$R(500) = \exp\left[-\left(\frac{500 - 100}{780}\right)^4\right] = 0.933$$

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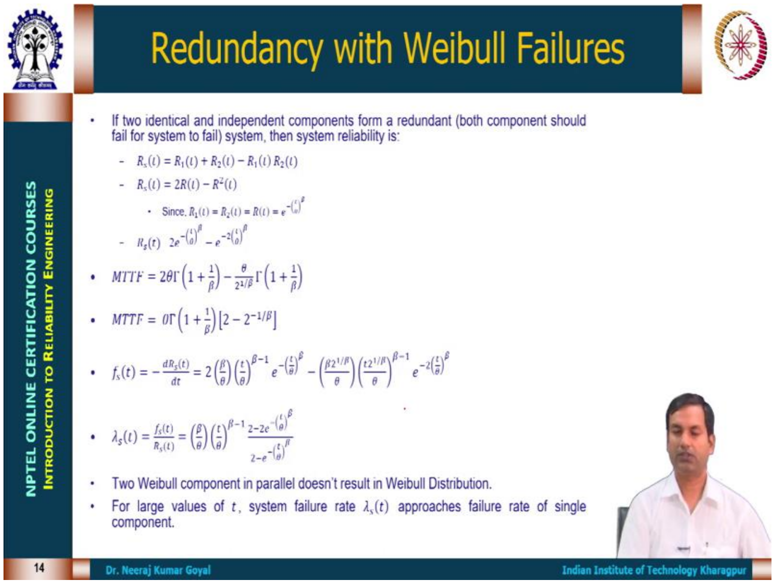

So, we will stop our discussion here and we will continue a little more discussion left for the Weibull distribution and then we will start discussing other distributions like normal and log normal distribution. Thank you.